Algorithmic Game Theory - handout 8-10

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Next week there in no class - faculty trip.

We presented the notions of *price of anarchy* and *price of stability*. For nonatomic selfish routing, we presented the Pigou network and the Braess paradox. We proved that for nonnegative affine functions, the price of anarchy of selfish routing (compared to the optimum min cost flow) is 4/3.

We then considered mechanism design, and specifically the VCG mechanism which involves money. We briefly discussed Generalized Second Price auctions (which are in commercial use), and combinatorial auctions.

Homework.

1. For integer $d \geq 1$, consider nonatomic selfish routing with edge cost functions that are non-negative polynomials of degree at most d (of the form $\sum_{i=0}^{d} a_i x^i$ with all $a_i \geq 0$).

   (a) Show that the price of anarchy is at most $d+1$.

   (b) Show that for the Pigou network with cost 1 on one edge and x^d on the other edge, as d grows, the price of anarchy grows (as a function of d) at a rate of $\Omega(d/\log d)$.

2. Let $G(V,E)$ be a connected graph with two distinguished vertices s and t, in which every edge e is owned by a different agent A_e. Initially, all edges are blocked. However, each agent A_e can unblock his edge e. Unblocking edge e has a cost c_e to agent A_e, and this cost is private information (known only to the respective A_e). One needs to design a direct revelation mechanism that leads to unblocking the edges along a least cost path between s and t, where the cost of a path is the sum of c_e values along its edges. The mechanism should work as follows. First, every agent A_e reports a cost c'_e, supposedly representing his cost for unblocking his edge. Then the mechanism computes the minimum cost s-t path with respect to the reported costs c'_e (this can be done in polynomial time), and payments p_e to be paid to every agent. The utility for an agent A_e is quasi-linear: $p_e - c_e$ if edge e is on the computed path, and p_e otherwise. The mechanism needs to be incentive compatible IC (it is a dominant strategy to report a value c'_e equal to the true c_e), and individually rational IR (no agent derives negative utility from the mechanism). Note: there is no requirement that the payments made by the mechanism are small.

   (a) Can such a mechanism be designed if there is a cut edge e for s and t (meaning that after removing e the vertices s and t lie in different connected components of the resulting graph)? Explain.

   (b) In the remaining questions we restrict our attention to graphs in which s and t are 2-edge-connected in G (meaning that if any single edge is removed from G, there still remains a path between s and t in G). For this case, design a mechanism satisfying the requirements above.

(c) Suppose that all true edge costs are either 0 or 1. How much might your mechanism need to pay (expressed as a function of $|V|$ and $|E|$)? Give both an upper bound and an example of a graph showing that your upper bound is tight up to a constant multiplicative factor. (One would expect the answer in this section to be $\Theta(|V|^2)$.)

(d) Design an IC mechanism (that is also IR) for the special case of 0/1 costs, in which the total payment of the mechanism is limited to $O(|V|)$.