

Graph coloring – Handout6

February 8, 2023

Homework: Hand in by Feb22 (to be checked and returned by Feb 26).

Present a polynomial time algorithm for coloring n -vertex 4-colorable graphs of maximum degree d (where d can be a function of n) by $\tilde{O}(\sqrt{d})$ colors, where the \tilde{O} notation hides factors that may be polylogarithmic in n . Then show for some $\delta < \frac{1}{2}$ a polynomial time algorithm that colors 4-colorable graphs while using at most $\tilde{O}(n^\delta)$ colors. In more detail:

1. Formulate an SDP relaxation for 4-coloring, and explain why the SDP is feasible when the graph is 4-colorable.
2. Present a randomized algorithm that rounds the solution that is approximately feasible for the SDP so as to get a coloring of the graph with $\tilde{O}(\sqrt{d})$ colors. Prove that it runs in expected polynomial time.
3. Show how to obtain a coloring with $\tilde{O}(n^\delta)$ colors for your choice of $\delta < \frac{1}{2}$. This breaks into two cases. In one $d \leq n^{2\delta}$, and then we are already done. In the other, there are vertices of degree larger than $n^{2\delta}$, and there you need to explain how to modify the above algorithm.