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EDITORIAL

Special issue on combinatorial game theory

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We are proud to present this first issue of the *International Journal of Game Theory* that is dedicated to papers in the field of Combinatorial Game Theory (CGT). Combinatorial games are traditionally two-player perfect-information games with compact rules (typically only win or loss as outcomes, and no chance moves), and a rich mathematical structure. As reflected in this special issue, activity in CGT accommodates a sprinkling of deviation from this tradition, such as infinite and cyclic games, games with a winning score, or one-player and multi-player games, which freshens up the theory.

The call for papers to submit CGT papers to IJGT given below gives an overview of what CGT is about. This call (slightly amended here) was posted in 2011 on the website of the Game Theory Society, by the Editor of IJGT, Shmuel Zamir, and co-

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Editor Bernhard von Stengel. Together with Aviezri Fraenkel as responsible Associate Editor, we invited the CGT community to send their substantial papers to IJGT—with success, as we think this special issue demonstrates, in addition to the many CGT papers already published previously. Aviezri Fraenkel as Associate Editor has since been joined by Thane Plambeck and Urban Larsson. In Lisbon, under the auspices of Associação Ludus, Carlos Santos has been organizing the "Combinatorial Game Theory Colloquium" taking place every two years starting in January 2015, 2017, and next in 2019. Some of the papers in this special issue have been presented at the first colloquium in 2015. It is the editorial policy of IJGT that all submissions are thoroughly reviewed and screened for quality, so this is not a proceedings volume, and the special issue has the same high publication standard of IJGT as any other. The majority of the papers in this issue have in fact been submitted independently of the colloquium.

We believe the basics of CGT are part of *what every game theorist should know*. To this end, Urban Larsson and Bernhard von Stengel have hassled the authors repeatedly to address their article not only to the CGT specialist, but to make it a bit more self-contained. We hope this will be appreciated by typical IJGT readers who are more often "classical" game theorists, and that they will find interest to use research ideas from CGT in their work—and to contribute to CGT in return, enticing combinatorial game theorists to incorporate classical game theory. CGT shows, sometimes with some algebra, more often with entirely new mathematical methods, which player is ahead in a game, and by how much. For theoretical computer scientists, CGT is the main proving ground for higher complexity studies, such as PSPACE- and EXP-completeness, due to the natural presence in games of universal rather than only existential quantifiers in optimization problems.

We hope that the fun and sophistication of studying combinatorial games comes across in these articles.

Call for papers for submission to the International Journal of Game Theory in the area of Combinatorial Games

(Posted on the website of the Game Theory Society by Shmuel Zamir, Editor, and Bernhard von Stengel, Co-Editor of IJGT, on 1 July 2011.)

The International Journal of Game Theory (IJGT) encourages submissions of significant papers in Combinatorial Game Theory, and has invited Aviezri Fraenkel to its Editorial Board to deal with these submissions.

IJGT, founded in 1971, has a long tradition of publishing papers in game theory with significant mathematical content. Combinatorial Game Theory has developed into a field with advanced mathematical and computational complexity techniques and a number of challenging open questions, where new results will nicely fit with and complement the scope of IJGT.

Combinatorial games are typically two-player games with perfect information and "win", "lose", and "draw" or "tie" as possible outcomes, and an underlying mathematical structure. This is in contrast to games involving chance and lack of information such as Poker, which are central to "classical" game theory.

A basic combinatorial game is Nim, given by a number of heaps of chips where players alternately remove some chips from one of those heaps, and the last player to move wins. Over a century ago, it was shown how to play Nim optimally using the binary representation of the heap sizes. This method can be extended to "impartial" games where the available moves from any position do not depend on the player to move. An important algebraic structure is the "sum" of games where a player can move in one of several independent parts of the game. Such decompositions are important, for example, to improve the complexity of algorithms for playing endgames of the board game Go where humans still highly outperform computers.[1]

Recent progress in Combinatorial Game Theory concern difficult questions on partizan (not impartial) games such as chess, misère play (where the last player to move loses) and interactions of game tokens, both of which conflict with the "sum" of games, and computational hardness questions.

The classic book on combinatorial games is "Winning Ways" by Elwyn R. Berlekamp,[2] John H. Conway, and Richard K. Guy from 1982, republished in 2009. More recently (2007) the reader-friendly enticing textbook "Lessons in Play" was produced by Michael Albert, Richard Nowakowski, and David Wolfe; and now (2013) also the authoritative graduate text "Combinatorial Game Theory" by Aaron Siegel. As shown by Conway in another classic book "On Numbers and Games" (ONAG, 1976), all two-player games can be constructed by a simple Dedekind-reminiscent cut, with a rich mathematical theory. The real numbers are a subset of the set of all games. Many challenging questions concern the computational complexity of optimal play, given a particular game specification.

Given the advanced development and mathematical depth of Combinatorial Game Theory, its significant papers will be a welcome contribution to IJGT. We hope that IJGT will be considered as the premier publication outlet by the Combinatorial Game Theory community.

Acknowledgements We thank the authors for their cooperation and patience during the long time this special issue took to appear, in particular with the sometimes long refereeing times, and our repeated requests for changing their introductions and notation. The more substantial improvements are due to the highly competent and detailed reports of the anonymous referees.
In Lisbon, the Combinatorial Game Theory Colloquia (taking place every two years in January 2015, 2017, and next in 2019) are organized by Associação Ludus, with gratefully acknowledged support from the Centro de Análise Funcional, Estruturas Lineares e Aplicações, Centro de Matemática Aplicada à Previsão e Decisão Económica, Centro Interuniversitário de História das Ciências e da Tecnologia, and the Biosystems and Integrative Sciences Institute.
We hope this special issue will encourage all CGT authors further to send their best work to the *International Journal of Game Theory*.

[1] This was written five years before the program AlphaGo by Google DeepMind convincingly beat the top Go player Lee Sedol in March 2016 in Seoul, South Korea.

[2] Elwyn Berlekamp and Richard Low are the authors of the article "Entrepreneurial Chess" in this special issue.