Algorithmic Game Theory – Final Exam

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General instructions. The exam has 2 parts. You have 3 hours. No books, notes, or other external material is allowed.

Part I (40 points)

Answer 4 of the following 5 questions. Each question is composed of a statement (question c is composed of two statements). Determine if the statement is TRUE or FALSE, and explain your answer in 1-3 sentences (i.e. sketch of the proof or a convincing argument why it is correct/wrong).

a. For every cost-sharing scheme $\xi : A \times 2^A \to \mathbb{R}$ in a cost-sharing game (A, c), if ξ is cross-monotone and γ-budget-balanced, then the cost allocation given by $\alpha_j = \xi(j, A)$ is in the γ-core of the game.

b. If a finite game has a dominant strategy and the game is repeatedly played by a player who is following a *regret minimization* strategy, then the difference between the average payoff of the player and the average payoff of the dominant strategy cannot become arbitrarily large as a function of time.

   Can the regret minimization strategy be better than the dominant strategy on the average?

c. Consider an auction of two identical items with $n \geq 3$ players who are each interested in getting one of the items. Each player submits a bid, and the items are awarded to the two highest bidders (breaking ties arbitrarily), who will be called the first winner and the second winner respectively. The following auctions are incentive compatible:

   (a) Charge the first winner with the second highest bid and the second winner with the third highest bid.

   (b) Charge each of the winners with the third highest bid.

d. In a 0-sum finite two person game, in every Nash equilibrium, the expected payoff of the row player is the same.

e. For every $\varepsilon > 0$, in every two-player game with n possible actions for each player and payoffs between 0 and 1, there exists an ε-Nash equilibrium where all players' probabilities are integer multiples of $\Omega(\varepsilon/n)$.

Part II (60 points)

Answer 3 of the following 4 questions.

1. A pure strategy s in a two player game is said to be *dominated* if for every mixed strategy t of the other player, strategy s is not a best response with respect to t. Clearly, a dominated strategy cannot be part of a Nash equilibrium. Show that there is a polynomial time algorithm for detecting whether a two player game (given in standard form) has a dominated strategy.

2. **Social Choice.**

   (a) Suppose that there are n alternatives and they are ordered on a line. There is an odd number of voters. Each voter has a most preferred alternative, called her peak, and for any two alternatives on the same side of the peak the voter prefers the one closer to her peak. Consider the following social choice function: each voter reports her peak and the median of those peaks is chosen.

   Prove that truth telling is a dominant strategy in this setting.

   (b) *Range voting*: in a range voting scheme each voter assigns scores to each of the candidates. The only restriction on the scores is that every score must be in the range 0-99. The scores for each candidate are summed up and the candidate with highest total score wins the election.

   Prove that in range voting there is no incentive for a voter to reverse the scores of candidates in the following sense: for any $i < j$, given the score of the ith favored candidate there is no incentive to assign the jth favored candidate with a score that is higher than i's. Is the reverse true as well, that is given the jth score there is no incentive to assign i with a lower score?

   (c) State the **Gibbard-Satterthwaite Theorem** that deals with incentive compatible social choice functions. Explain why neither of the voting schemes presented above contradicts this theorem.

3. Let (G, r, c) be a routing game whose cost functions c_e are *affine*, and suppose $\hat{G}$ is the network obtained by removing from G some edges (but the source-sink pairs (s_i, t_i) and requirements r_i are the same). Let f and $\hat{f}$ be Wardrop equilibria for G and $\hat{G}$, respectively. Recall that the cost of f is $\sum_{e \in E(G)} c_e(f_e) \cdot f_e$, and similarly for $\hat{f}$.

   (a) Show an example where the cost of $\hat{f}$ is strictly smaller than that of f.

   Hint: Use the following network. There is one source-sink pair (s, t), two (disjoint) directed paths of length 2 that go from s to t, and in addition, the middle vertices in these two paths are connected to each other with a directed edges whose latency function is constant 0.

   (b) Prove that the cost of $\hat{f}$ must be at least 3/4 times that of f.

   (In words, we see that removing edges might improve the cost of the equilibrium, but by no more than a factor of 4/3.)

4. There is an n by n grid in which each edge e is "owned" by a different player called A_e. The true cost of an edge e is denoted by $c_e \geq 0$, and is known only to the player who owns e. An administrator has the goal of constructing a spanning tree of minimum (true) cost on the grid. For this purpose, the administrator commits to using a mechanism in which each player A_e submits a bid b_e, and based on the bids the mechanism outputs a list of edges that form a spanning tree and a list of payments to the players, where the payment to A_e is denoted by p_e. Player A_e's utility (i.e. payoff) is $p_e - c_e$ if e is in the spanning tree (gets payed p_e but losses her edge) and p_e otherwise (gets to keep her edge). Assume that (i) the players' goal is to maximize their utility (ii) there is no collusion among the players (iii) the mechanism is publicly known.

   (a) Suggest a mechanism (namely, a function from the bids to the list of edges to be bought and the payments) such that it is incentive compatible for every player A_e to bid her true cost, namely, to have $b_e = c_e$. The mechanism should be such that (i) the payment to a player A_e whose edge was chosen for the tree satisfies $p_e \geq b_e$; and (ii) the payment to every other player is 0.

   Note: the administrator's goal is to construct a spanning tree of minimum true cost, but there is no requirement to minimize the payments p_e.

   (b) Point out the property of the grid graph that allows you to get an incentive compatible mechanism with the above properties. Give an example of a connected graph where this is not possible.

THE END.