Algorithmic Game Theory - handout5

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Homework. (No need to hand in.)

1) Consider the following three player game. Player A has strategies a1 and a2, player B has strategies b1 and b2, and player C has strategies c1 and c2. The payoffs are described below. The name of a player appearing in a strategy profile means that the player gets a payoff of 1. Otherwise the payoff is 0. For example, on profile (a1,b2,c2) players A and B each gets a payoff of 1 and player C gets a payoff of 0.

```
           b1         b2                          b1         b2
     |----------|----------|                |----------|----------|
a1   |          |    B     |           a1   |    C     |  A; B    |
     |----------|----------|                |----------|----------|
a2   |    A     |  A; C    |           a2   |  B; C    |          |
     |----------|----------|                |----------|----------|

                c1                                     c2
```

Equivalently, the payoff for each player can be described as follows: if a player plays his first strategy he gets a payoff of 1 iff the two other players play their second strategy. If a player plays his second strategy, he gets a payoff of 1 iff the player preceding him (in the cyclic order A-B-C-A) plays his first strategy.

Find *all* Nash equilibria of this game (and prove that no other Nash equilibrium exists).

2) Recall that problems in PPAD are problems whose input includes an implicit description of a directed graph with at most exponentially many nodes. There is a polynomial time algorithm that given the name of a node figures out from the implicit description the edges incident with the node. Every node has at most one incoming edge and at most one outgoing edge. One is given a source node (has no incoming edge), and the goal is to find any sink node (has no outgoing edge). The *matching-sink* problem is more specific and requires one to output the sink node that lies on the end of the path of the given source node. Prove that *matching-sink* is NP-hard. Remark: *matching-sink* is in fact PSPACE-complete.

Hints.

1. Find first all pure Nash equilibria. Then for mixed Nash, the equilibrium condition for a player playing a mixed strategy gives an algebraic equation on the strategies of the other players. Inspecting this equation and a case analysis leads to two cases: either both other players play pure strategies, or both other players play mixed strategies. The latter case leads to additional algebraic equations. The system of algebraic equations has a symmetric irrational solution, and one should argue (e.g., by making a perturbation to this solution) that it does not have other solutions. (Thanks to Dima Kogan for pointing out an error in a earlier version of this hint.)

2. Given an NP-hard problem, the process of finding a solution by exhaustive search can be represented using the terminology of the question.