A 1'st order phase transition via property T

May 2007

A model proposed by Gromov [1] in 1992 consists in picking uniformly and independently $3^n d$ realtors among the 3^n possible reduced words of length n on the generators a, a^{-1}, b, b^{-1}, repetition is allowed. This is called the density model for random groups. Gromov showed that with high probability the group is trivial if $d > 1/2$, nontrivial and in fact, infinite hyperbolic, if $d < 1/2$.

So lets add the random realtors sequentially one after the other to get a sequence of random quotient groups collapsing to the trivial group at some point. Look at the resulting sequence of non isomorphic groups obtained this way. It may be the case that by adding one random relation an infinite group collapses to the trivial group. But we can not exclude the possibility that the infinite group will collapse to a finite non trivial group, and then you will get a sequence of finite group till they collapse to the trivial group.

Question 1. *How many such non isomorphic finite groups one can see before collapsing to the trivial group? Provide an upper bound on the size of this sequence? We* **conjecture** *that the sequence of finite group is a.s. uniformly bounded in n.*

Zuk [3] proved that if $d > 1/3$ then a.s. the random group has property (T), with a uniform Kazhdan constant. Zuk [3] gives a lower bound for the displacement of unit vectors by the generators in any unitary representation. Applying this to the regular representation gives a.s. a uniform constant C in a linear isoperimetric inequality, provided the group in *infinite.*

From phase transition theory view point the model exhibits a *first order* phase transition. Rather then volume growth and isoperimetric inequality tend to 0 in a window of groups of length going to infinite with n, (as in second order transition), *the last infinite group* a.s. admits a uniformly bounded isoperimetric constant and exponential growth, a discontinuity.

For background on random groups see [2].

Acknowledgements: Thanks to Pierre Pansu and Gady Kozma for a useful discussion.

References

[1] M. Gromov, Asymptotic invariants of infinite groups, in *Geometric group theory*, Cambridge University Press (1993).

[2] Y. Ollivier, A January 2005 invitation to random groups, *Soc. Brasileira de Matematica* (2005).

[3] A. Zuk, Property (T) and Kazhdan constants for discrete groups. *Geom. Funct. Anal.* **13**, No.3, 643-670 (2003).