Summary and practice questions

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1 Summary

We recall topics studied in the course.

1. Dynamic programming. How it can be applied to problems that have a linear structure or a tree structure.

2. Tree decomposition of graphs. Use of dynamic programming for graphs of bounded treewidth.

3. Graph minors. Existence of polynomial time algorithms for minor closed properties.

4. The greedy algorithm. Matroids.

5. Intersection of matroids. Alternating paths/trees algorithms for bipartite matching.

6. Maximum matching in arbitrary graphs. Gallai-Edmonds decomposition.

7. Maximum weight matching in bipartite graphs. A primal-dual view. Egervary's theorem.

8. Fast matrix multiplication.

9. Computing the permanent via inclusion-exclusion. Detecting the existence of a matching using a randomized determinant computation. The tree-matrix theorem.

10. Basics of spectral graph theory. Hoffman's bound on maximum independent set. Refutation of random 4CNF formulas.

11. The Lovasz local lemma. Moser's algorithmic version for satisfying sparse k-CNF formulas.

12. Today: Randomized streaming algorithms for words over free groups (Lipton-Zalcstein). String pattern matching (Karp-Rabin).

2 Some practice questions

Exam will be with no books. At least one question will be from the homework assignments. There will be some choice.

No guarantee that questions in the exam will be similar to the questions below (and no guarantee that they will be different).

1. Show a polynomial time algorithm for finding a maximum independent set in a series-parallel graph.
2. Given a graph with n vertices and an edge (i, j), show a polynomial time algorithm that produces a matrix of order $n-2$ whose determinant equals the number of spanning trees containing the edge (i, j).
3. Let d be large, $\epsilon > 0$, and $1/2 < \alpha < 1$. (As an example, think of $d = 10^6$, $\epsilon = 10^{-3}$ and $\alpha = 0.7$.) Let G be a graph in which the degree of every vertex is between $(1-\epsilon)d$ and $(1+\epsilon)d$. Assume that G has a cut with $\alpha dn/2$ edges (a partition of its vertices into two disjoint sets such that the number of edges with endpoints in different sets is $\alpha dn/2$). What does this imply (as a function of ϵ, α and d) on the most negative eigenvalue of G's adjacency matrix? Discuss possible implications of this to the design of a heuristic for refuting the existence of large cuts in random graphs.
4. Show a polynomial time algorithm (and prove its correctness) that given an n by m matrix with integer entries, $n > m$ and rank m, finds an m by m submatrix of full rank with the largest number of 0 entries.

For some hints, see next page.

2.1 Hints

1. Review material on treewidth.
2. Show first that the number of spanning trees contain edge (i, j) is the difference of the determinants of two matrices of order $n - 1$.
3. To bound the most negative eigenvalue, use Rayleigh quotients with ± 1 vectors. Recall what was said in class regarding eigenvalues of random graphs.
4. Review material on Matroids.