[1]

POINTS IN EQUILIBRIUM

Given locally finite configuration of points on the real line $\mathbb{R}$, define a potential $F(x,y)$ to be the force between two points x, y, for example

$$F(x,y) := |x-y|^{-2}.$$

Letting $(a_n)_{n\in\mathbb{Z}}$ be the positions of the points, we say that the system is in *equilibrium* if

$$\sum_{i\neq n} F(a_i, a_n) sign(a_i - a_n) = 0 \qquad \forall n.$$

For a sequence $(a_n)_{n\in\mathbb{Z}}$ in arithmetic progression the total force is 0, this suggests the following question:

Question 0.1. *Assume $(a_n)_{n\in\mathbb{Z}}$ is in equilibrium, does $a_n = an+b$ for all n and some $a, b \in \mathbb{R}$?*

Non symmetric configurations in equilibrium on the sphere $\mathbb{S}^2$

Minimal energy configurations are in equilibrium.

A finite subset of the sphere is called *symmetric* if there is a finite group that acts transitively by isometries on the subset viewed as a finite metric space. We know ([1] using [2]) that on the sphere $\mathbb{S}^2$, there is a universal upperbound on the size of finite symmetric sets, which are not on a grand circle or two parallel circles. Maybe the soccer ball (truncated icosahedron) with 60 points is the largest such symmetric configuration. We still lack an explicit upperbound.

It is easy to see that minimal energy configurations are not on a grand circle. Therefore on the sphere we have non symmetric configurations in equilibrium.

References

[1] I. Benjamini, H. Finucane and R. Tessera, On the scaling limit of finite vertex transitive graphs with large diameter. *Combinatorica* to appear, arXiv:1203.5624

[2] E. Breuillard, B. Green and T. Tao, The structure of approximate groups. *Publ. Math. IHES.* to appear, arXiv:1110.5008

[1]Itai B. , May 2015