Algorithmic Game Theory - handout1

Uriel Feige

9 April 2024

Course webpage: https://www.wisdom.weizmann.ac.il/∼feige/agt2024.html

Homework assignments are an integral part of the course and will be a significant part of the grade. Please hand in the written assignments two weeks after they are given.

Teaching assistant: **Yotam Gafni**.

Homework. (Please keep the answers short and easy to read.)

1. The family of *and-or* games is (equivalent to) a family of two-player 0-sum games given in extensive form (as a game tree). There are two players, *min* and *max*. The game tree is a full binary tree of depth n and $N = 2^n$ leaves. Every leaf has a value, which is the payoff given to *max*, which can be either $+1$ (win) or -1 (loose). The payoff to *min* is the negative of the payoff to *max* (when *max* looses *min* wins). The game starts at the root of the tree. Number the layers of the tree from 0 at the root to n at the leaves. At even numbered layers, *min* chooses a move (an edge leading from the current node to the next layer), and at odd layers *max* chooses a move. After n moves a leaf is reached, each player gets his respective payoff, and the game ends.

   (a) How would an *and-or* game be represented in normal form? In particular, how many rows and how many columns will the game matrix have?

   (b) Prove that in every *and-or* game one of the players has a strategy that forces a win for that player regardless of the strategy of the other player.

   (c) As a function of n, what is the smallest number of leaves that can have value $+1$ and still *max* will have a winning strategy? Prove your answer.

   (d) Prove that in every *and-or* game, at least one of the players has a dominant strategy.

   (e) Show an example of an *and-or* game in which *min* does not have a dominant strategy.

   (f) Show an algorithm of time complexity at most polynomial in N for computing a subgame perfect Nash equilibrium for an *and-or* game.

2. Recall the second price auction. The highest bidder wins, and pays the highest bid among those who did not win. The utility (payoff) of an agent is 0 if he does not win, and his value for the item minus his payment if he wins. Suppose that there are two bidders, and the auction is implemented with "open bids" instead of sealed bids. That is, bidder 1 bids first, and bidder 2 bids second, after observing the bid of bidder 1. In case of tie in the bids, agent 1 is declared as the winner (and pays his bid).

   Let a and b denote the value of the item to bidders 1 and 2 respectively, and suppose that both bidders know both values.

   (a) Does bidder 1 have a dominant strategy? Does bidder 2 have a dominant strategy?

   (b) Is there a subgame perfect equilibrium for the game in which $a < b$ and bidder 1 wins the item? Is there one in which $a > b$ and bidder 2 wins the item? Prove your answer.

3. Prove that in the game with the following payoff matrix (in each cell, the first/second entry is the payoff for the row/column player, respectively)

2; 1	1; 2	0; 0
0; 0	2; 1	1; 2
1; 2	0; 0	2; 1

   there is no mixed Nash equilibrium in which the mixed strategy of the row player is supported on only two strategies.

References

[1] Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay V. Vazirani (Editors), Algorithmic Game Theory, Cambridge University Press, 2007. Okay to browse free online copies for non-commercial use.

[2] Tim Roughgarden, Twenty Lectures on Algorithmic Game Theory, Cambridge University Press, 2016.