Optimal Distance Labeling for Trees

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Joint work by
Ofer Freedman, Paweł Gawrychowski,
Patrick Nicholson and Oren Wieman

Distance labeling scheme

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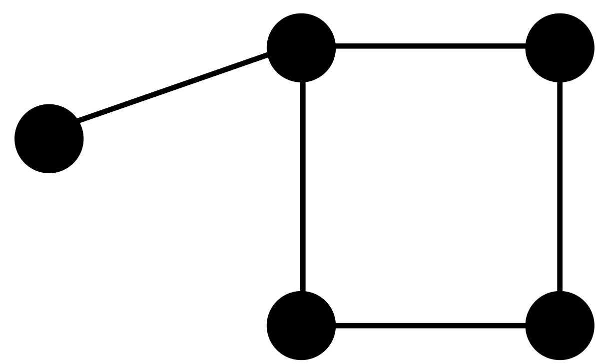
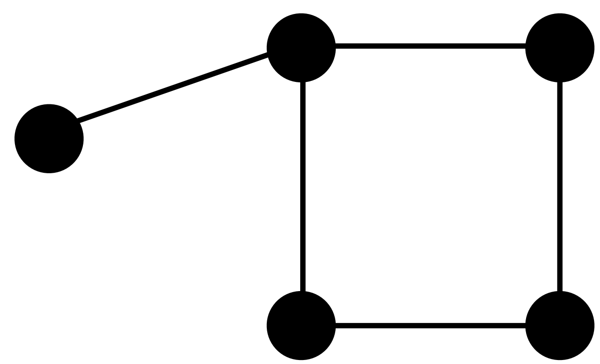
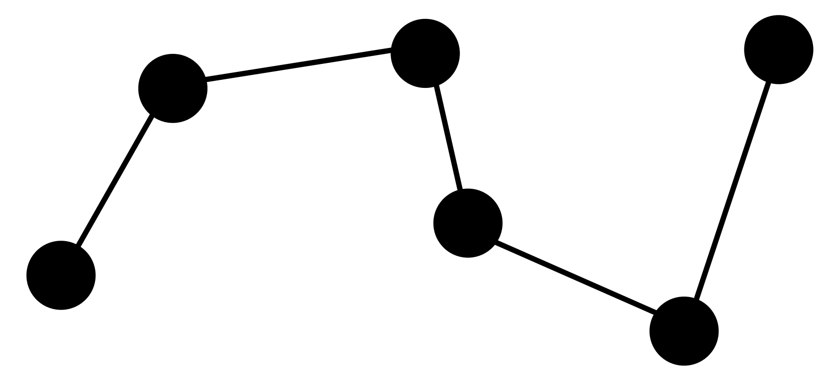


Distance labeling scheme





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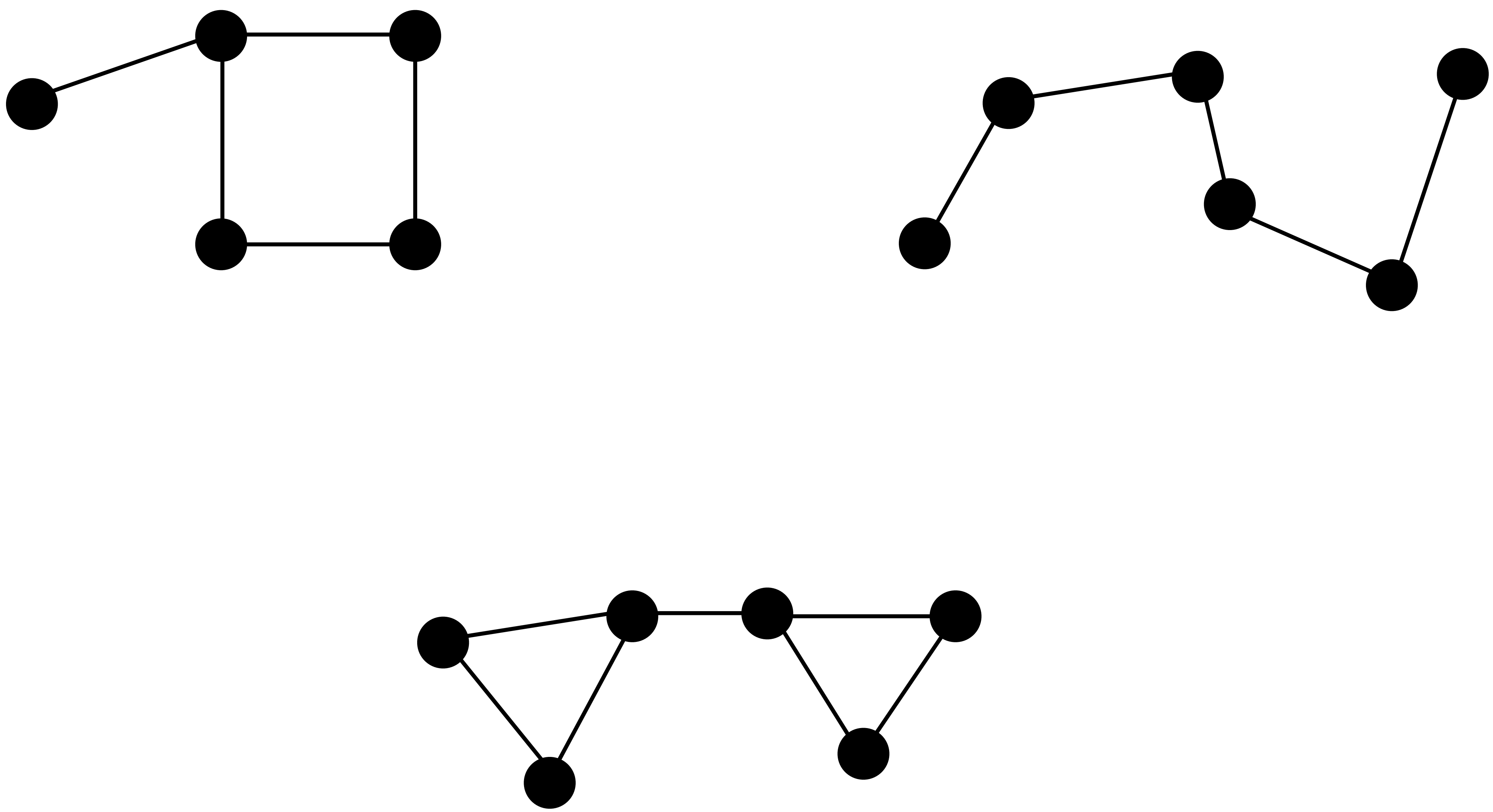
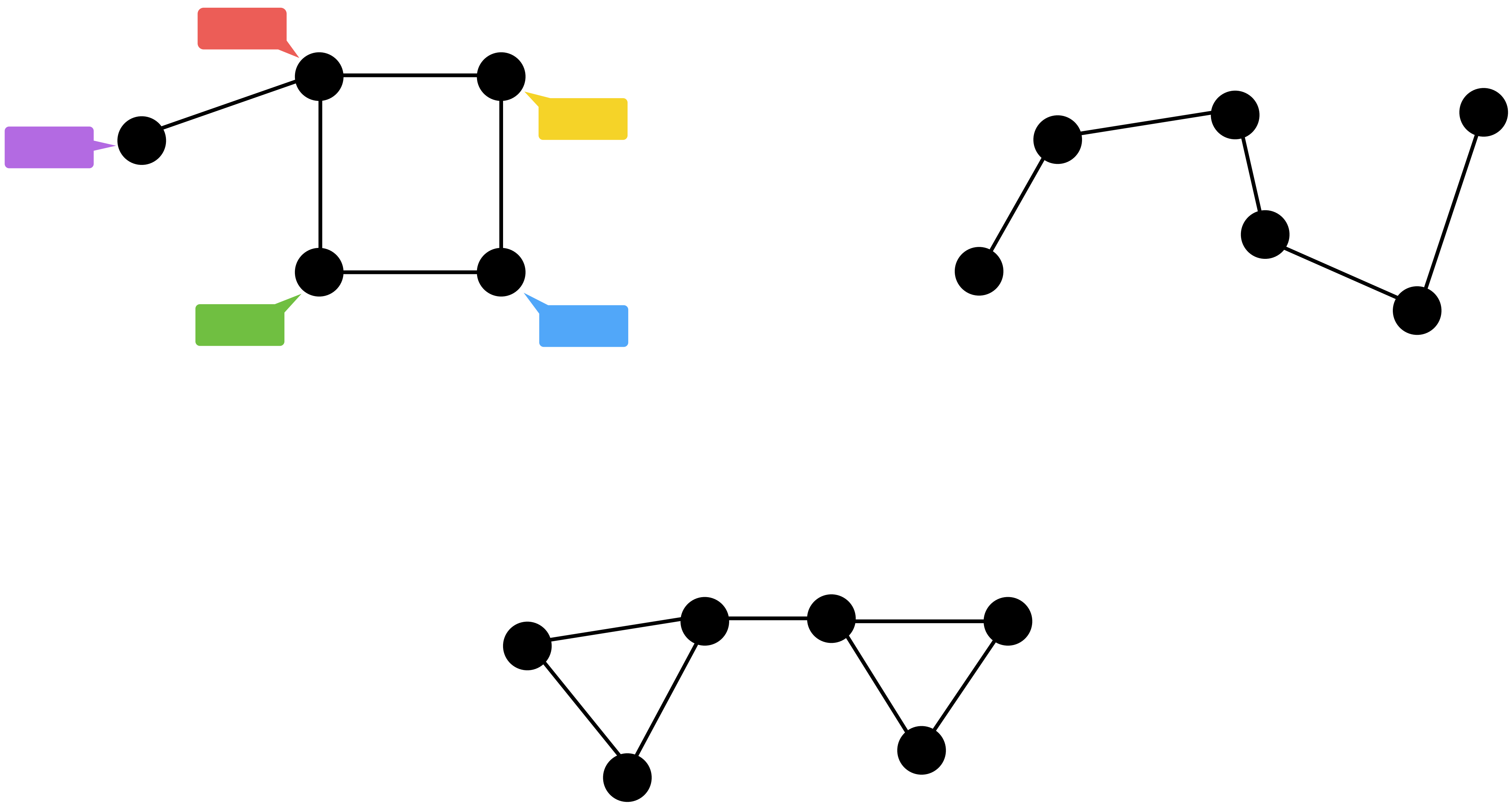
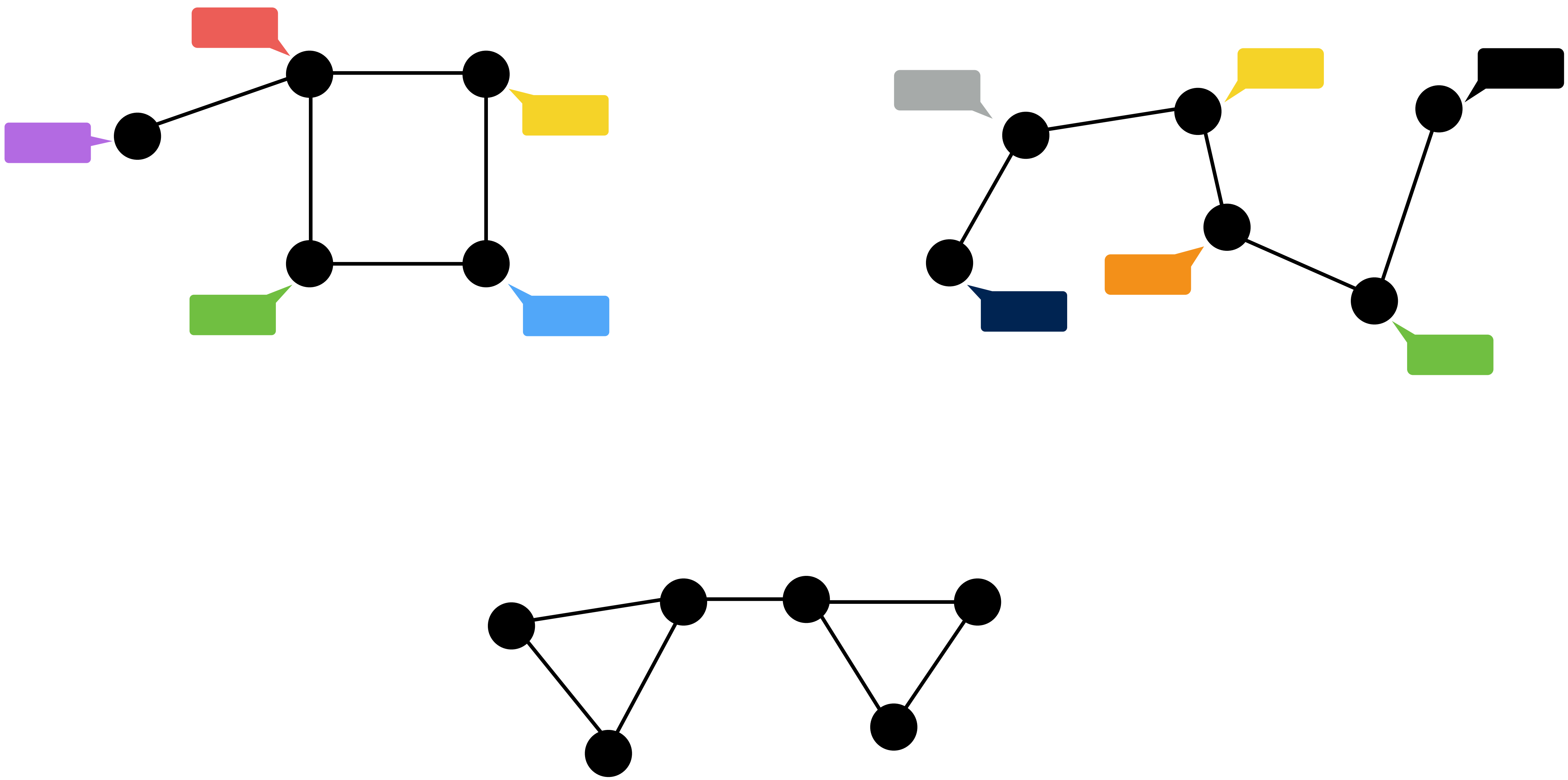


Distance labeling scheme



Distance labeling scheme



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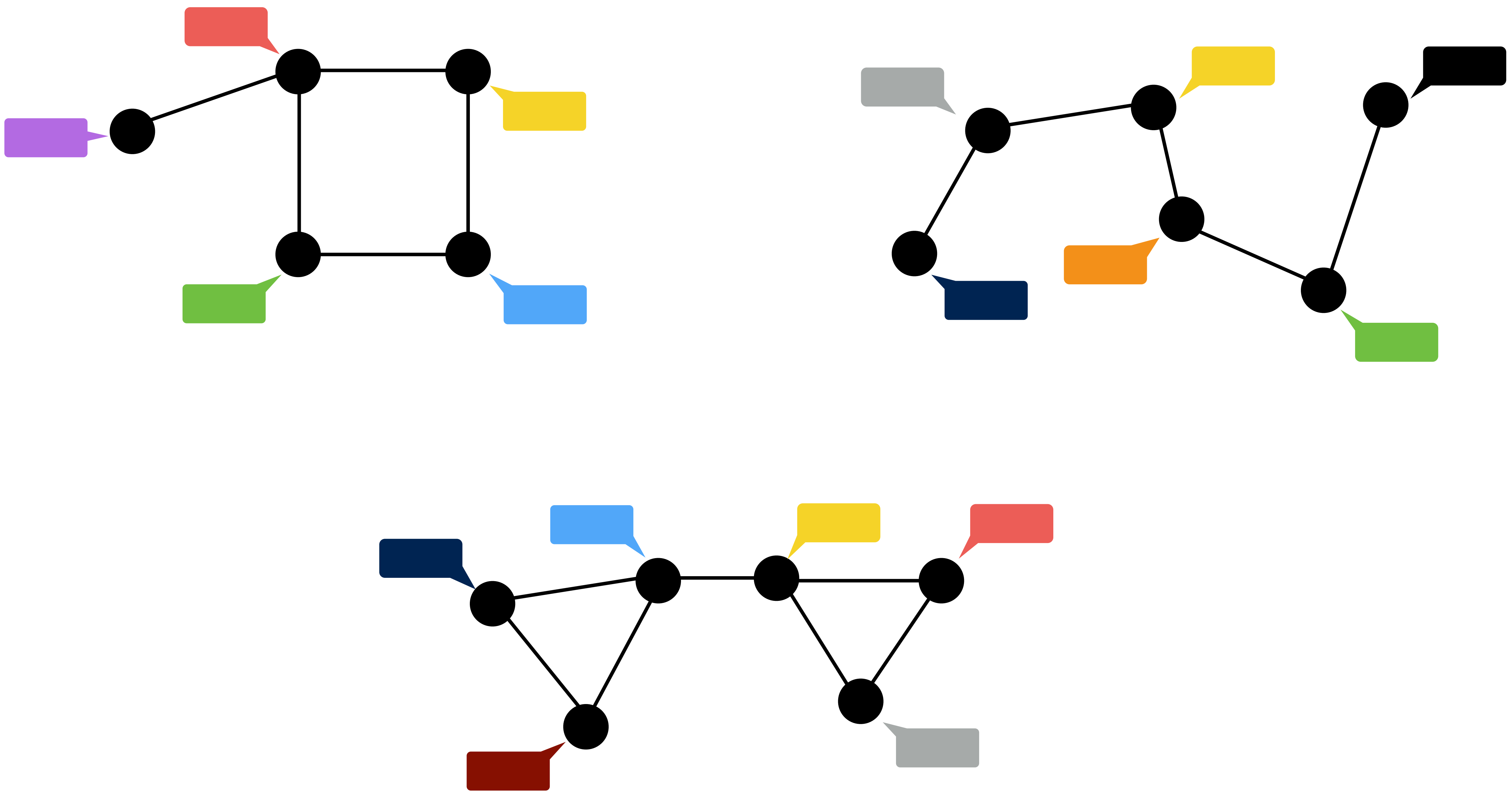

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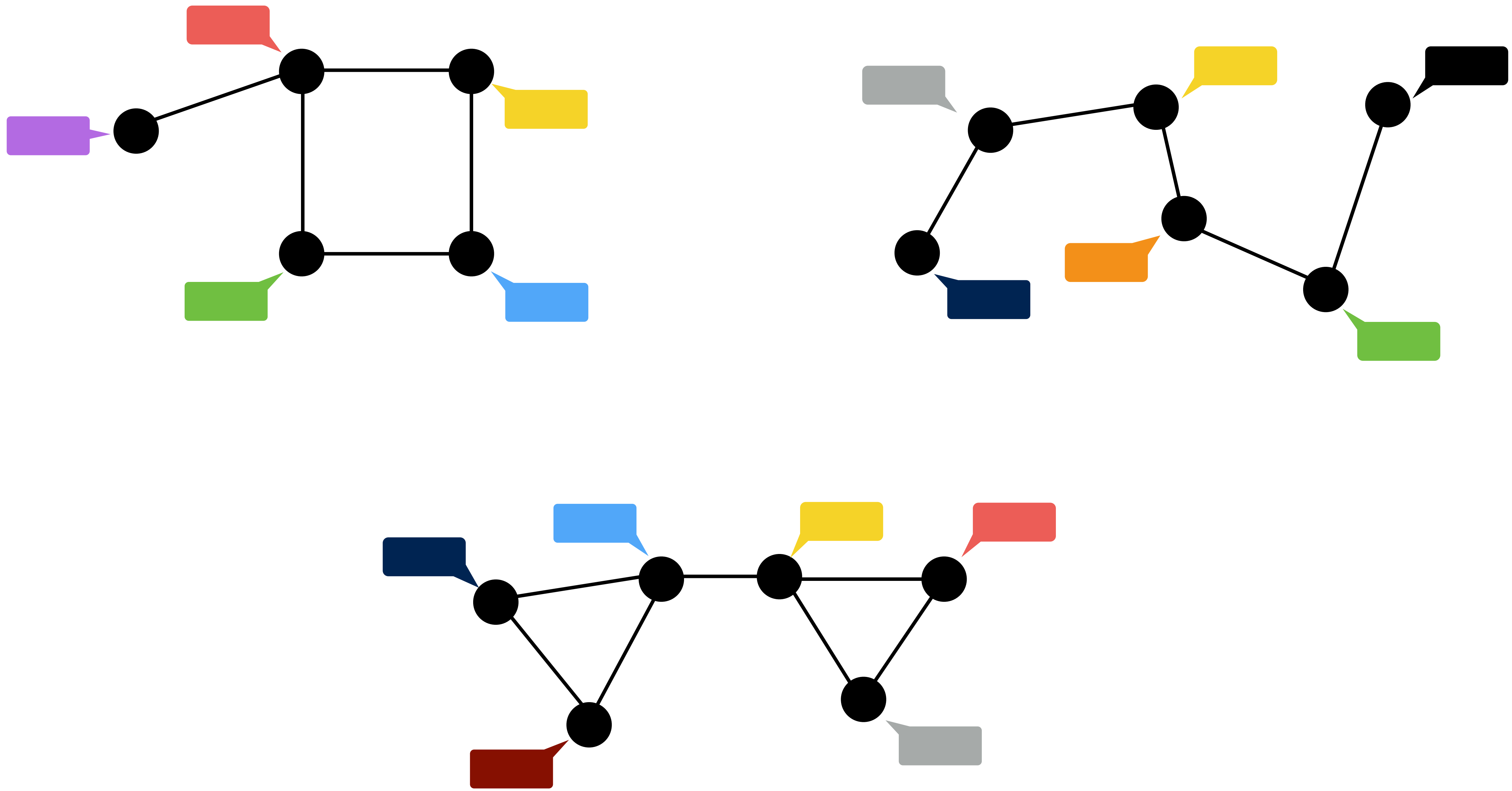
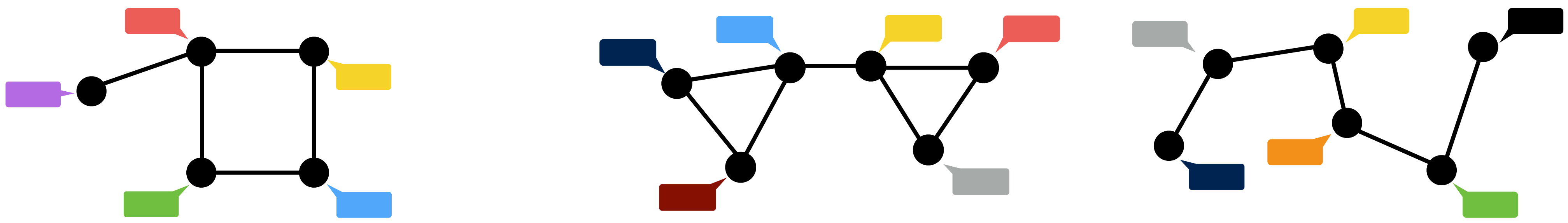
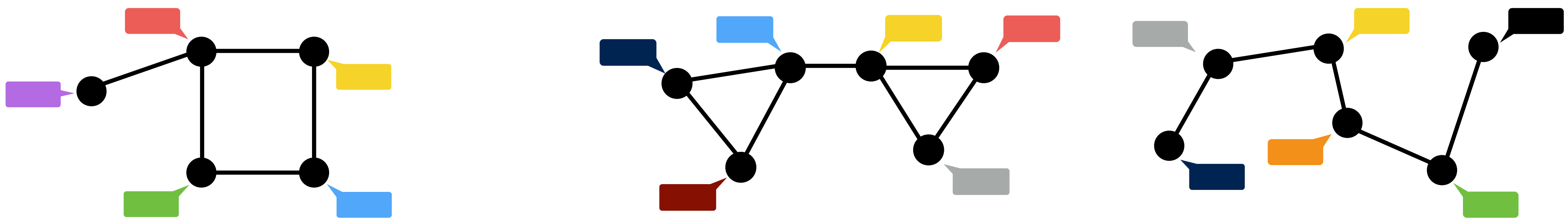


Distance labeling scheme

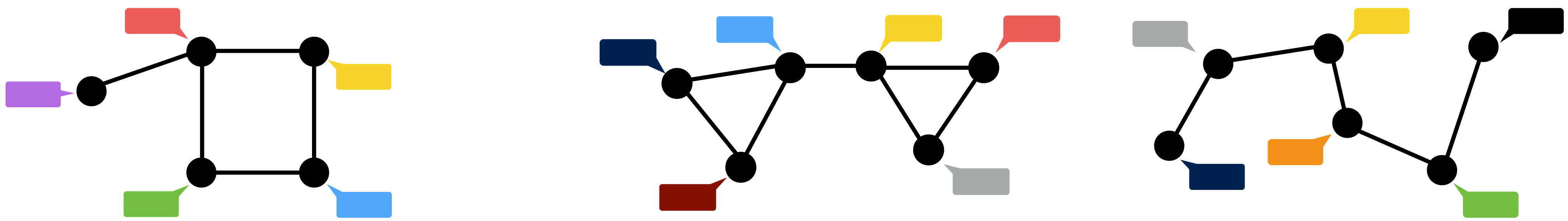


Distance labeling scheme



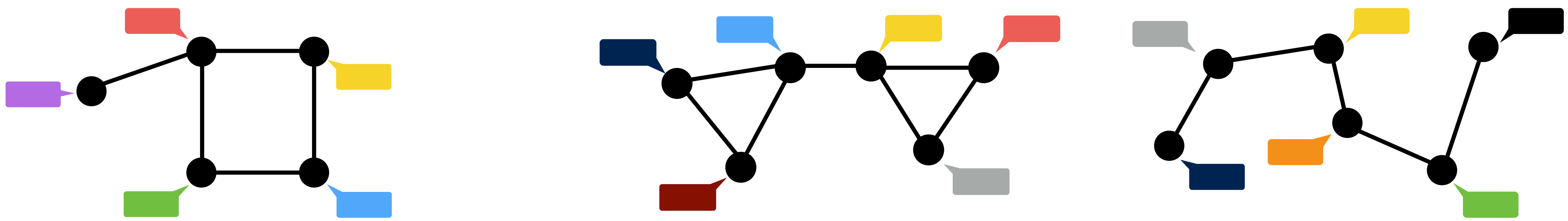
	exact		

Distance labeling scheme



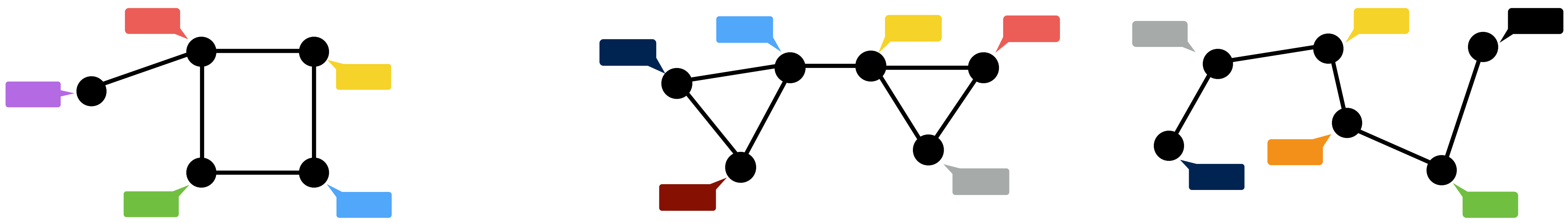
	exact	1+ε-Approximate	

Distance labeling scheme



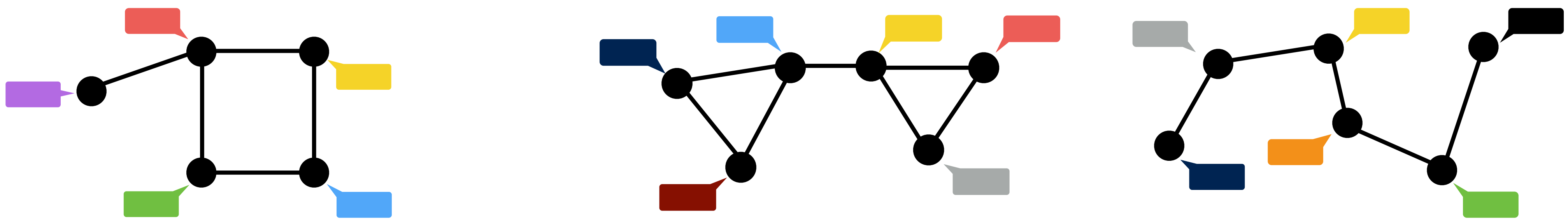
	exact	$1+\varepsilon$-Approximate (set $\varepsilon=1$)	

Distance labeling scheme



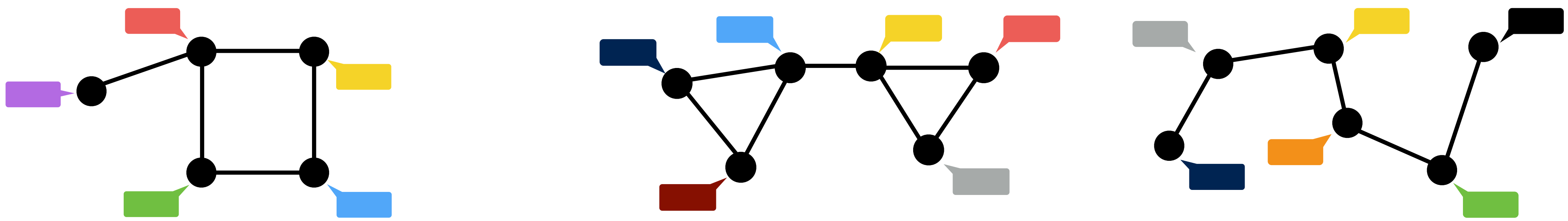
	exact	1+ε-Approximate (set ε=1)	k-Distance

Distance labeling scheme



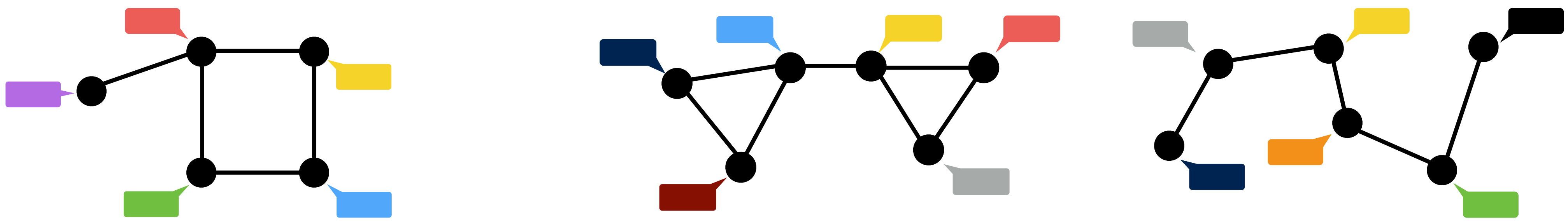
	exact	$1+\varepsilon$-Approximate (set $\varepsilon=1$)	k-Distance (set $k=3$)

Distance labeling scheme



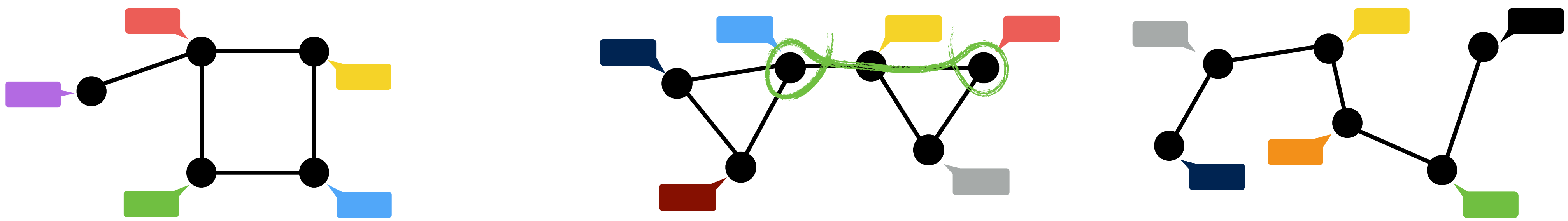
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
			

Distance labeling scheme



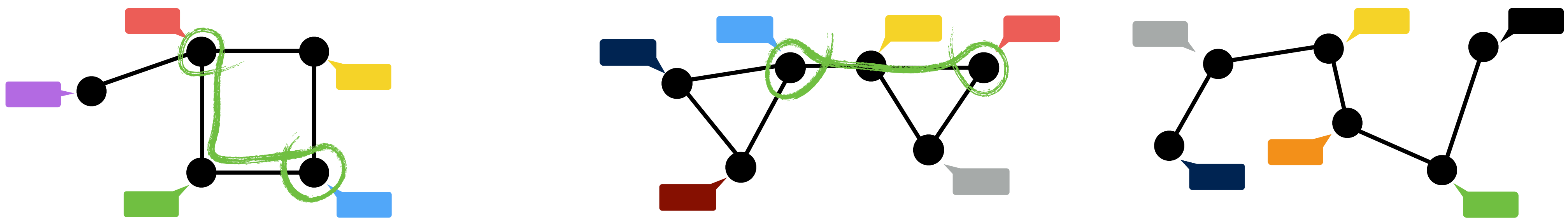
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2		

Distance labeling scheme



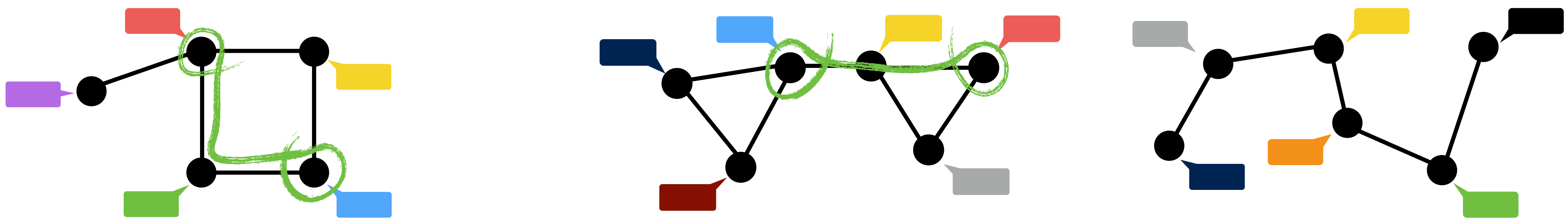
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2		

Distance labeling scheme



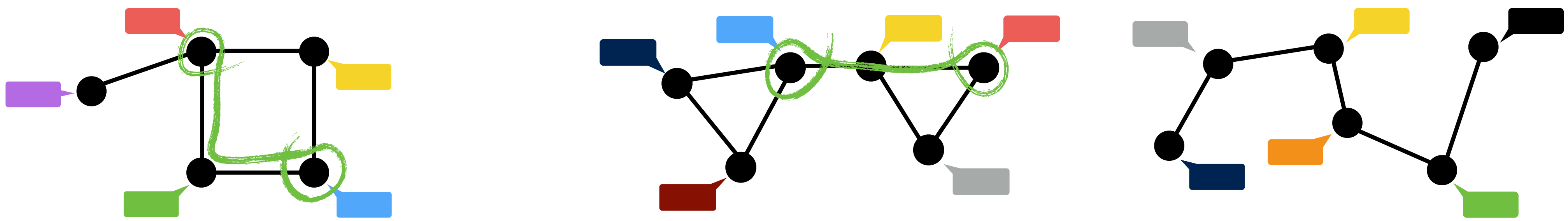
	exact	1+ε-Approximate (set $\varepsilon=1$)	k-Distance (set $k=3$)
	2		

Distance labeling scheme



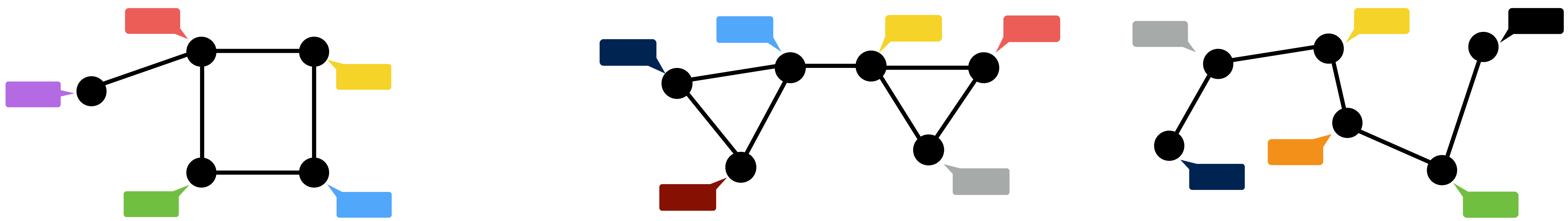
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	

Distance labeling scheme



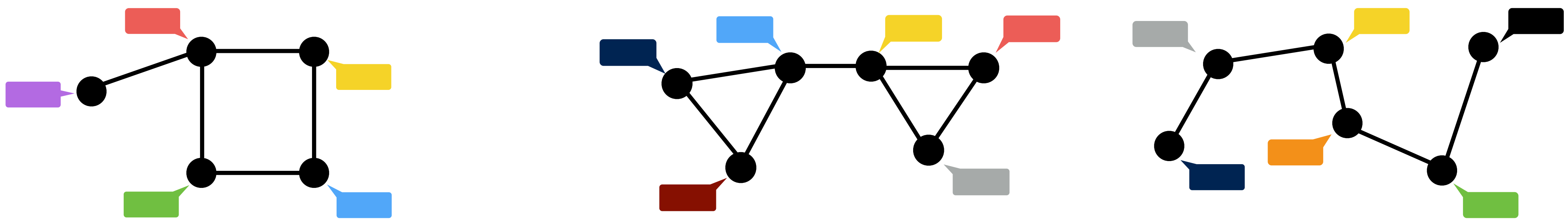
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2

Distance labeling scheme



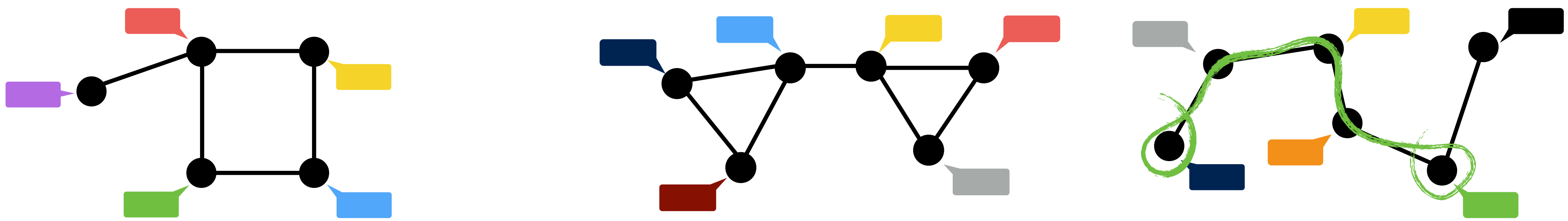
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2

Distance labeling scheme



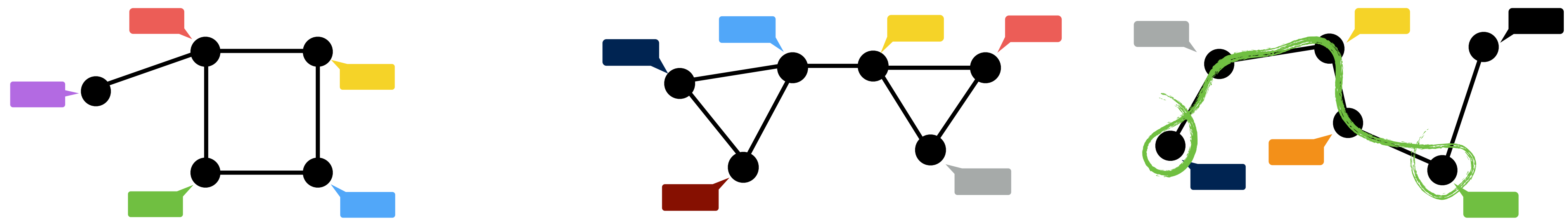
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
			

Distance labeling scheme



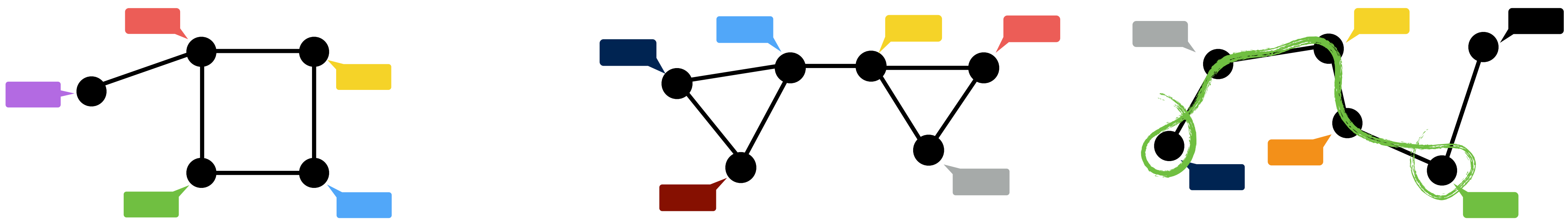
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
			

Distance labeling scheme



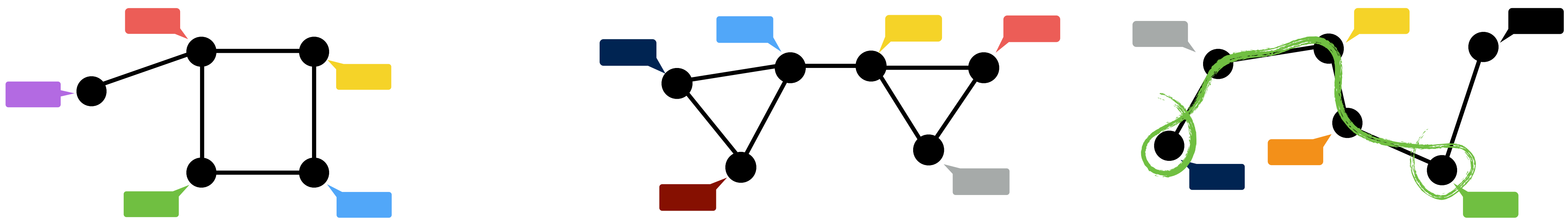
	exact	$1+\varepsilon$-Approximate (set $\varepsilon=1$)	k-Distance (set $k=3$)
	2	3	2
	4		

Distance labeling scheme



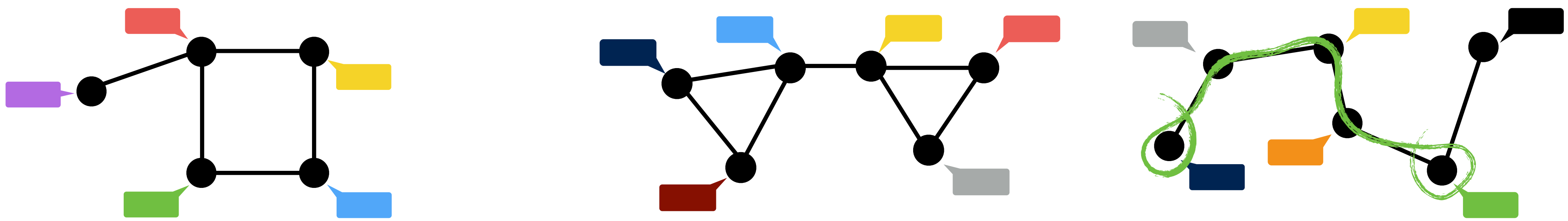
	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	

Distance labeling scheme



	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	> k

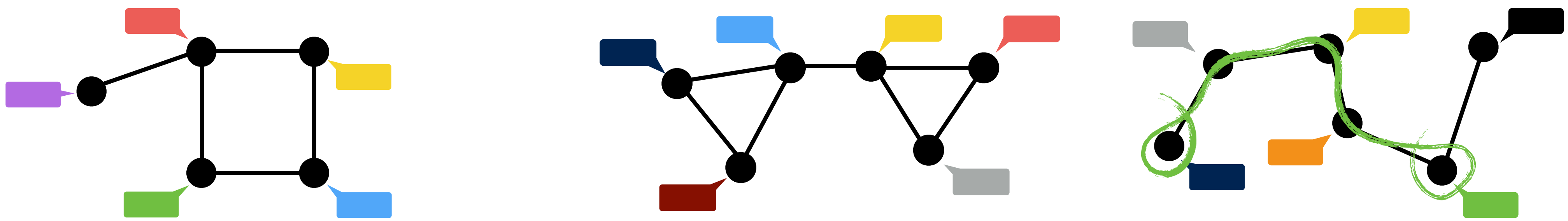
Distance labeling scheme



	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	> k

unique labels?

Distance labeling scheme

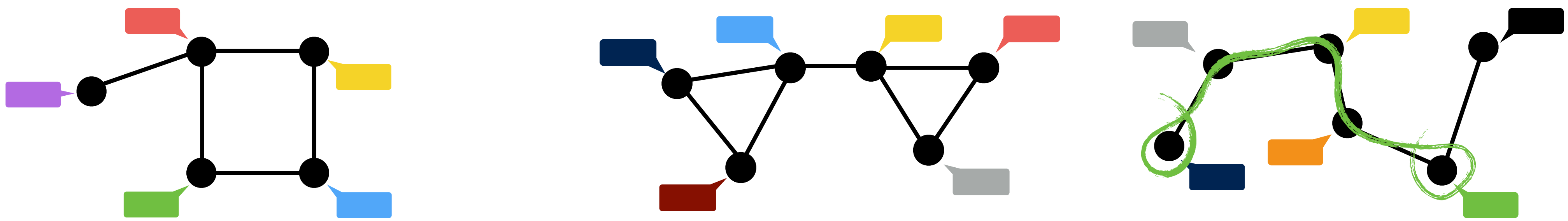


	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	> k

unique labels?

bits in a label?

Distance labeling scheme

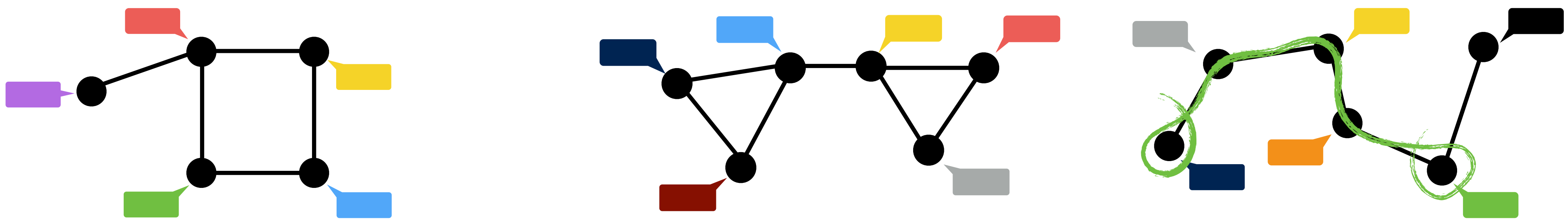


	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	> k

\# unique labels? x

\# bits in a label?

Distance labeling scheme



	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	> k

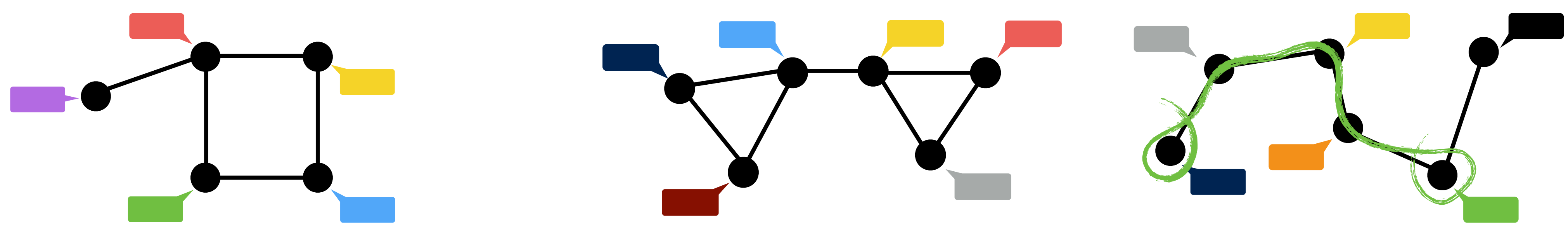
unique labels?

×

⇕

bits in a label?

Distance labeling scheme



	exact	1+ε-Approximate (set ε=1)	k-Distance (set k=3)
	2	3	2
	4	7	> k

unique labels? x

⇕

bits in a label? log(x)

Trees

Results

	Upper			Lower		
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	=>	$\frac{1}{4}\log^2 n$	$\frac{1}{8}\log^2 n$ [Gavoille et al. 2001]	=>	$\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	=>	$\log\frac{1}{\varepsilon}\log n$			$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	=>	$\log n$ $+$ $O(k\log(\log n/k))$			$\log n$ $+$ $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$	$O(\log n\cdot\log(k/\log n))$			$\Omega(\log n\cdot\log(k/\log n))$		

Results

	Upper	Lower
Exact lower order terms excluded		
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Trees

Exact Distance

Results

	Upper	Lower
Exact lower order terms excluded		
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$	
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ [Gavoille et al. 2001]
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => [Gavoille et al. 2001]
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001]
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Exact Distance

Universal Tree

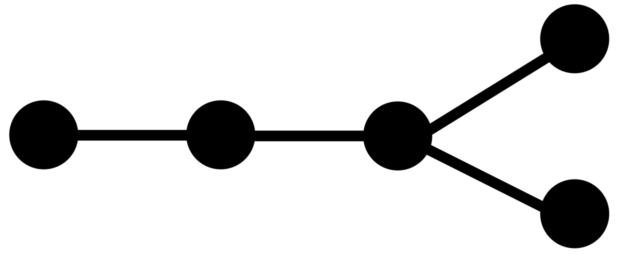
Universal Tree

Universal Tree

T_4:

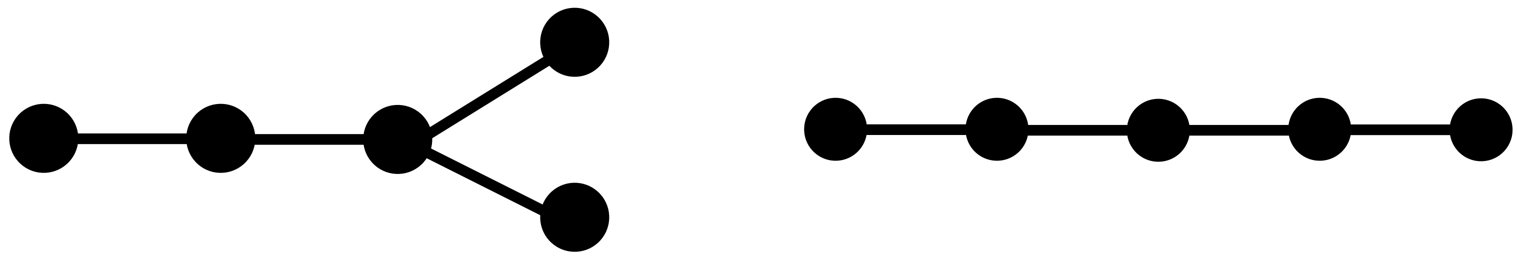
Universal Tree

T_4:



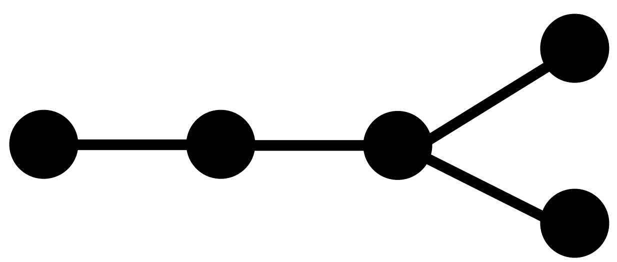
Universal Tree

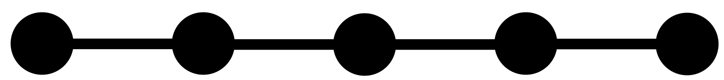
T_4:

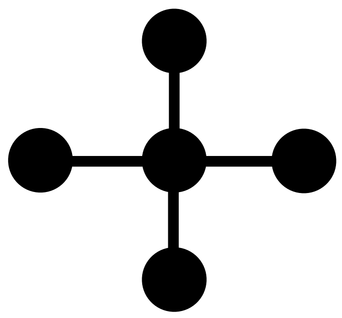


Universal Tree

T_4:

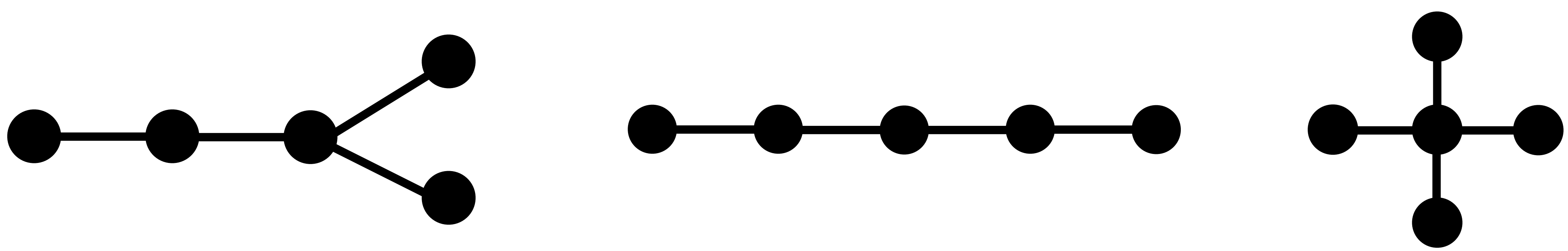






Universal Tree

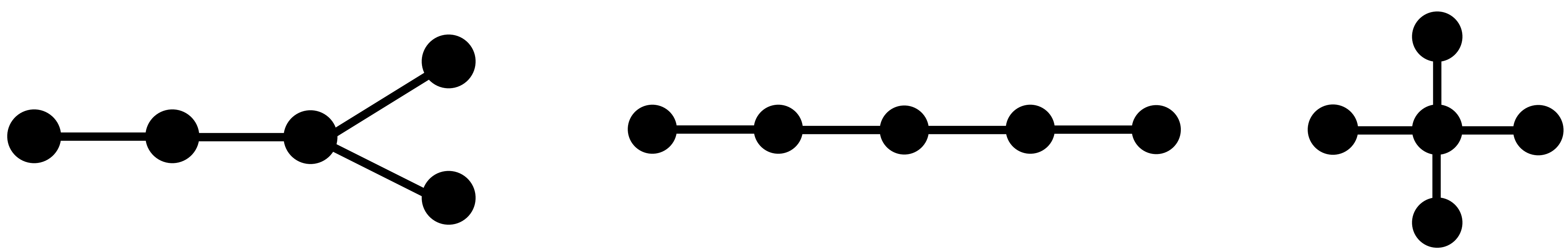
T_4:



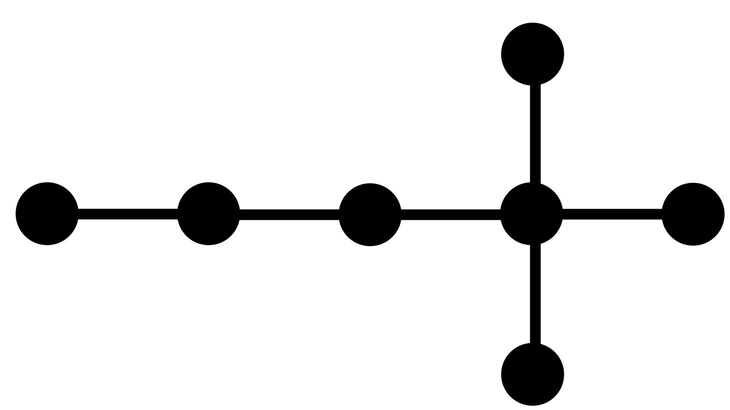
universal tree:

Universal Tree

T_4:



universal tree:



Universal Tree

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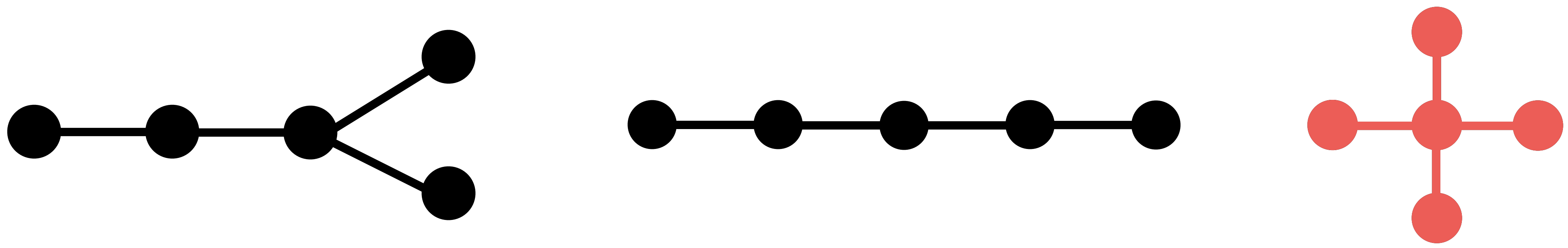
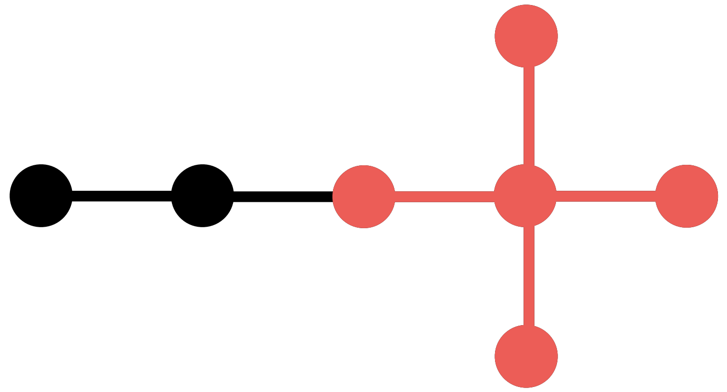


universal tree:



Universal Tree

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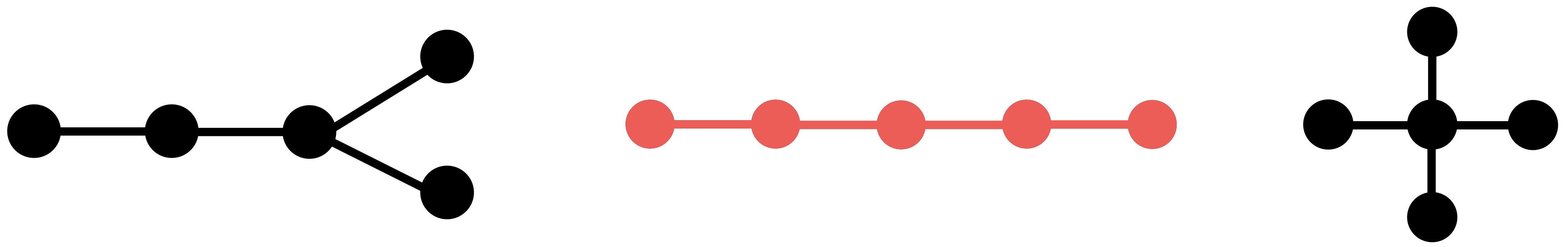
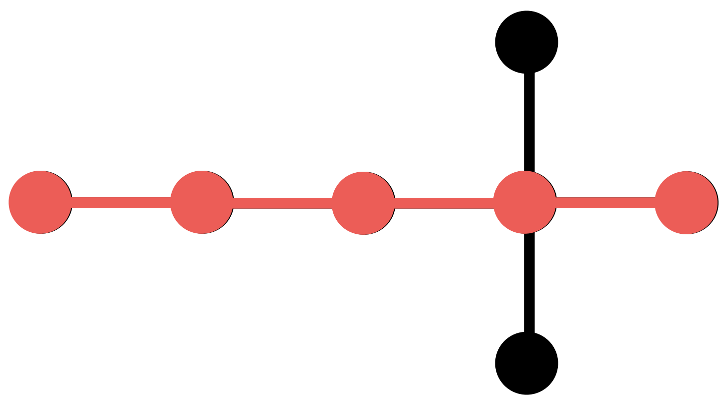


universal tree:



Universal Tree

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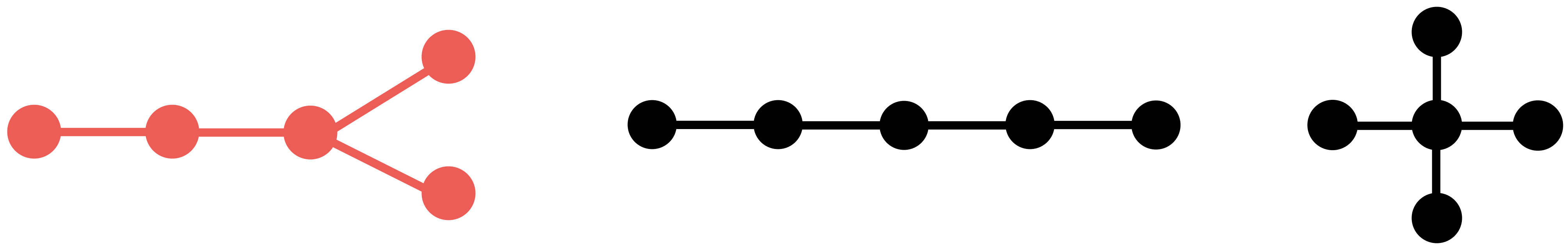
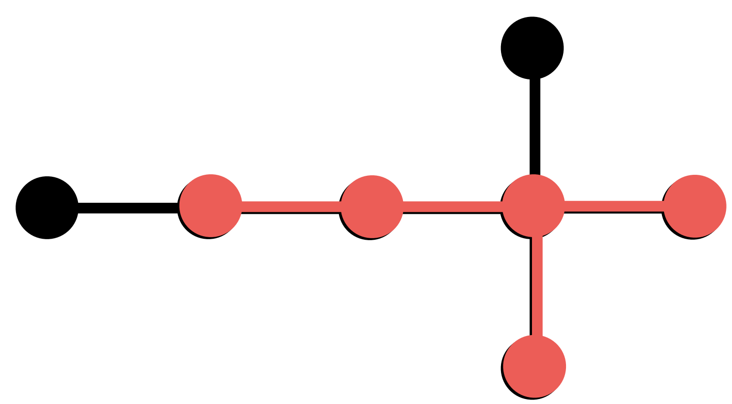
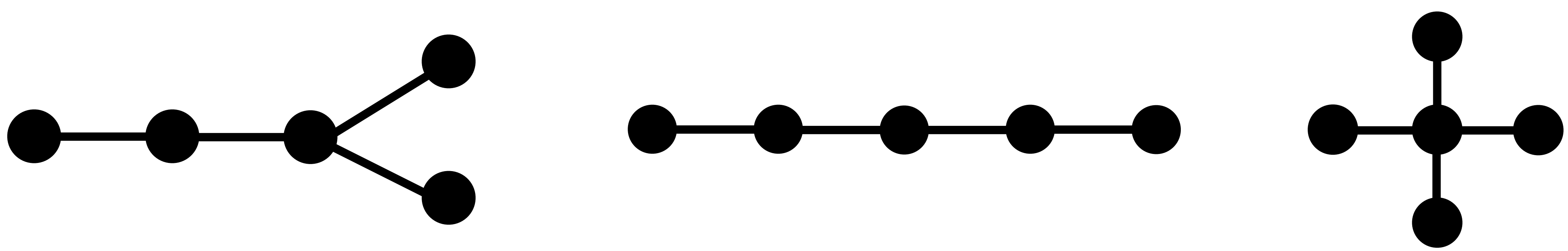


universal tree:

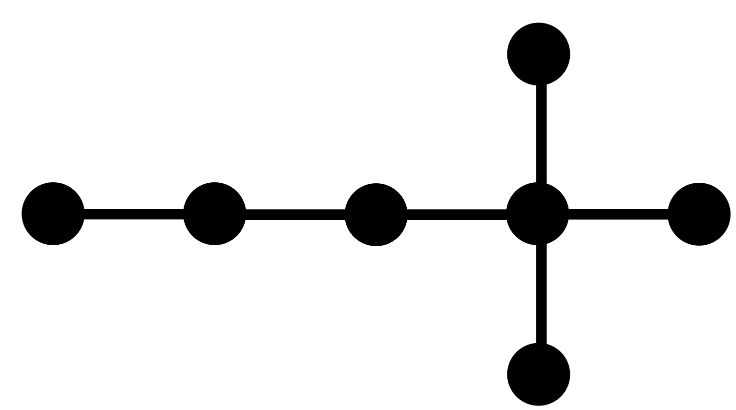


Universal Tree

T_4:



universal tree:

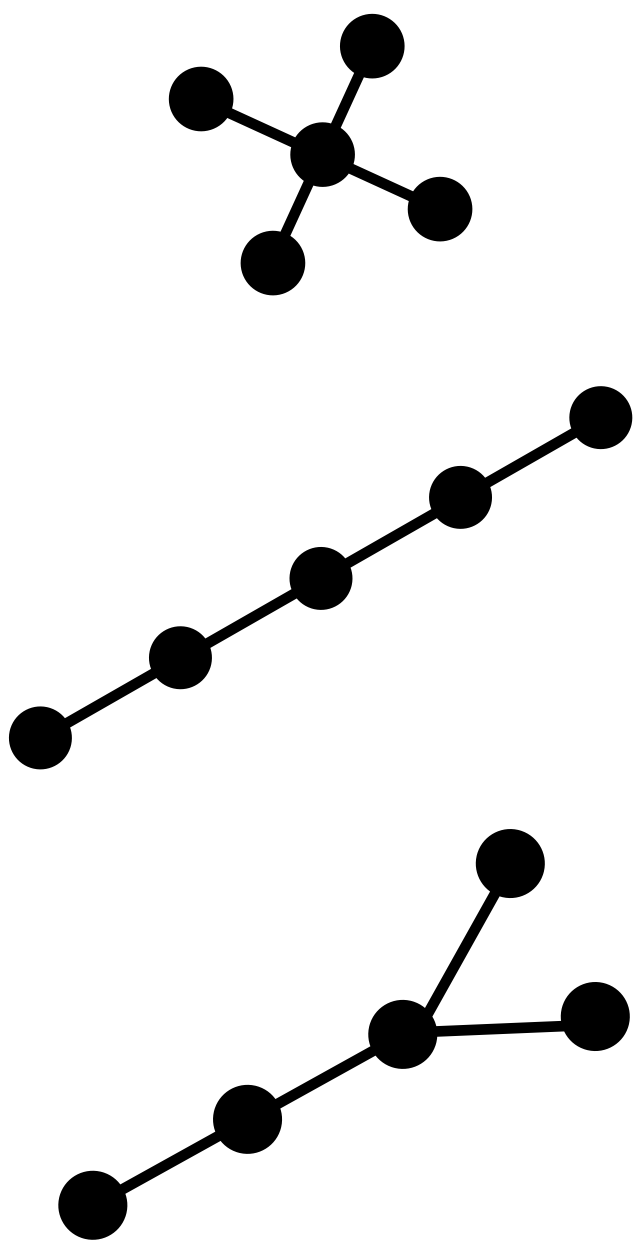


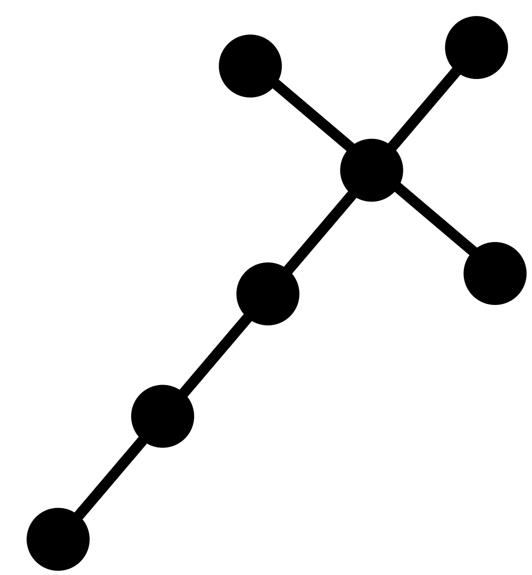
Exact Distance

Exact Distance **Universal Tree**

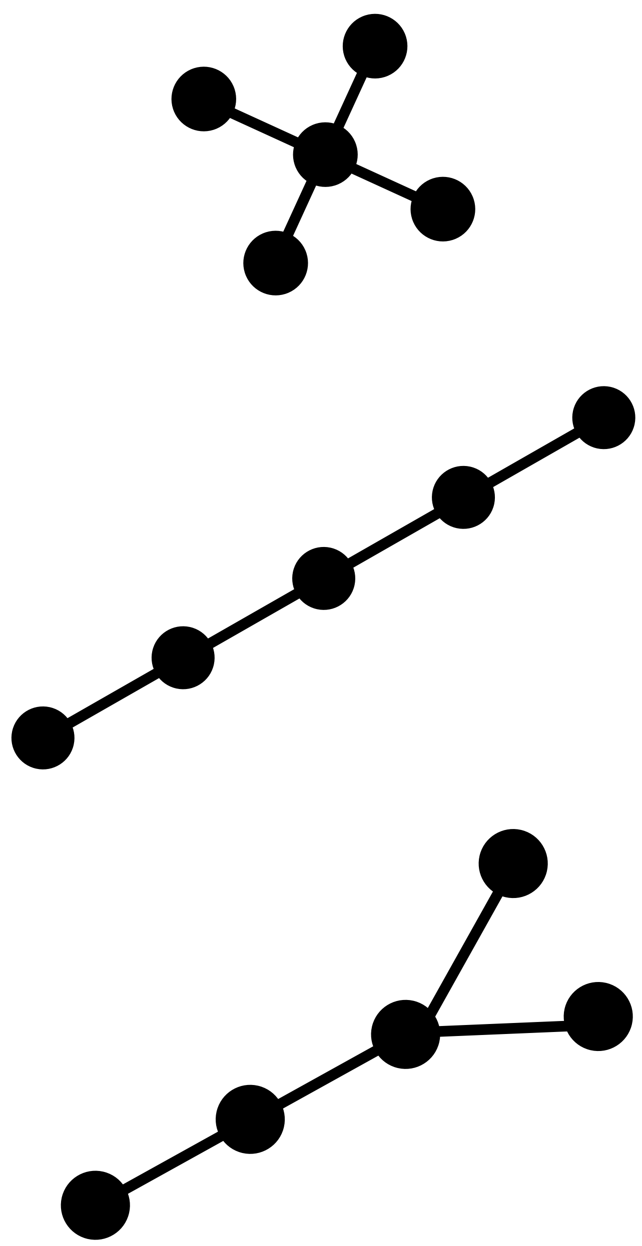
Exact Distance <= **Universal Tree**

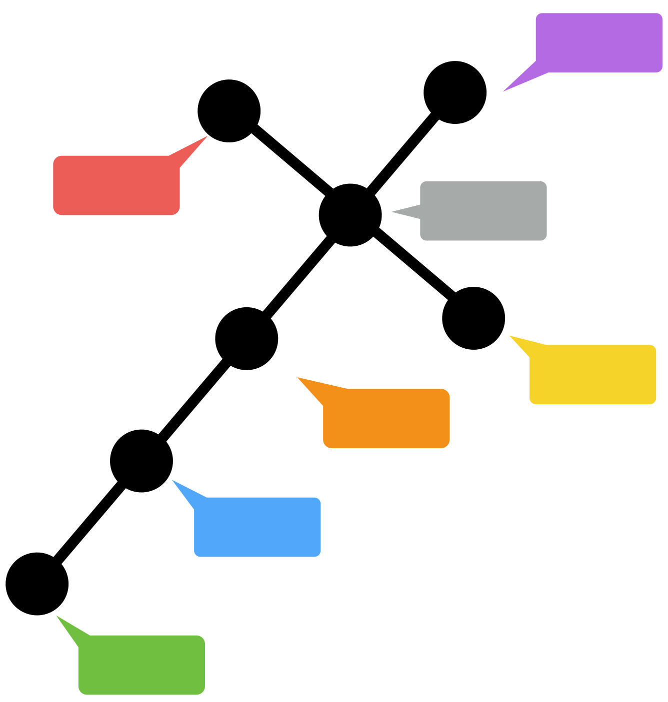
Exact Distance <= **Universal Tree**



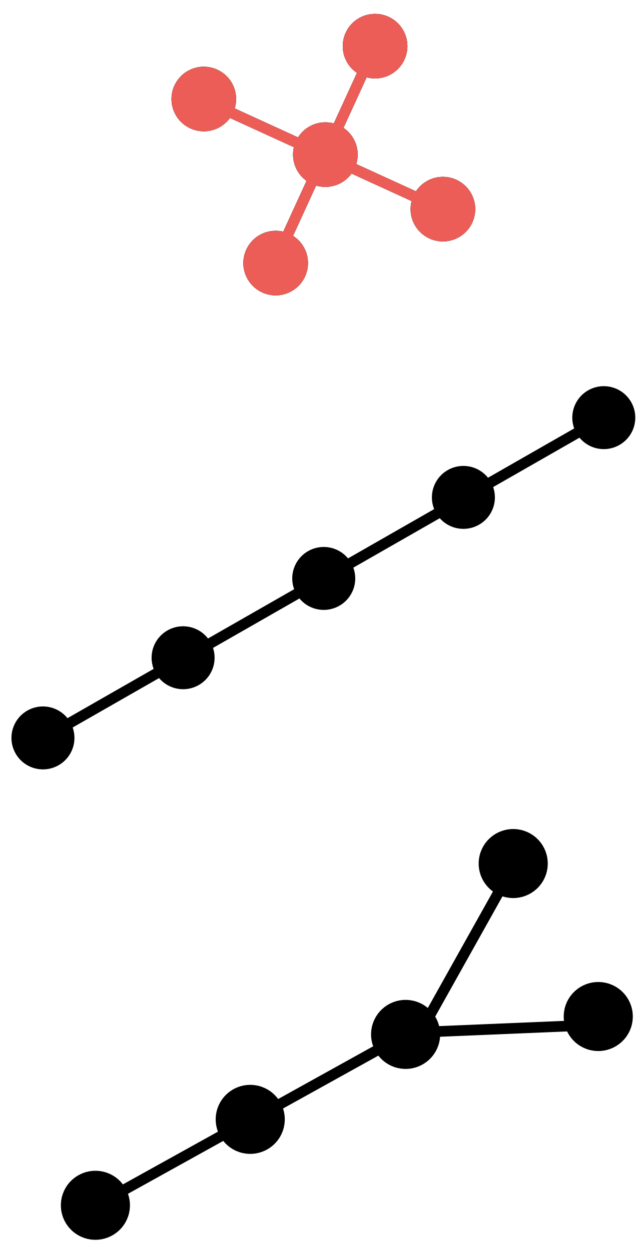


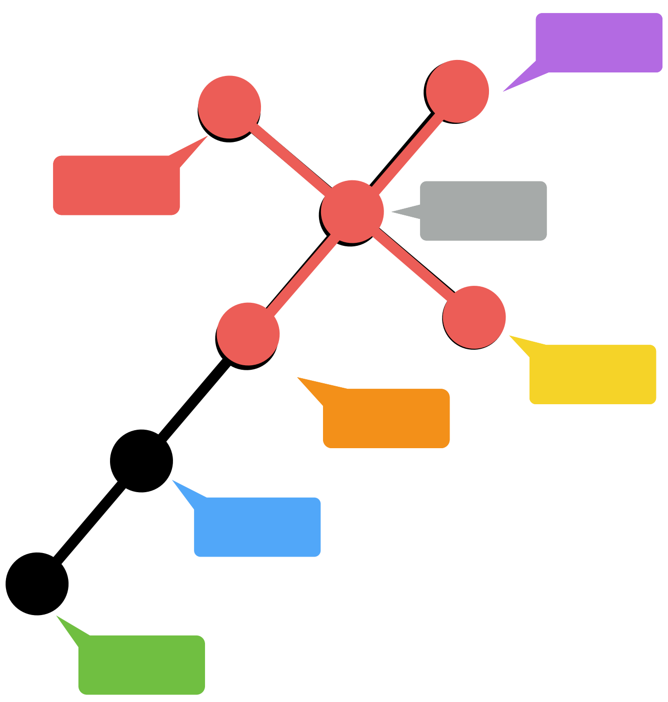
Exact Distance <= Universal Tree



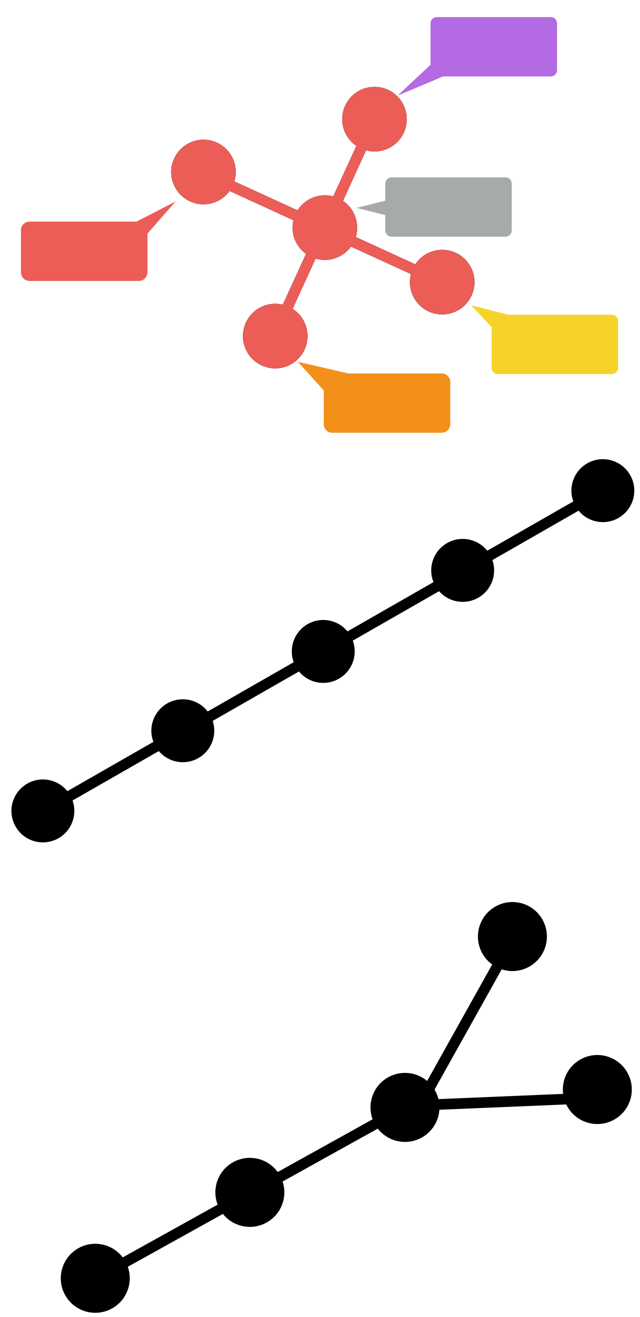


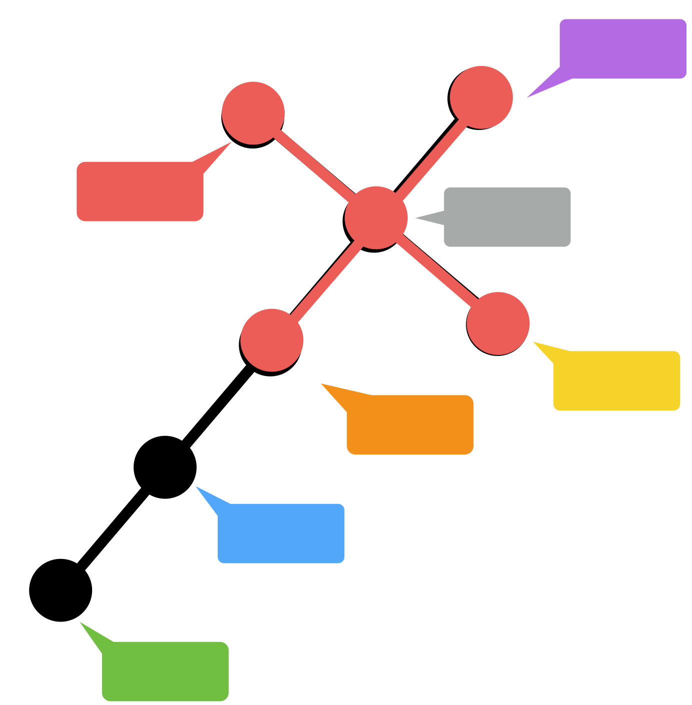
Exact Distance <= Universal Tree





Exact Distance <= Universal Tree





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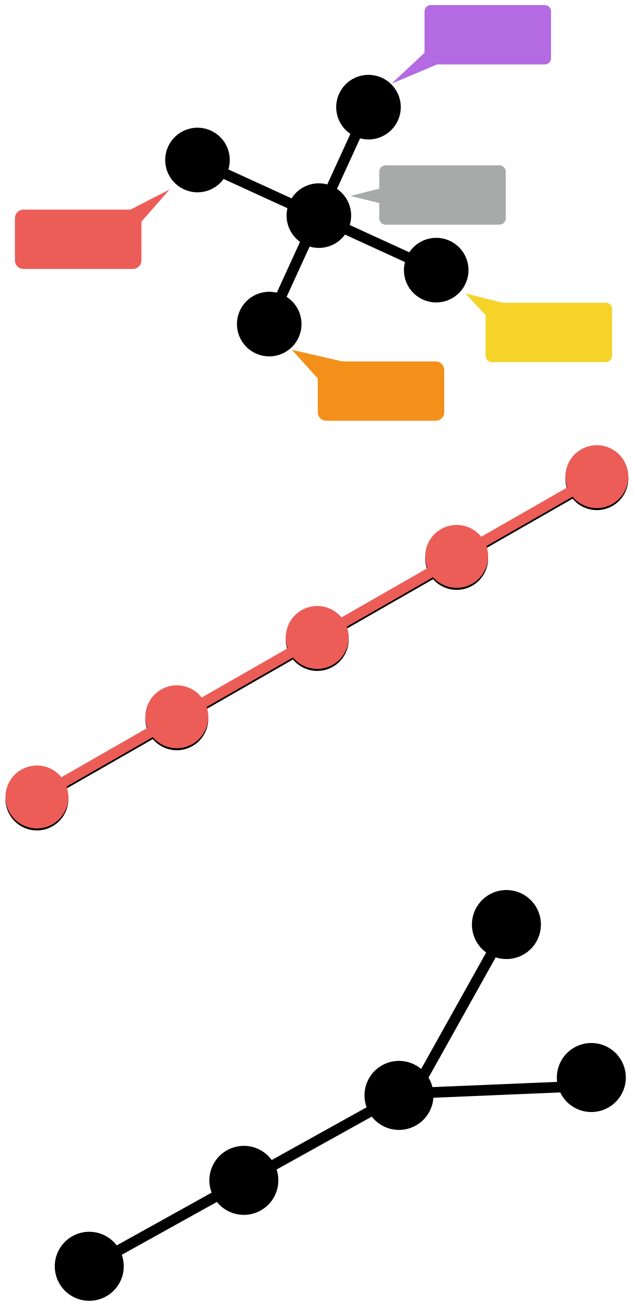
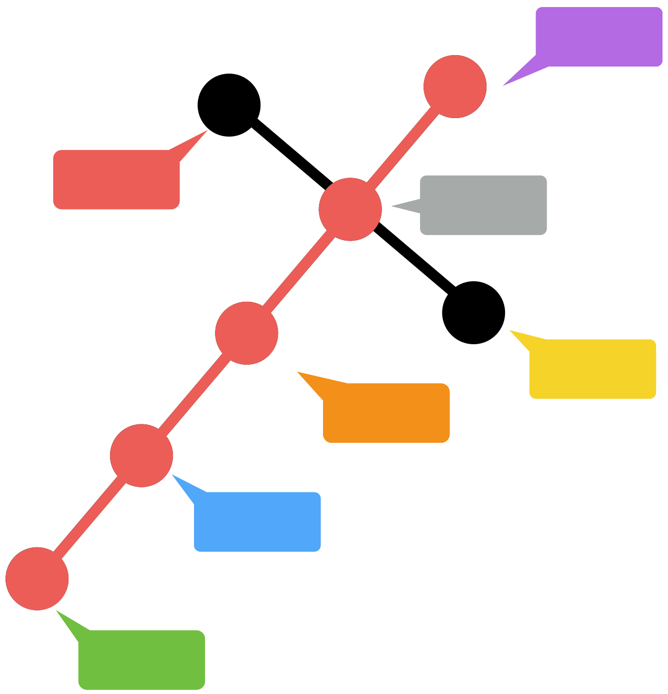
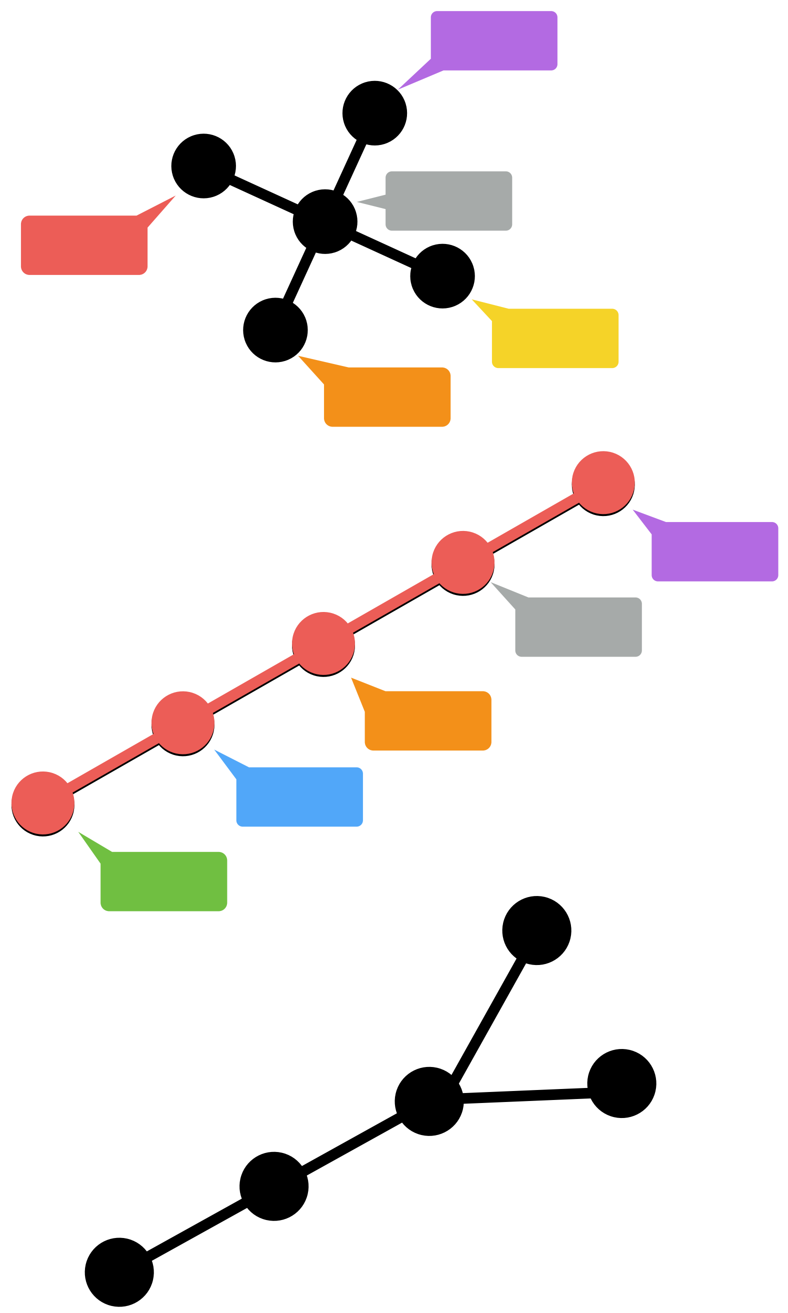
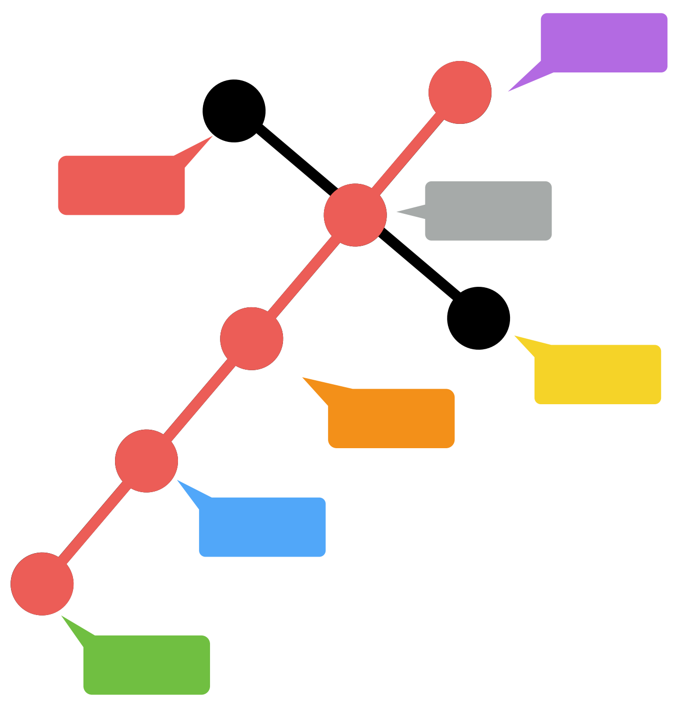


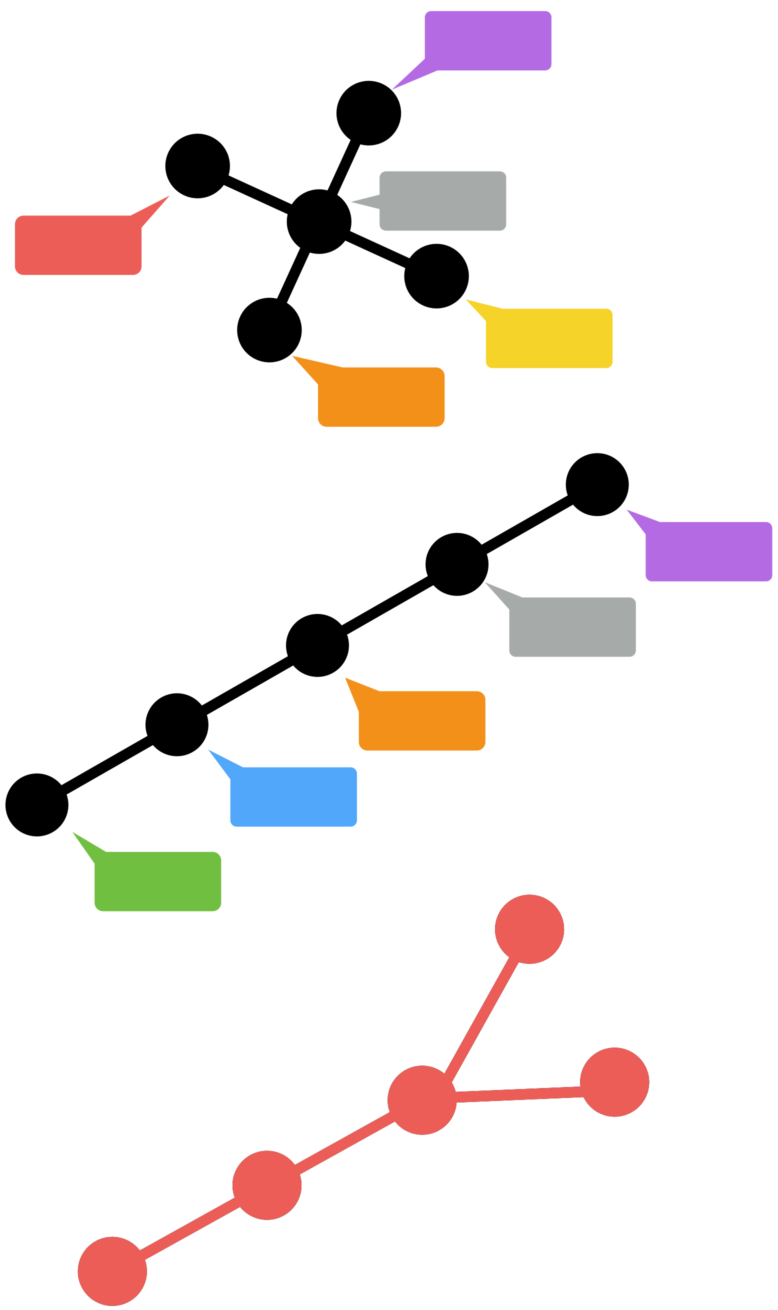


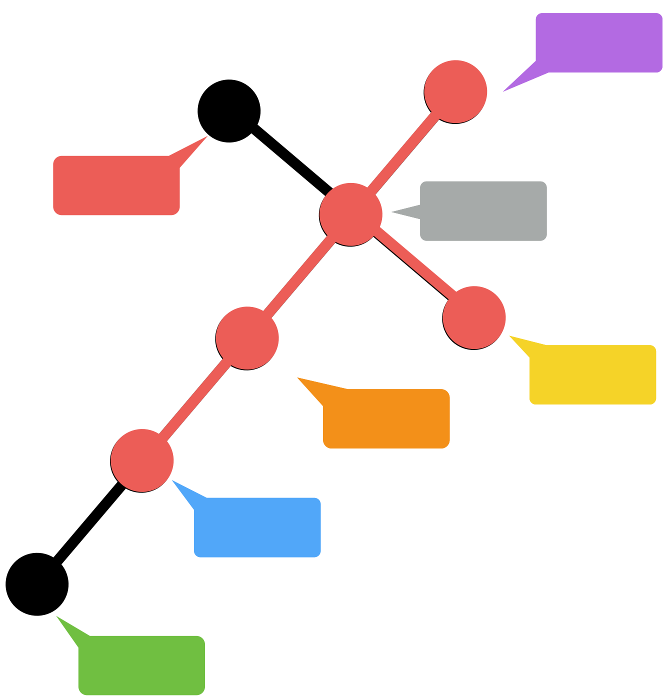
Exact Distance <= **Universal Tree**



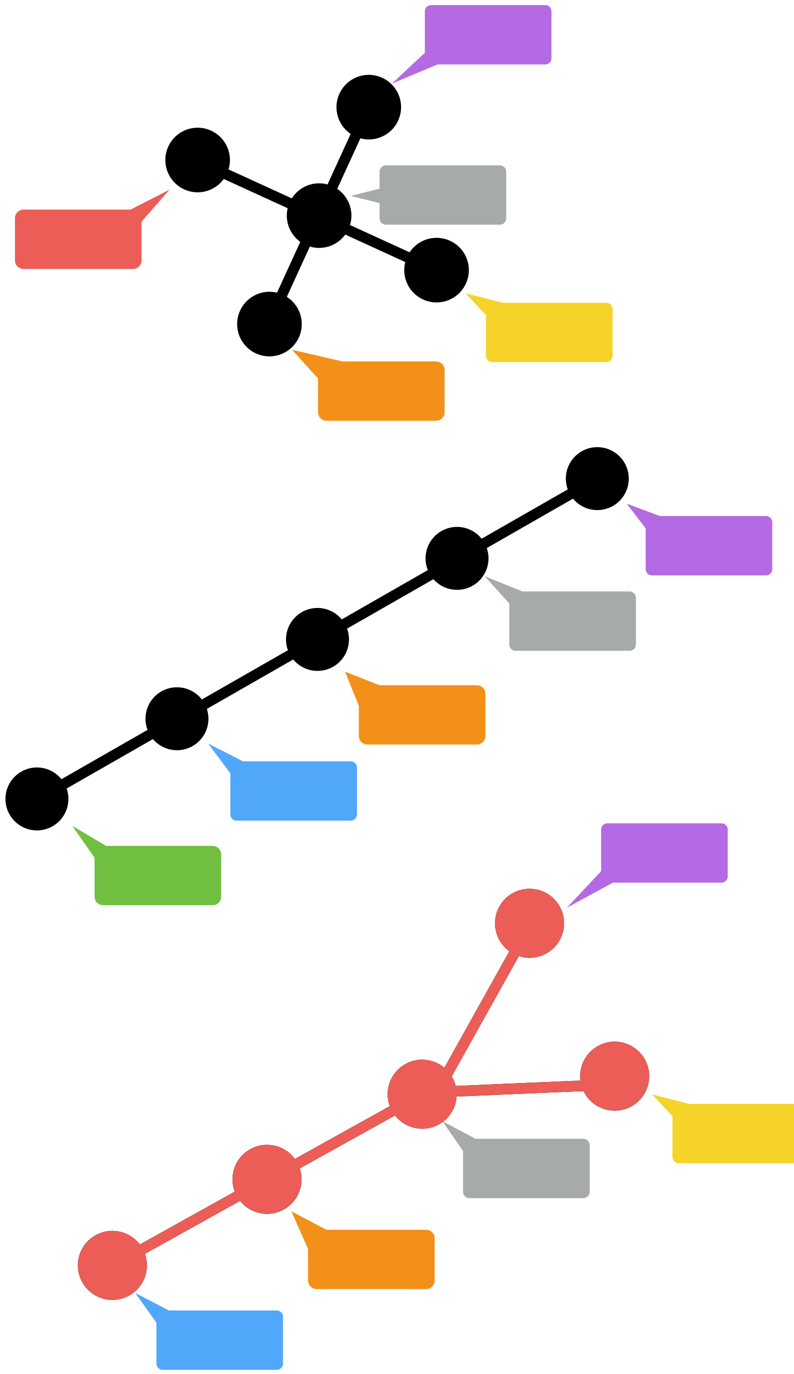


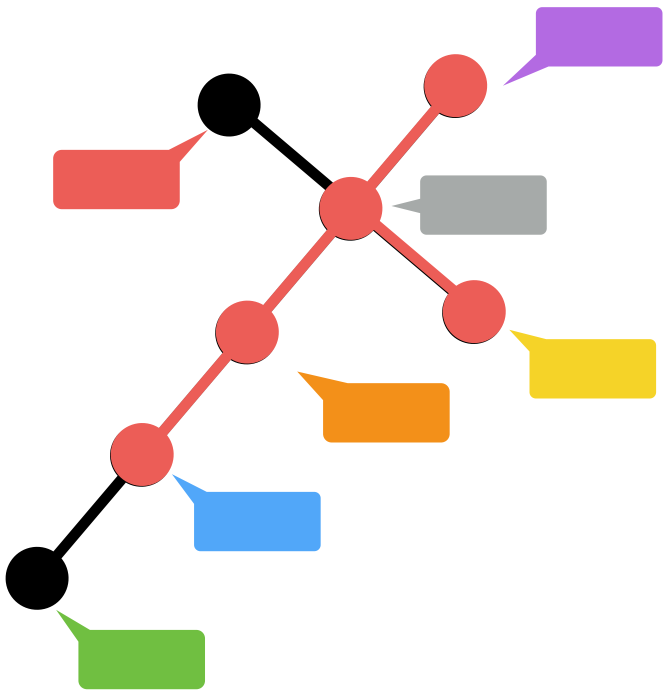
Exact Distance <= Universal Tree



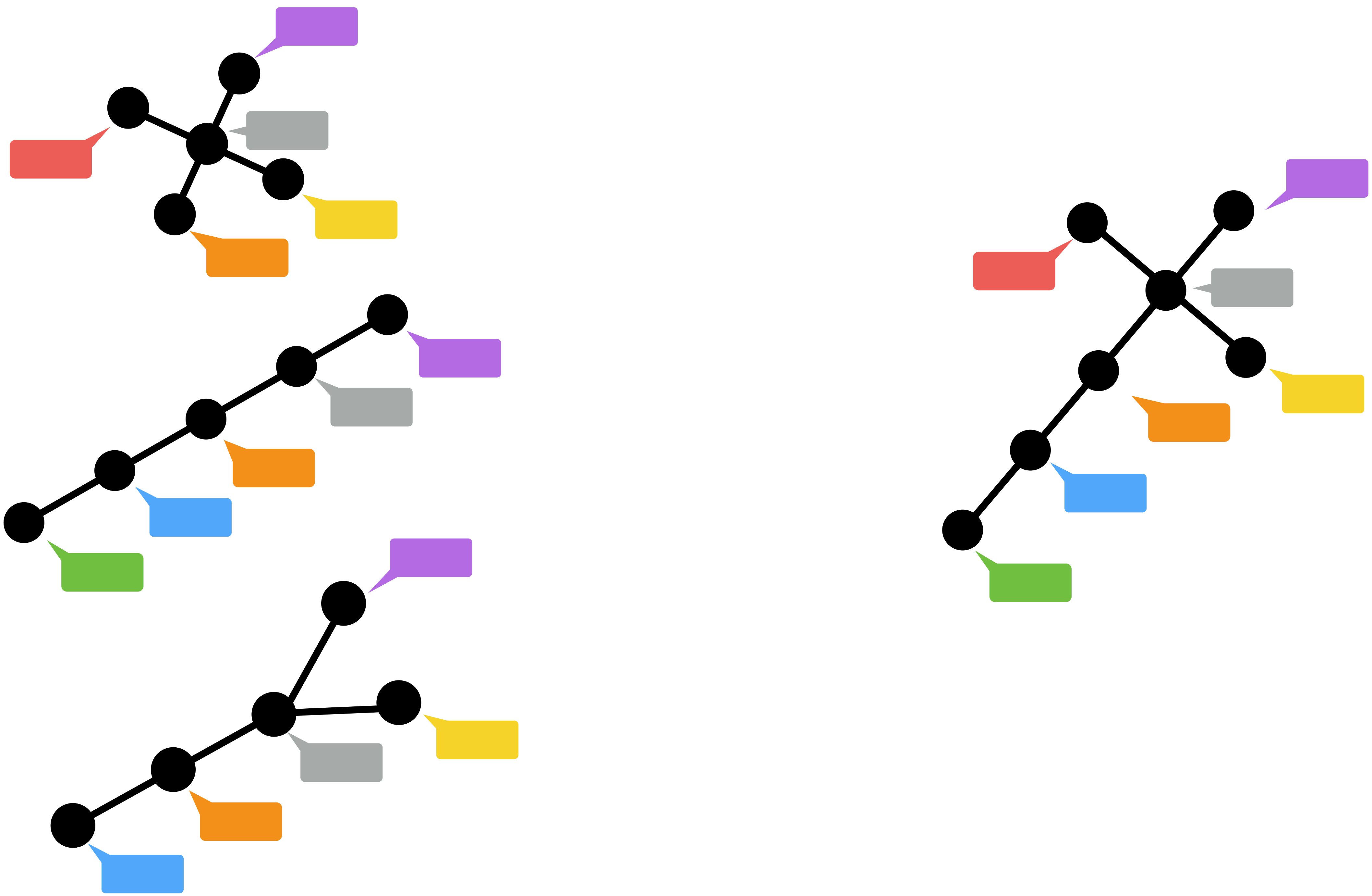


Exact Distance <= Universal Tree

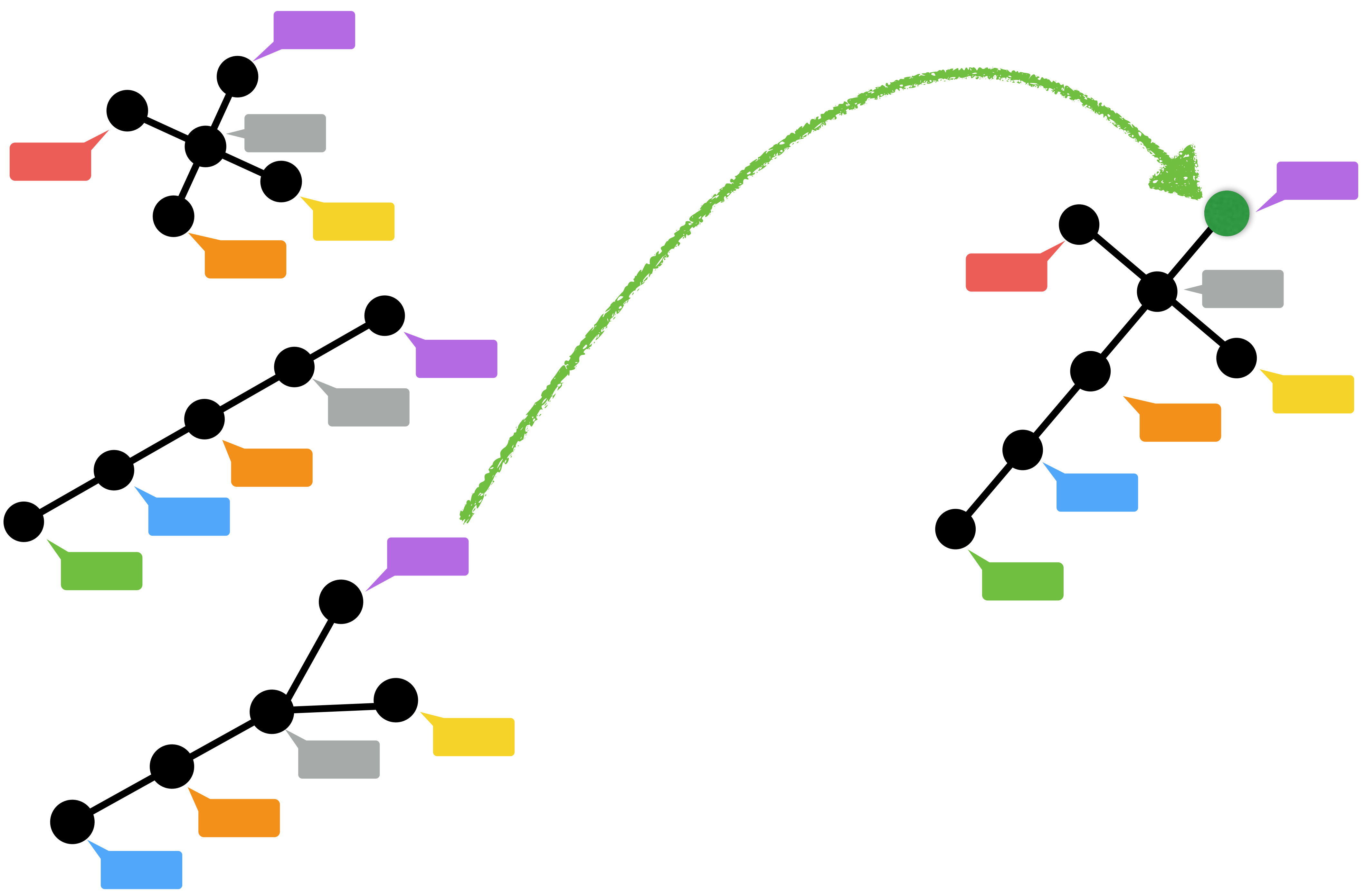




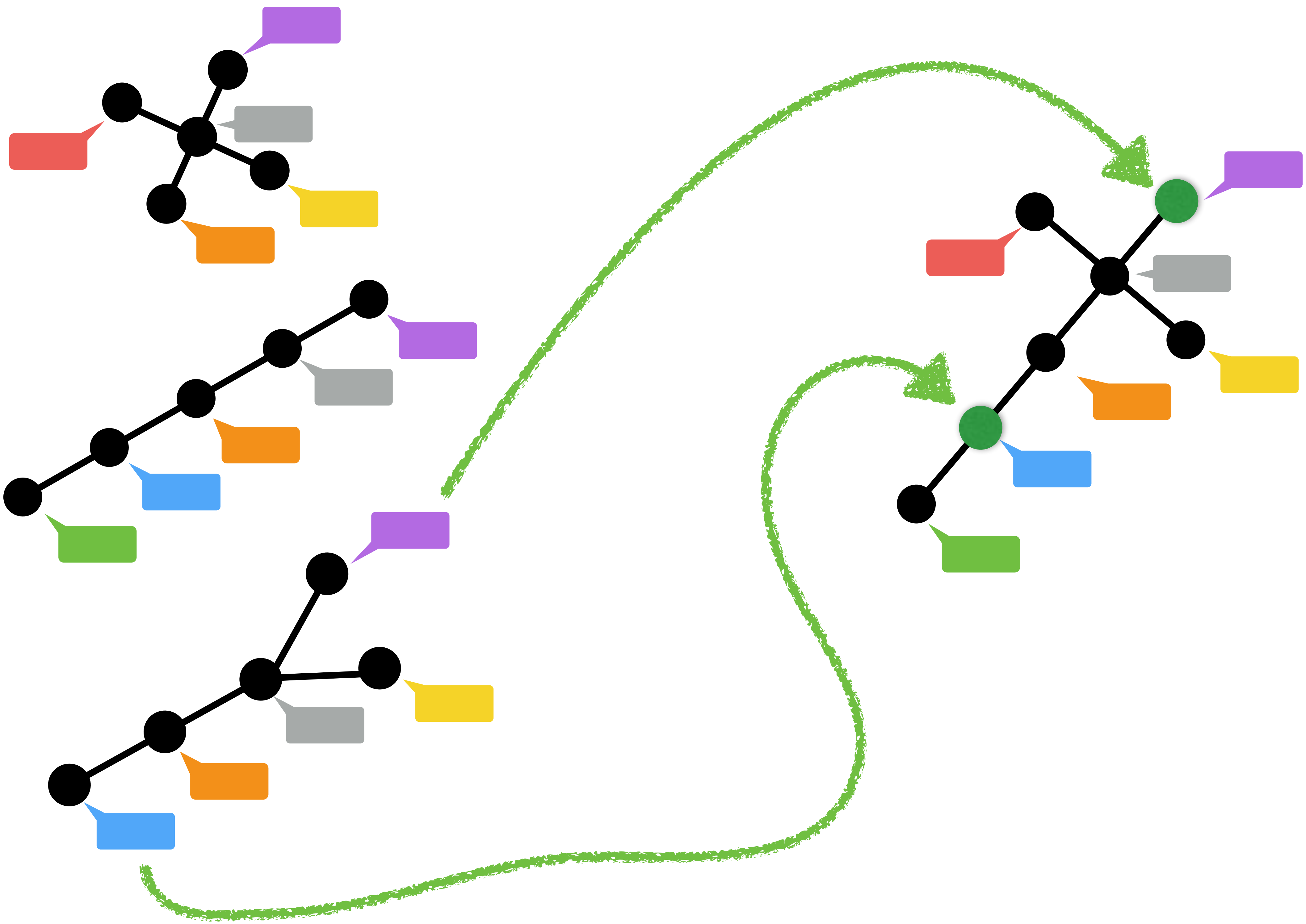
Exact Distance <= Universal Tree



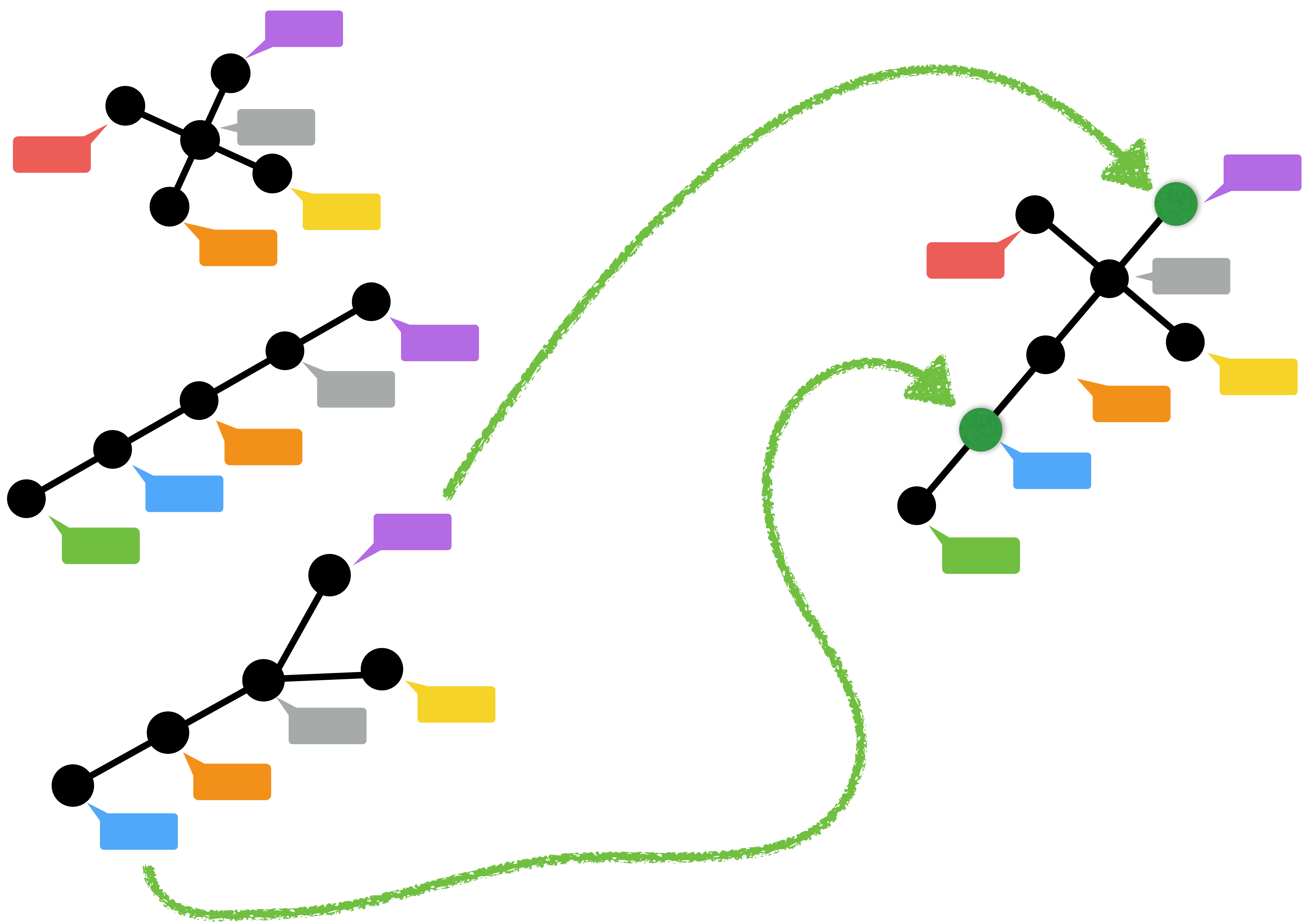
Exact Distance <= Universal Tree



Exact Distance <= **Universal Tree**



Exact Distance <= **Universal Tree**



Exact Distance <= **Universal Tree**

Exact Distance <= **Universal Tree**

Exact Distance <= **Universal Tree**

upper:

Exact Distance <= **Universal Tree**

upper: $n^{\frac{1}{2}} \log n$

	Exact Distance	<=	**Universal Tree**
upper:	$\frac{1}{2}\log^2 n$		$n^{\frac{1}{2}}\log n$

Exact Distance => **Universal Tree**

upper: $\frac{1}{2}\log^2 n$ $n^{\frac{1}{2}\log n}$

Exact Distance ?=> **Universal Tree**

upper: $\frac{1}{2}\log^2 n$ $n^{\frac{1}{2}}\log n$

Exact Distance ? => **Universal Tree**

upper: $\frac{1}{2}\log^2 n$ $n^{\frac{1}{2}}\log n$

lower:

Exact Distance ? => **Universal Tree**

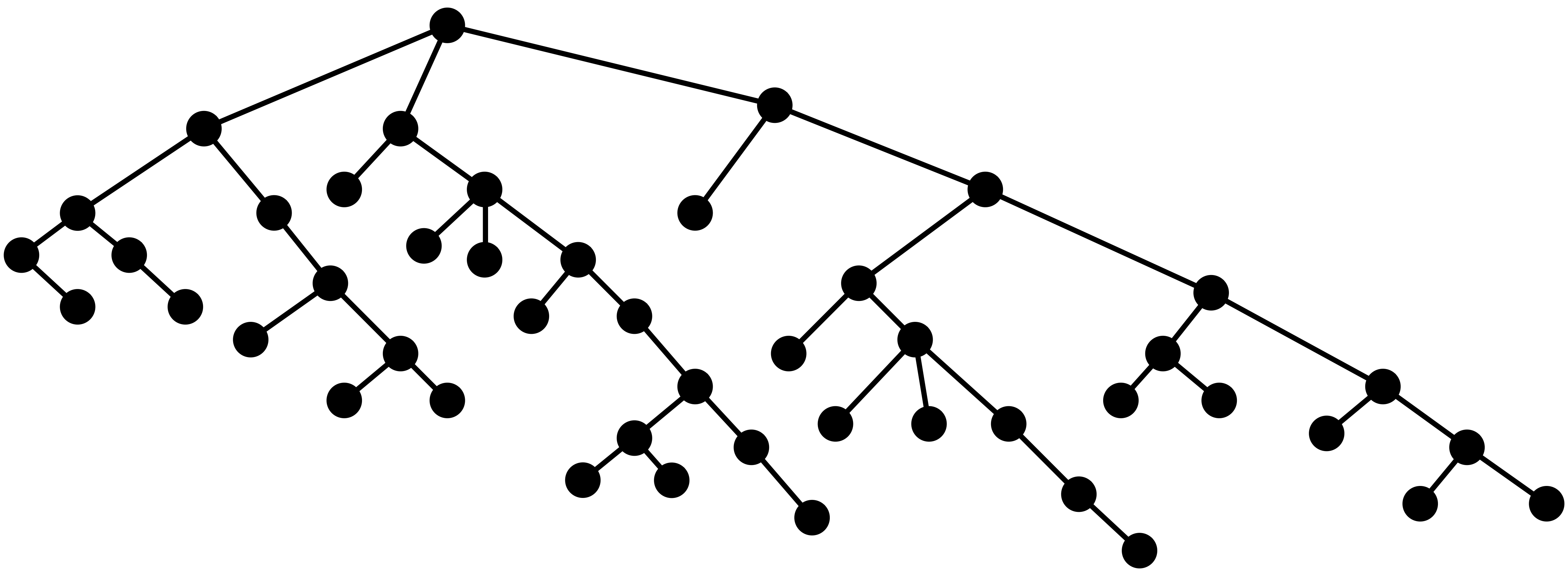
upper: $\frac{1}{2}\log^2 n$ $n^{\frac{1}{2}}\log n$

lower: $n^{\frac{1}{2}}\log n$

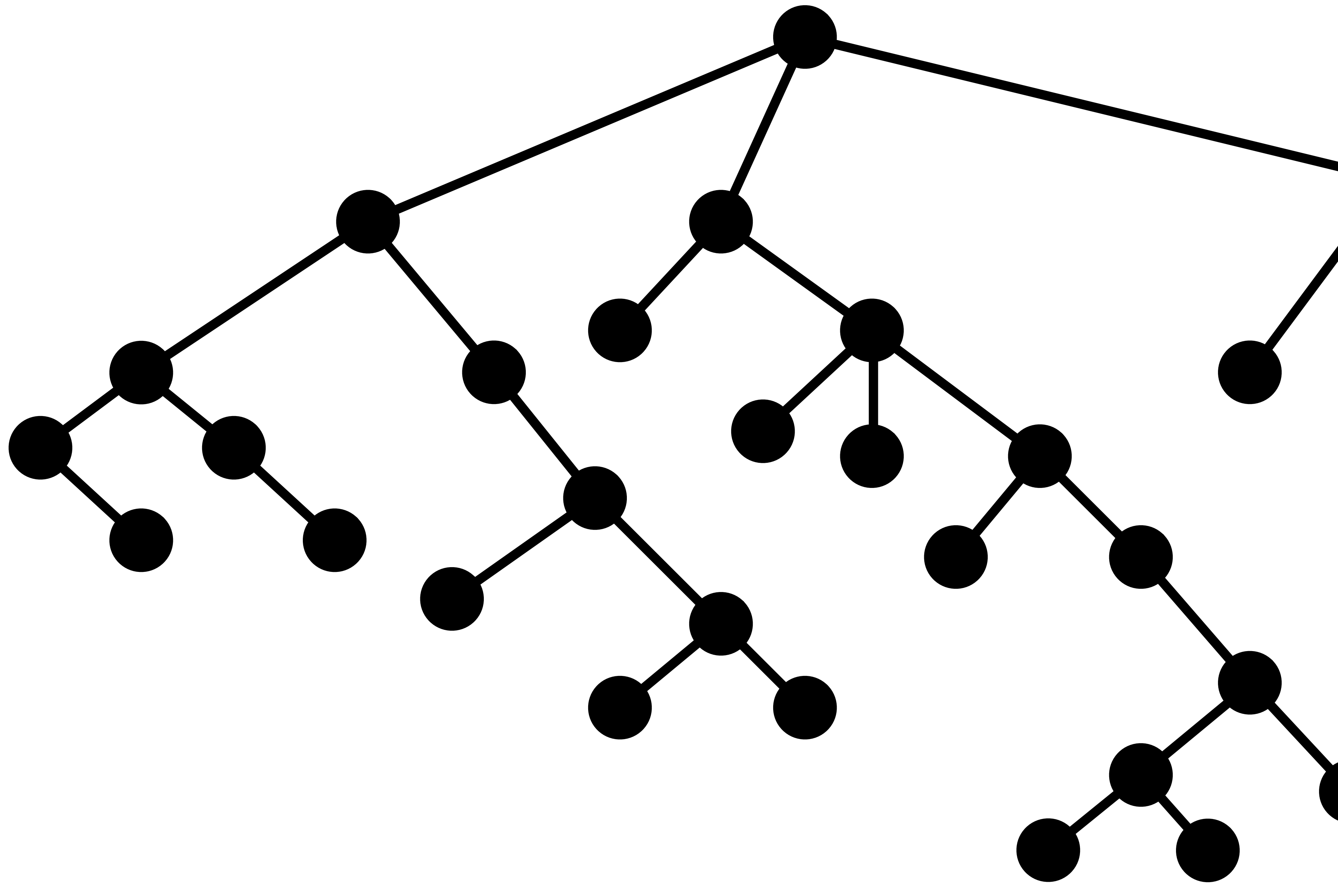
Exact Distance	? =>	**Universal Tree**
upper: $\frac{1}{2}\log^2 n$		$n^{\frac{1}{2}}\log n$
lower: $\frac{1}{4}\log^2 n$		$n^{\frac{1}{2}}\log n$

Level-Ancestor

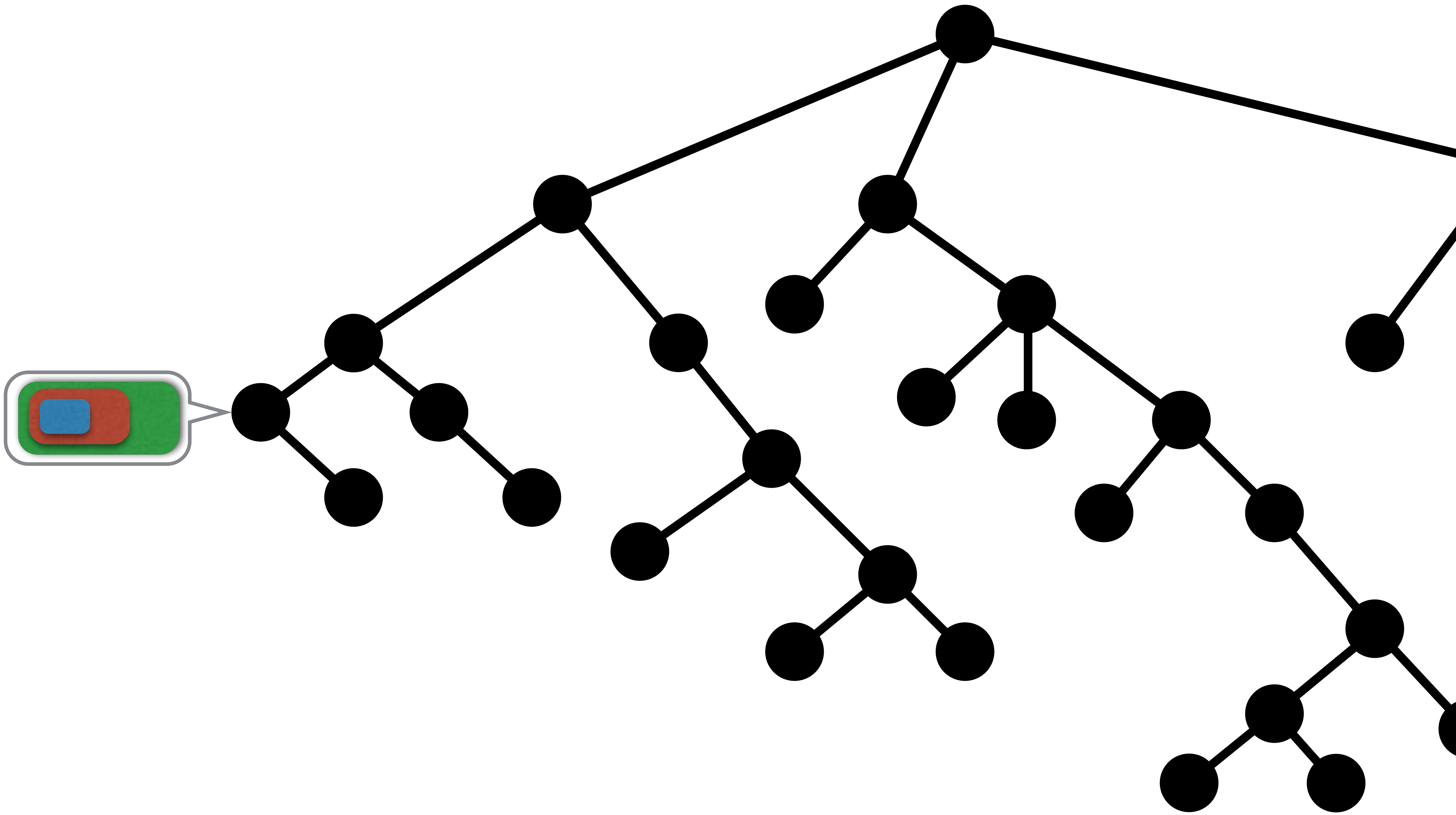
Level-Ancestor



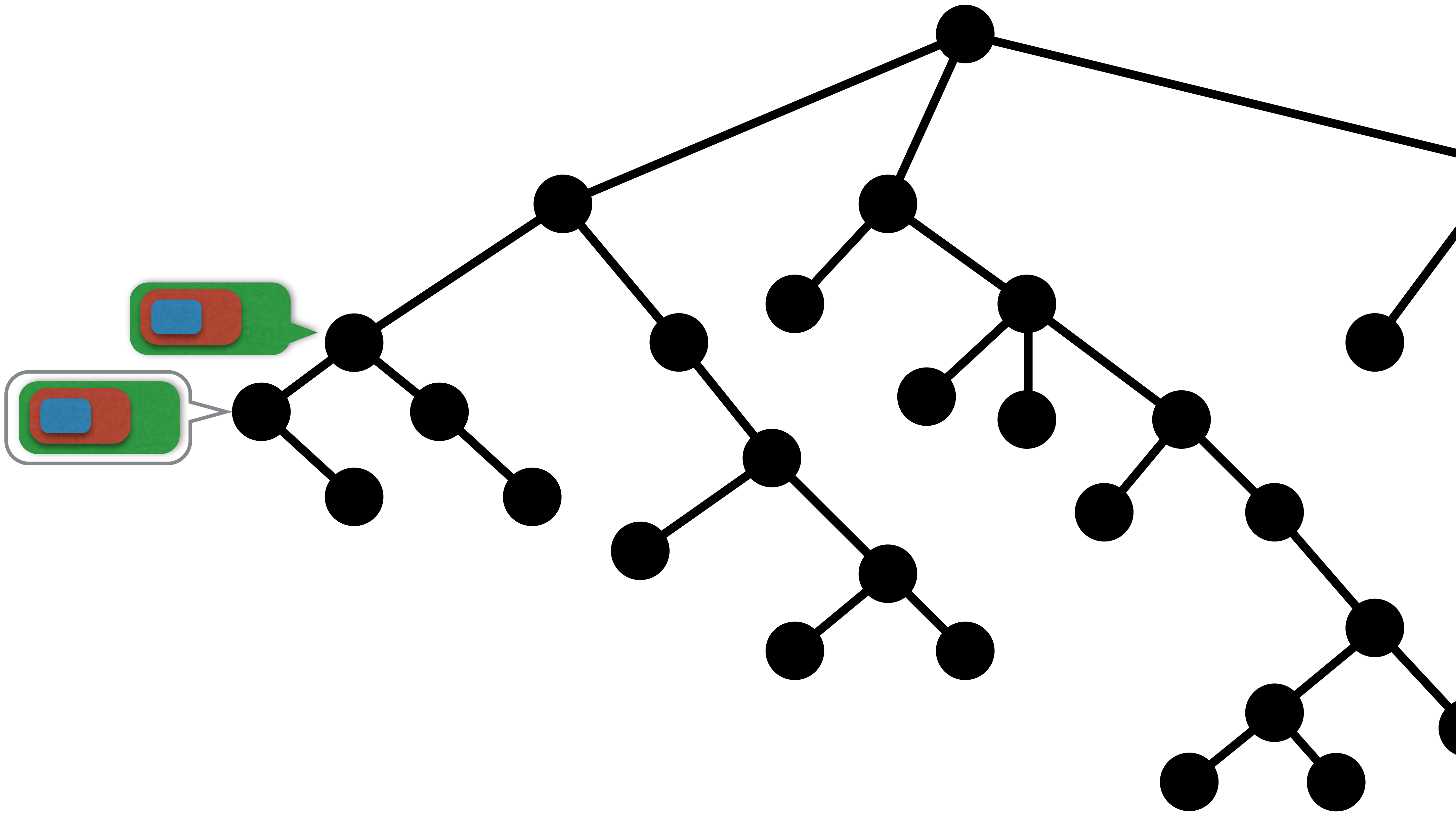
Level-Ancestor



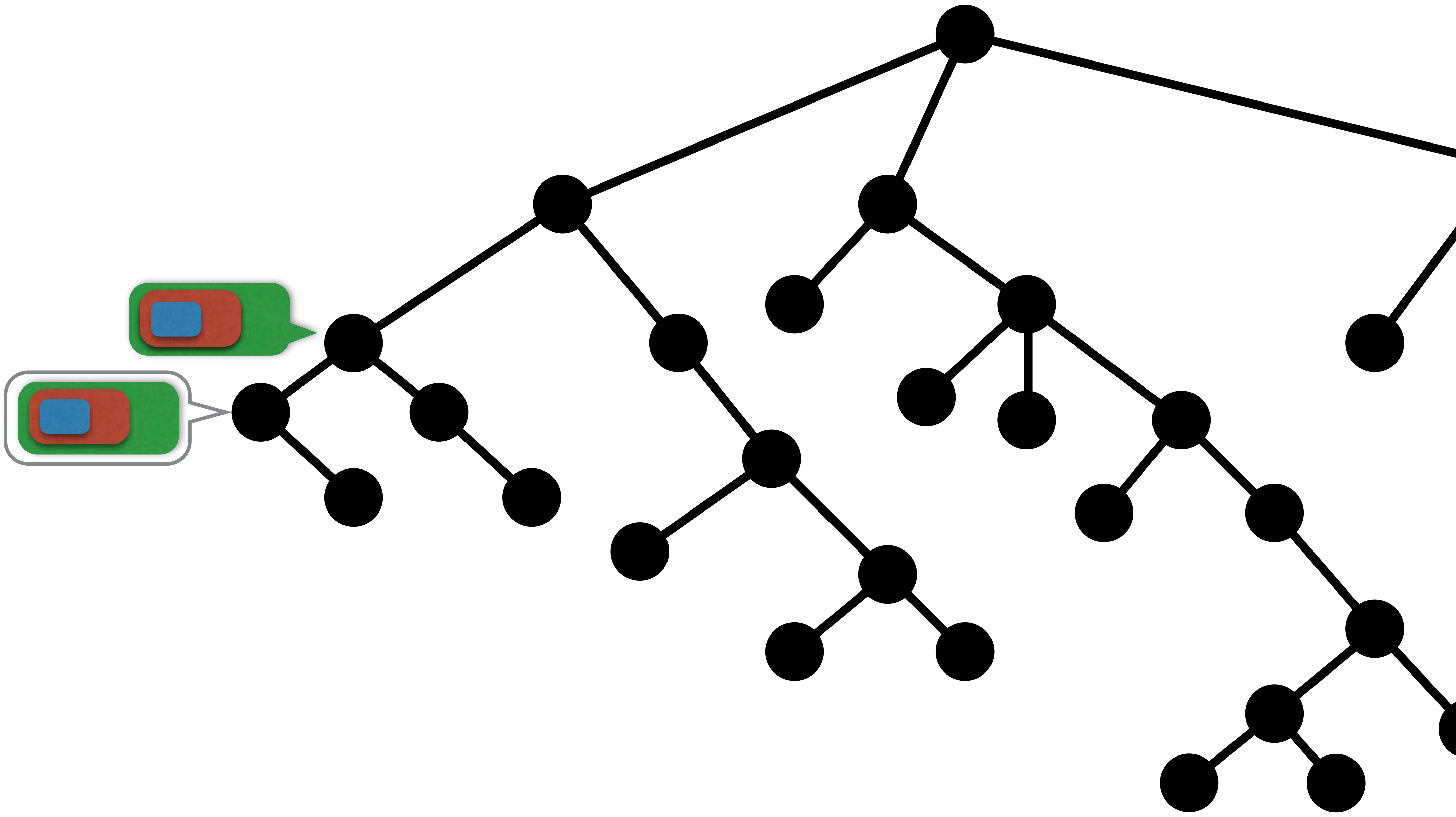
Level-Ancestor



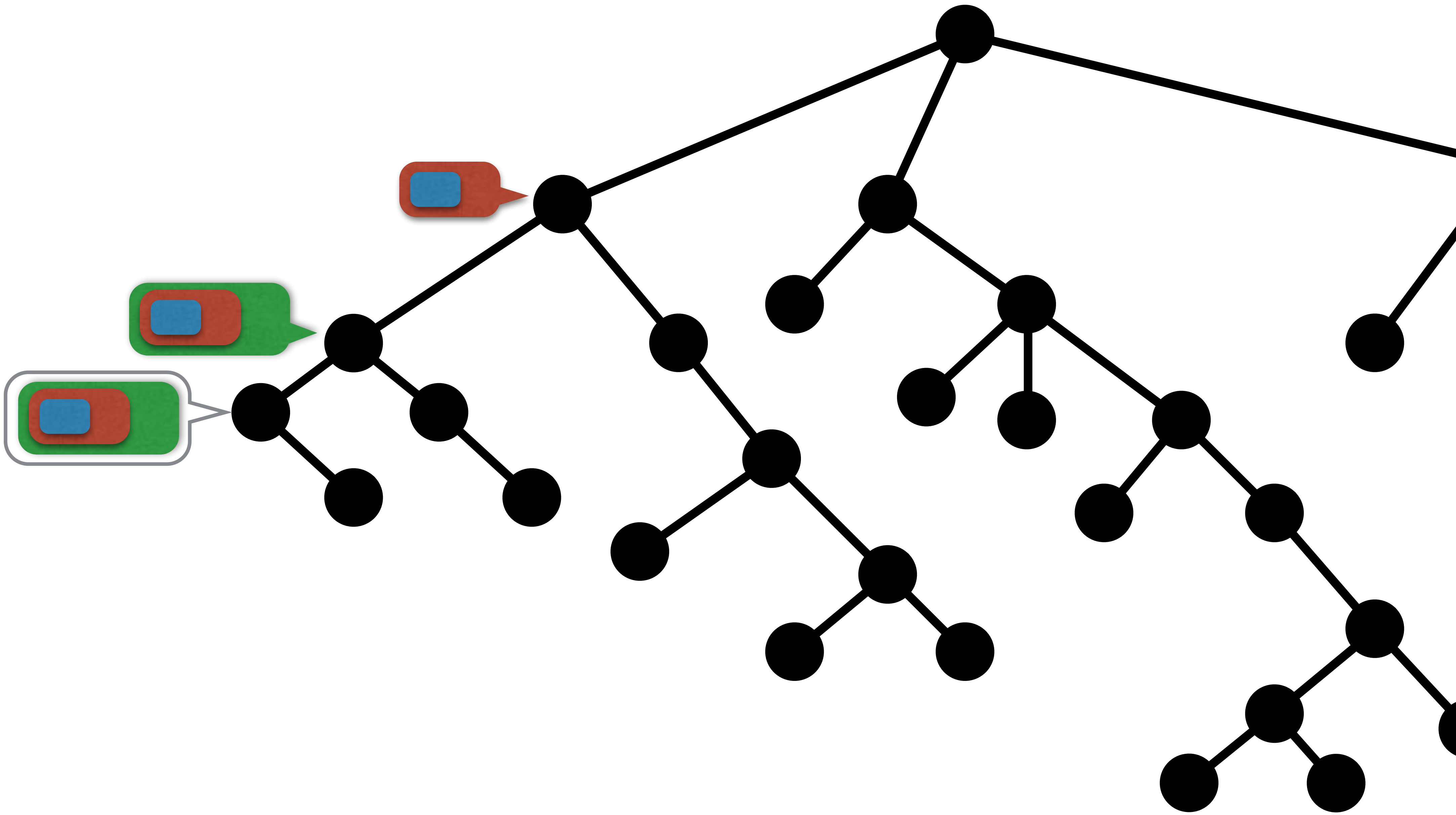
Level-Ancestor



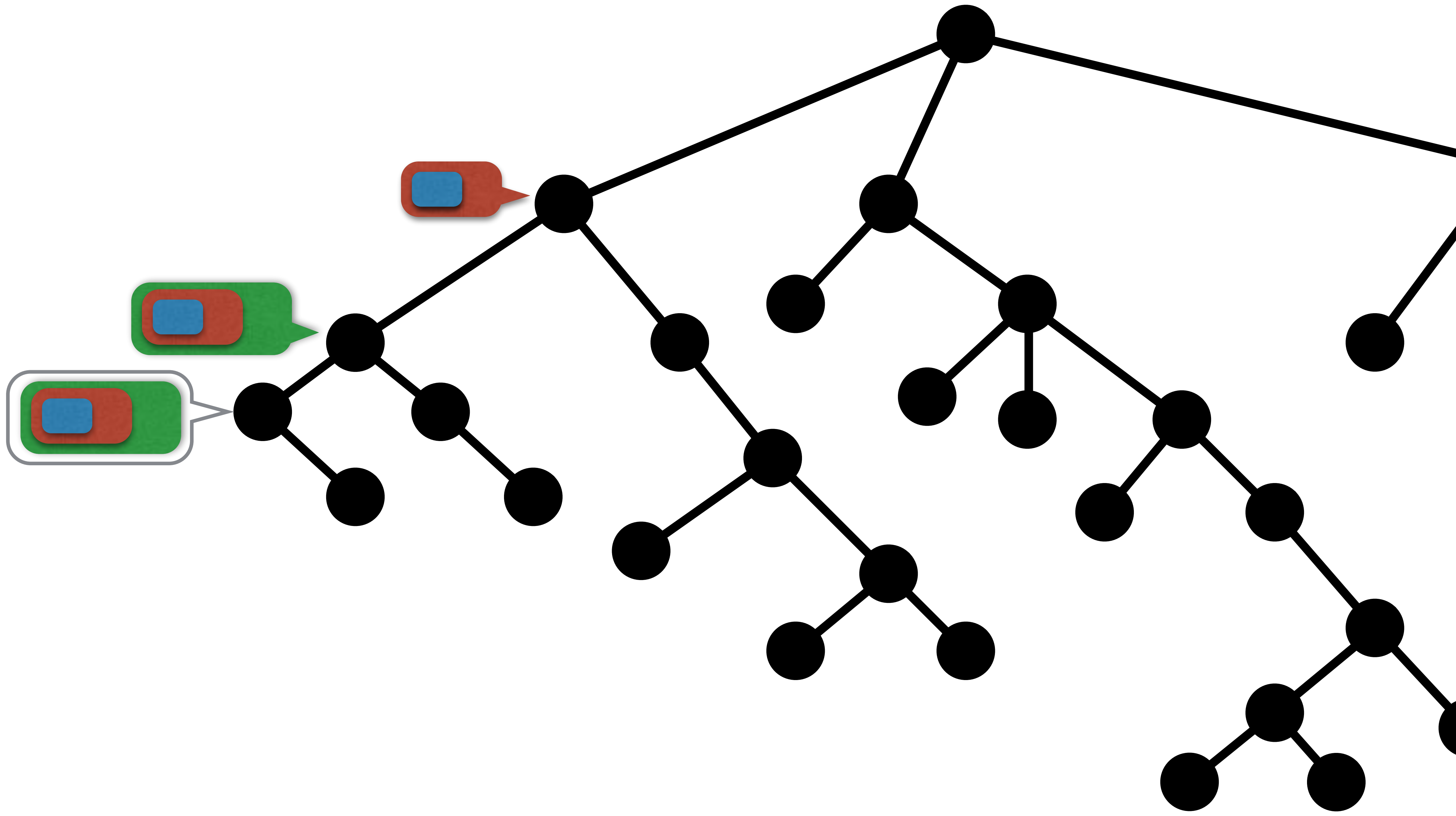
Level-Ancestor



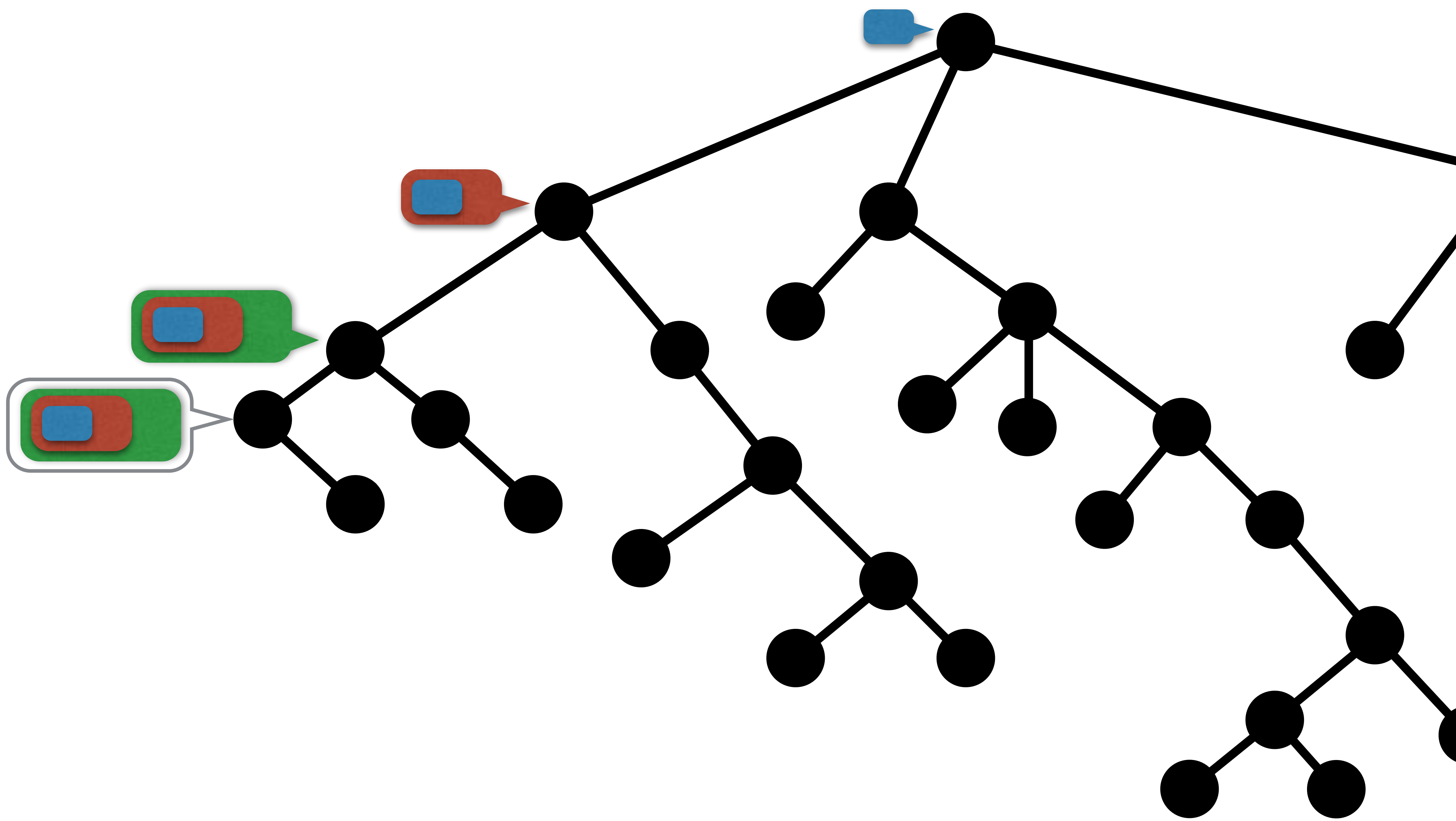
Level-Ancestor



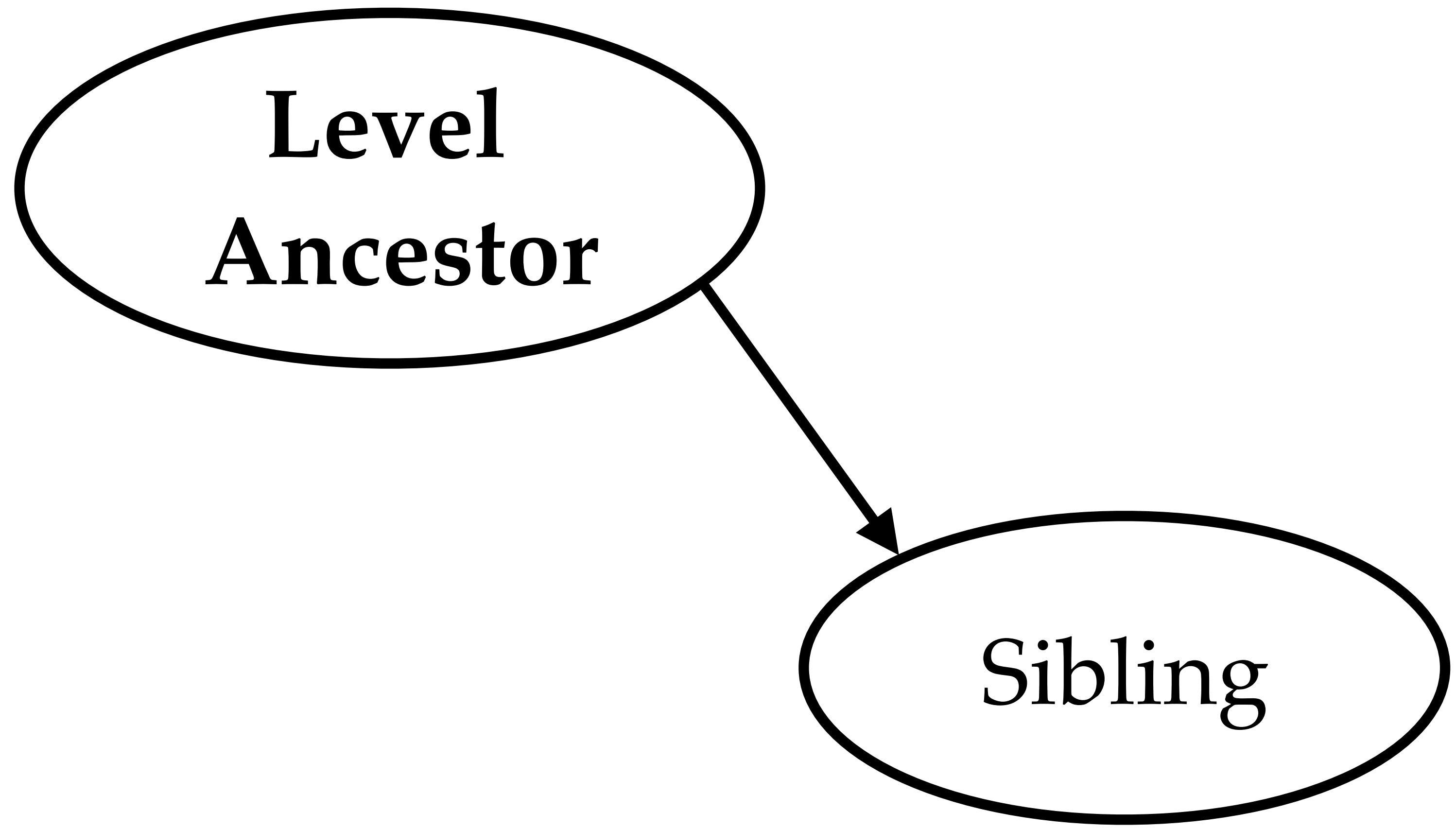
Level-Ancestor

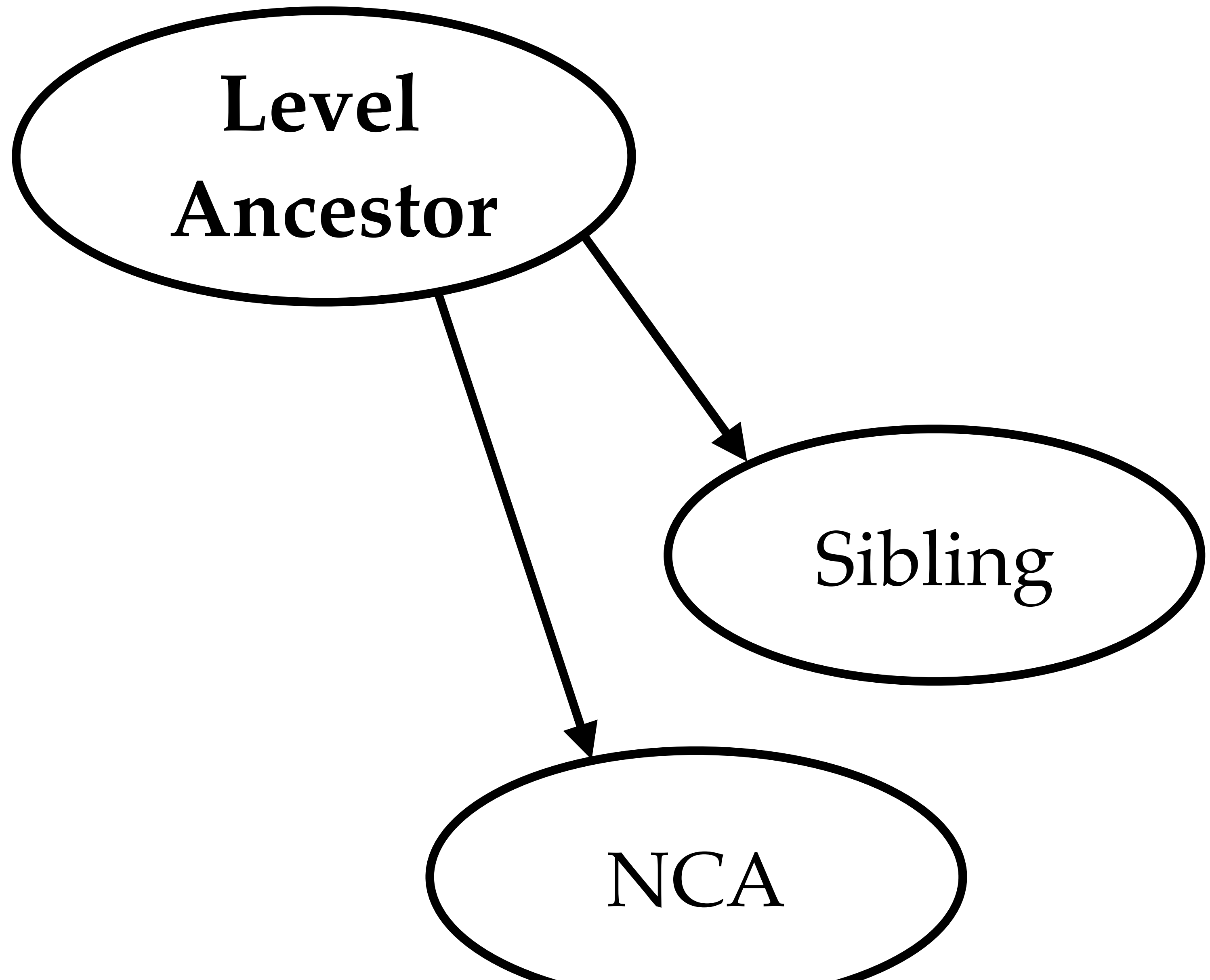


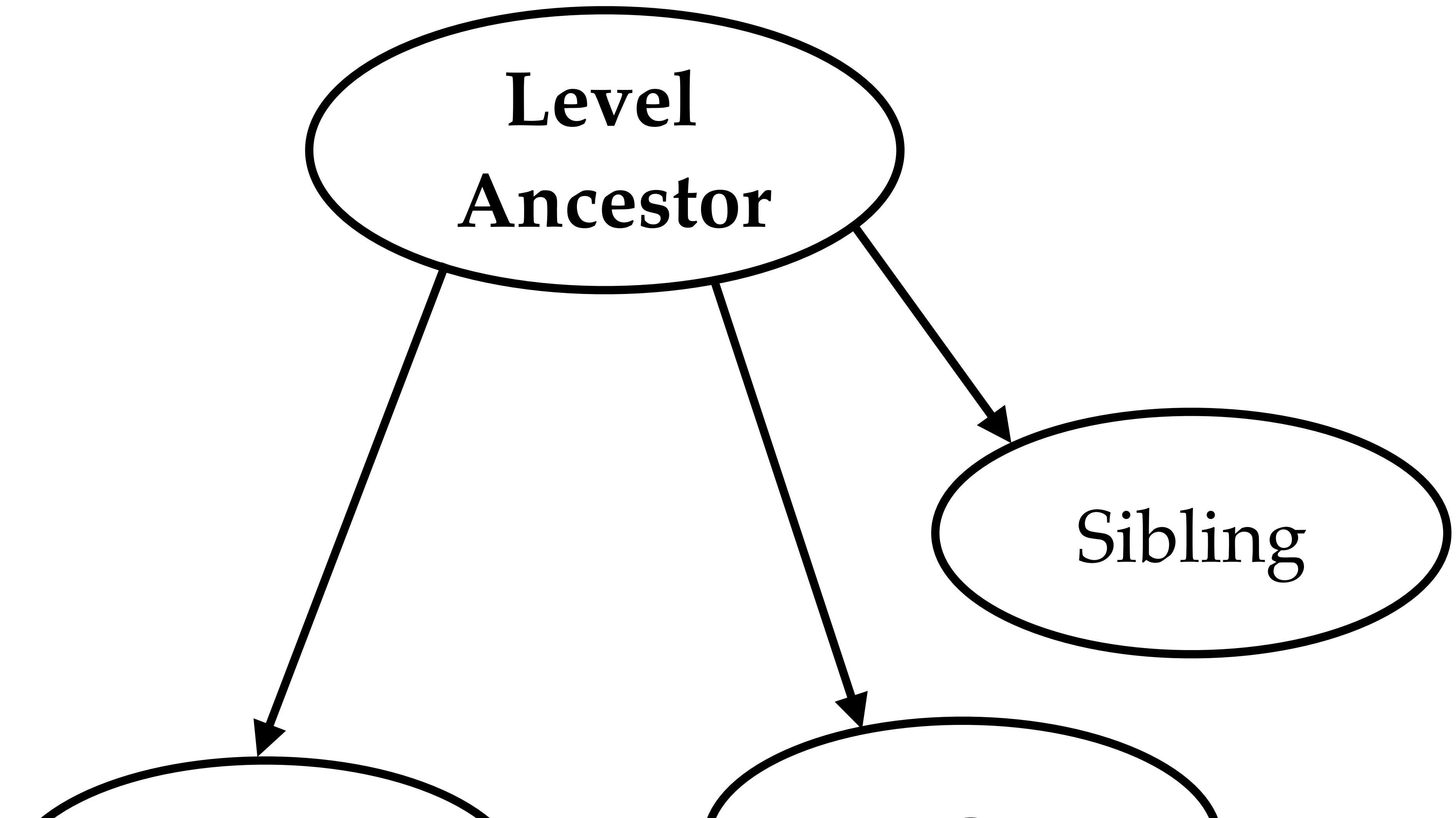
Level-Ancestor

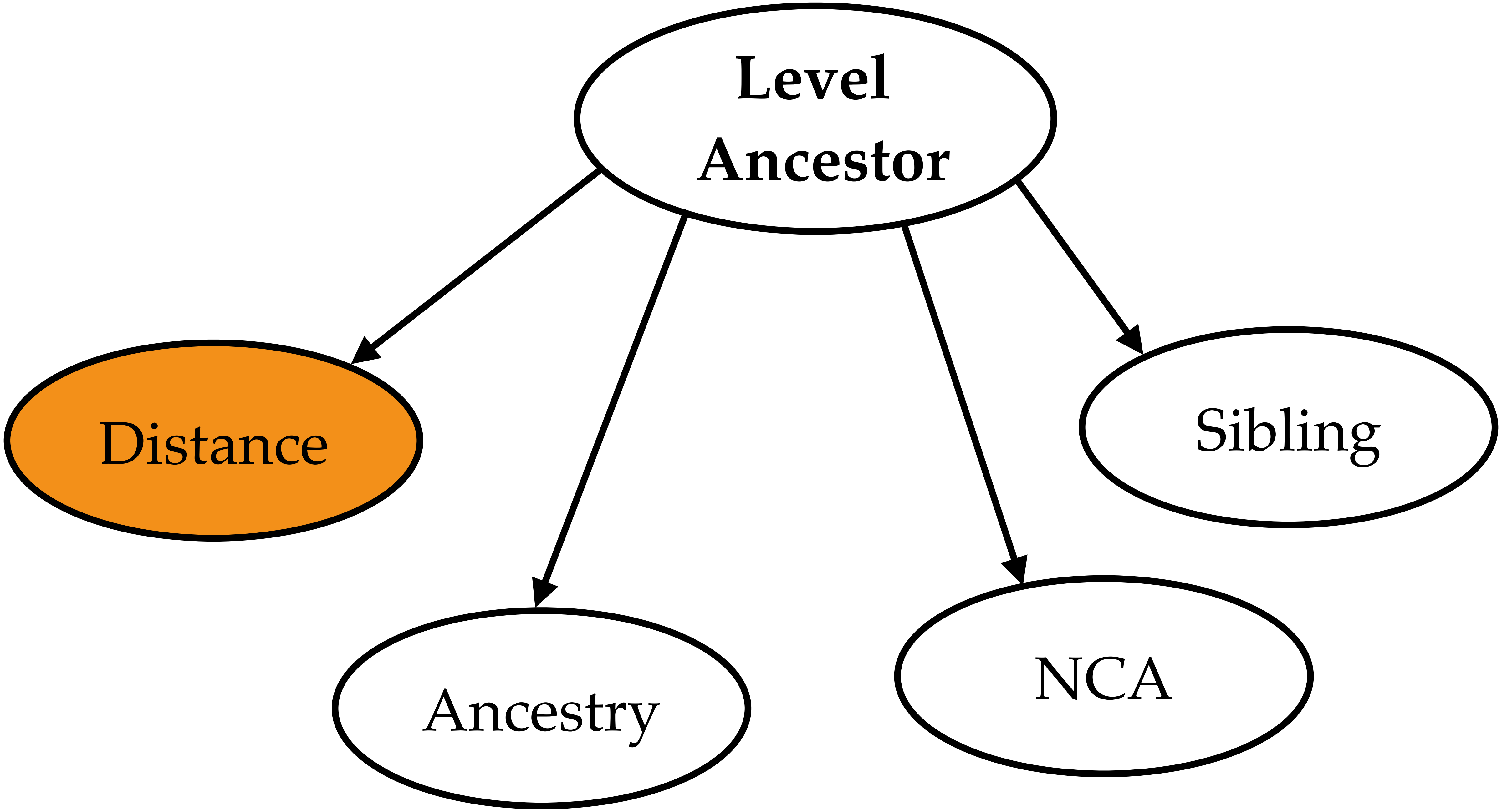


Level Ancestor


Level Ancestor
Sibling


Level Ancestor
Sibling
NCA


Level
Ancestor
Sibling
NCA
Ancestry



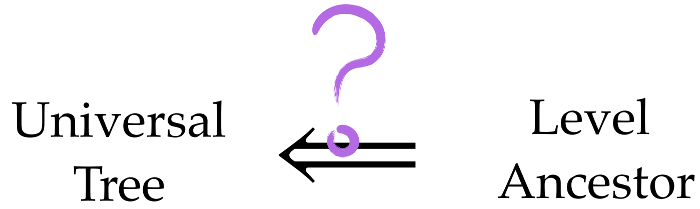
Level Ancestor
Distance
Ancestry
NCA
Sibling

Universal Tree $\Longrightarrow$ Level Ancestor

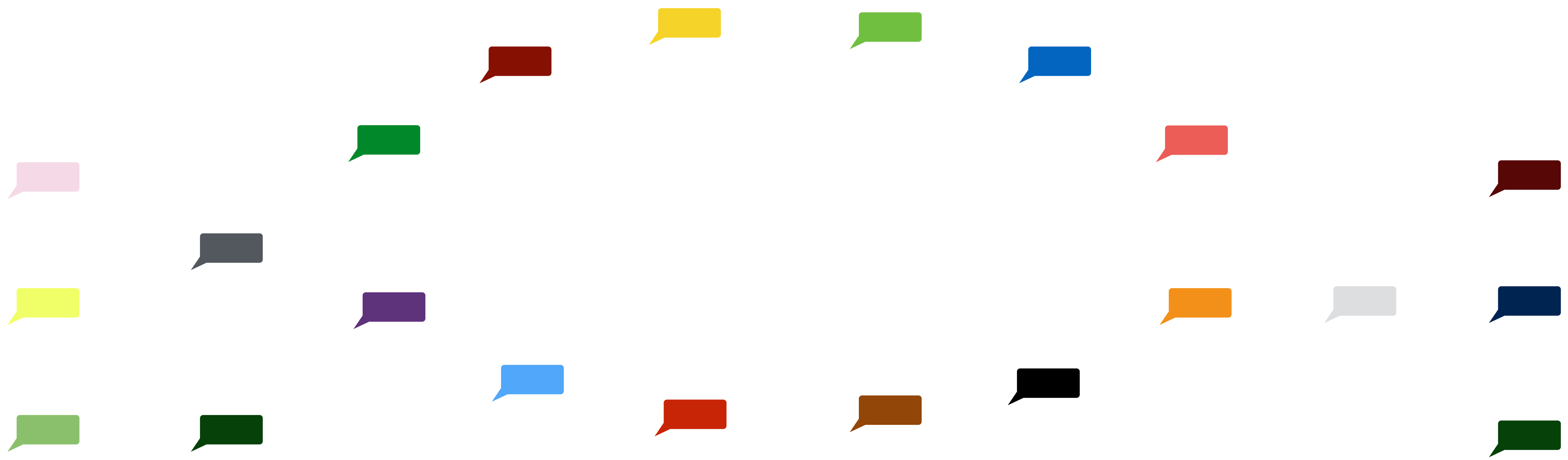
Universal Tree $\Rightarrow$ ✓ Level Ancestor

Universal Tree $\Longleftarrow$ Level Ancestor

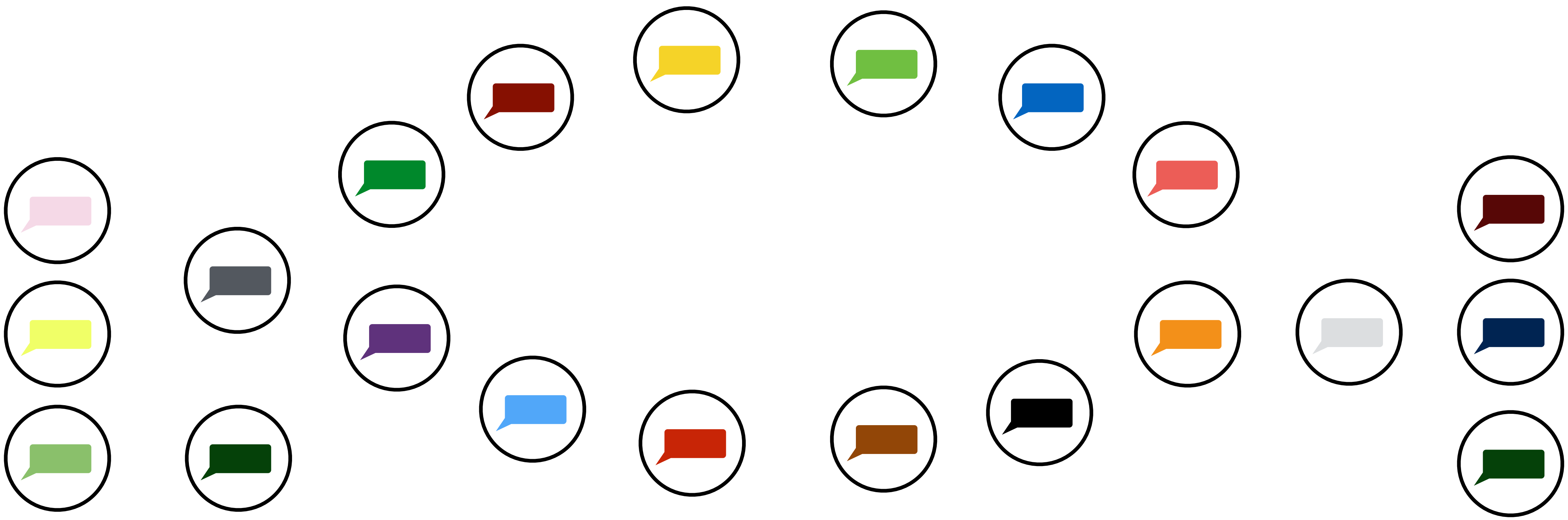
Universal Tree $\overset{?}{\Longleftarrow}$ Level Ancestor


Universal Tree
Level Ancestor

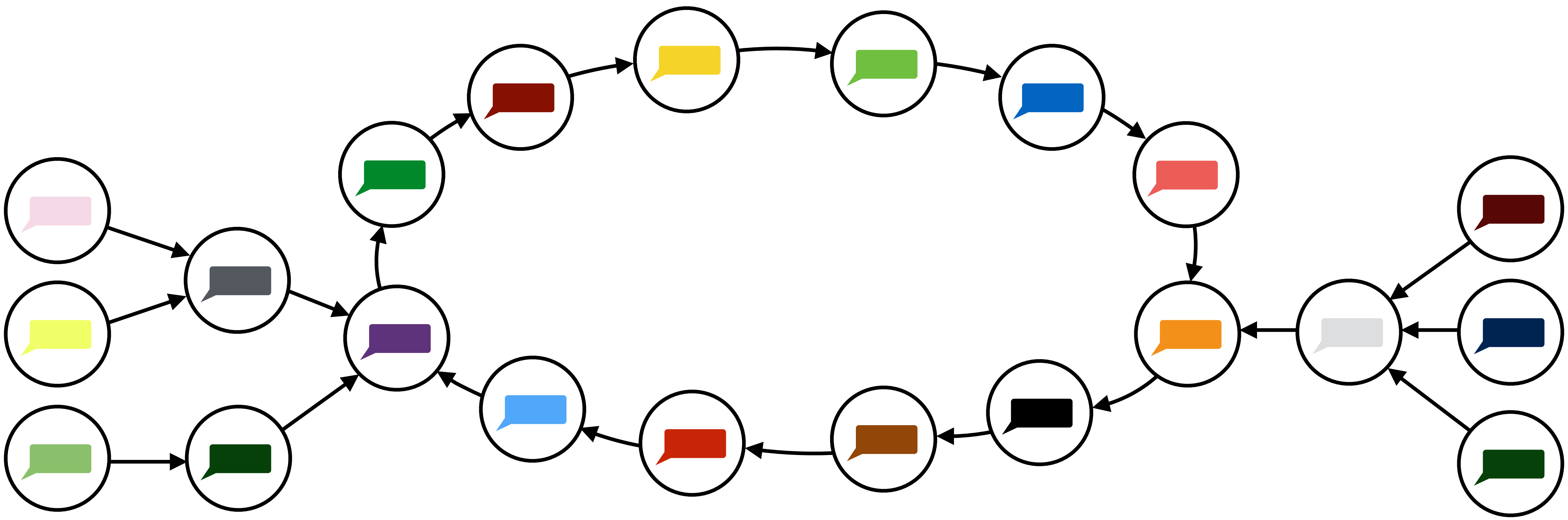
Universal Tree $\Leftarrow$? Level Ancestor



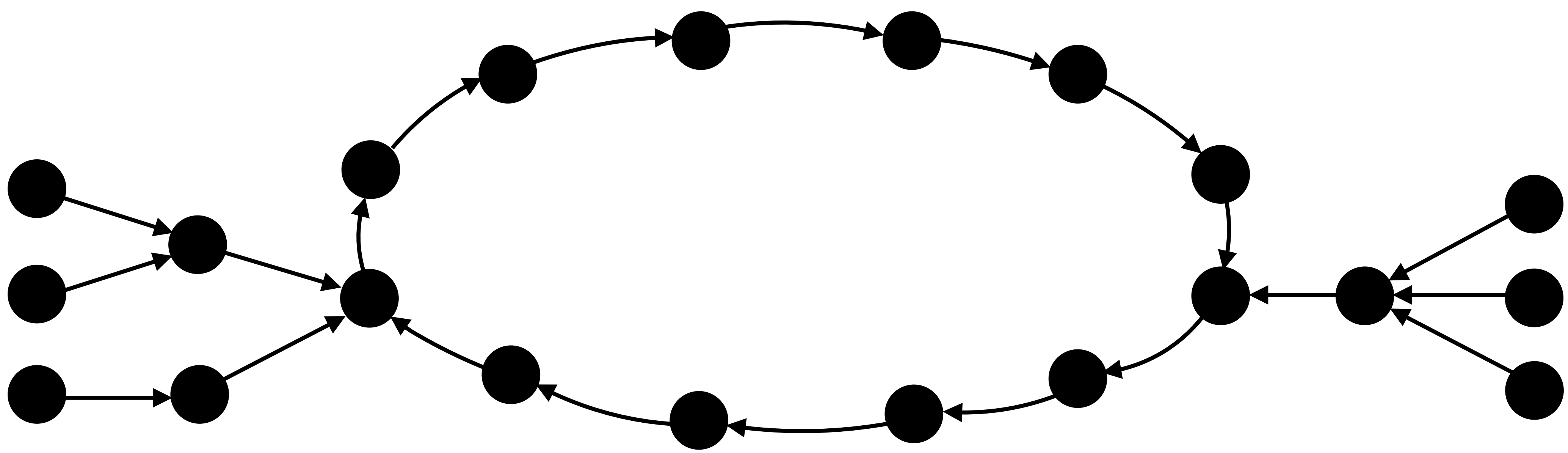
Universal Tree ⇐ Level Ancestor



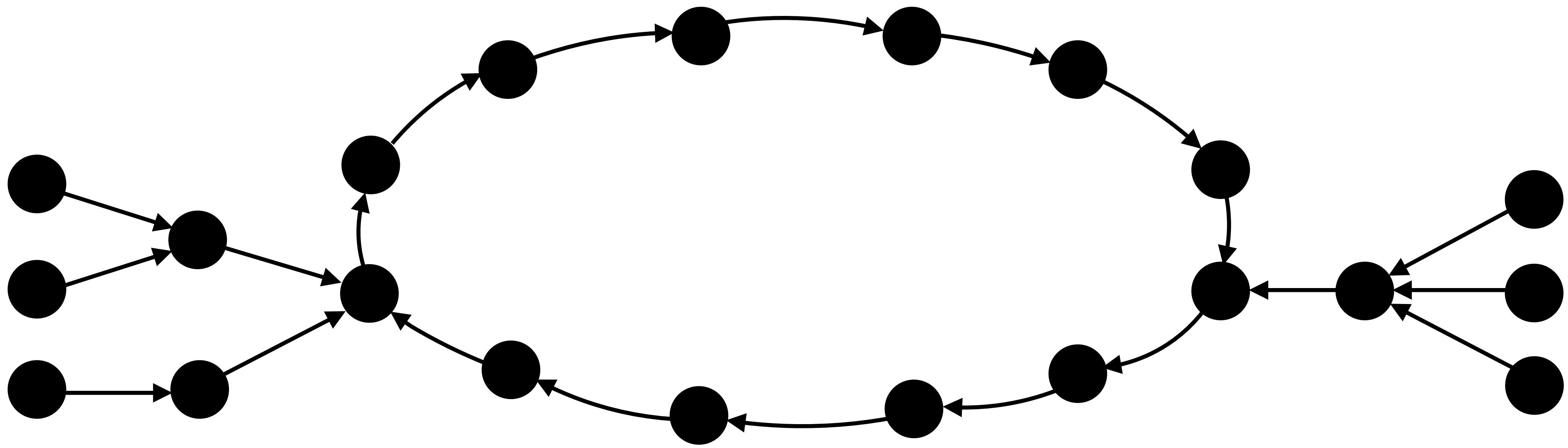
Universal Tree $\overset{?}{\Leftarrow}$ Level Ancestor



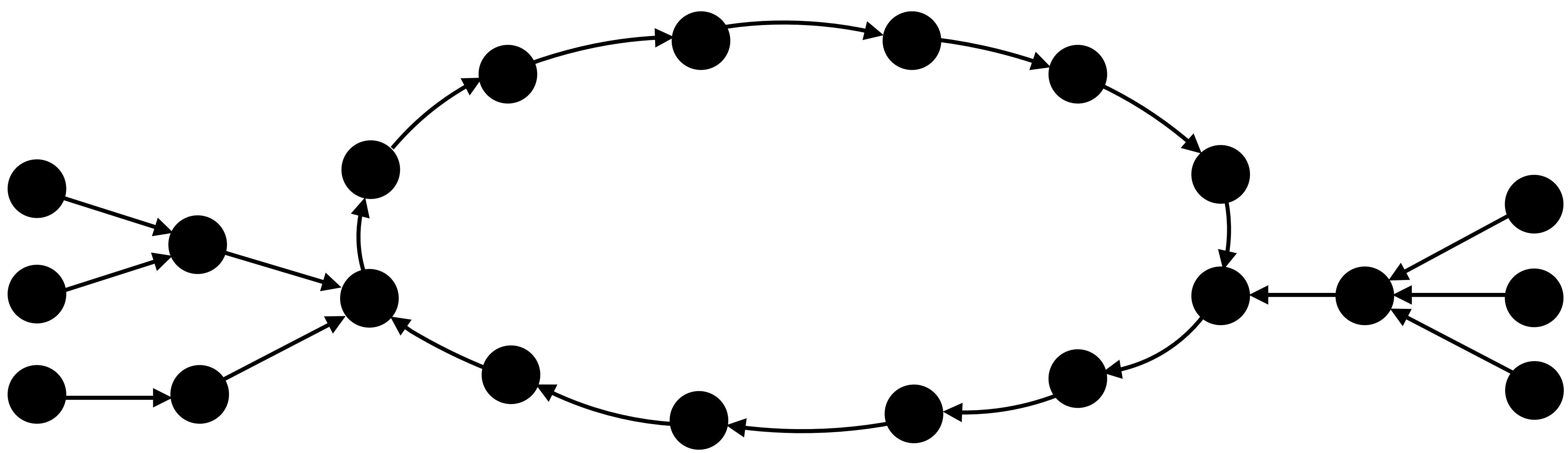
Universal Tree $\overset{?}{\Longleftarrow}$ Level Ancestor



Universal Tree ⟸ ? Level Ancestor

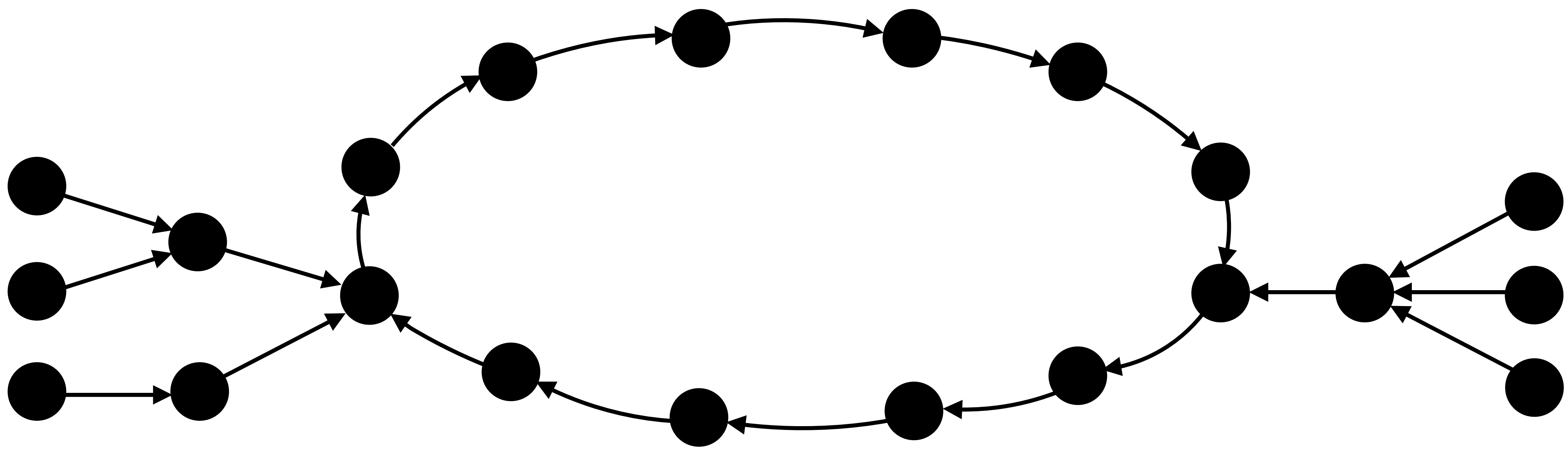


Universal Tree $\overset{?}{\Longleftarrow}$ Level Ancestor

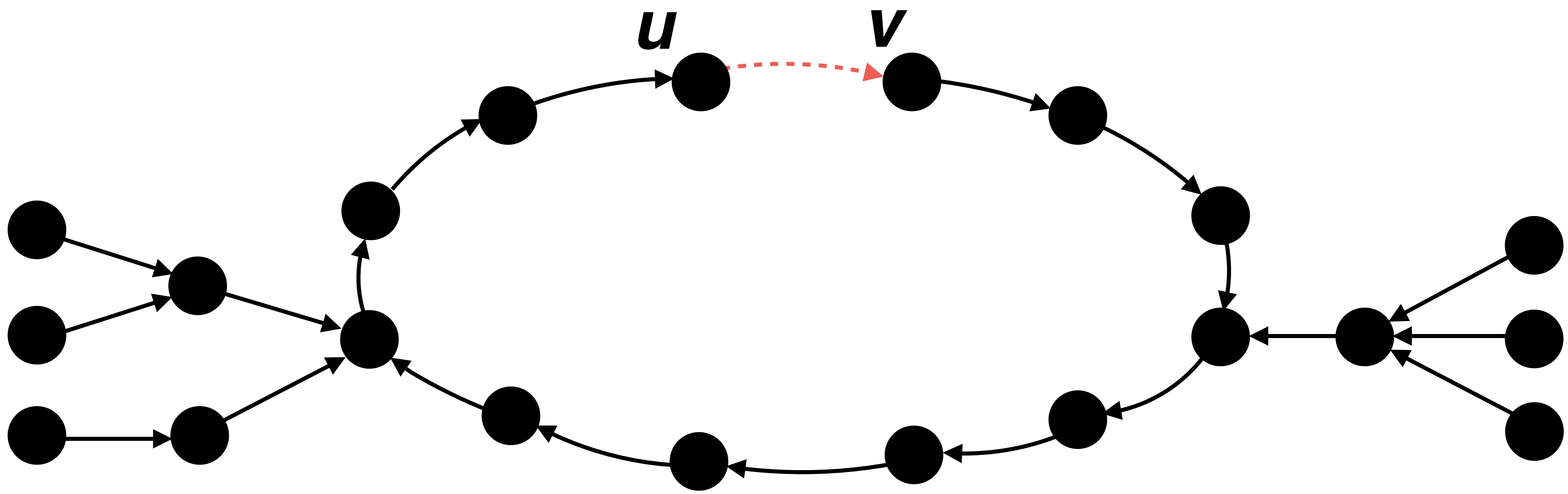


Unicyclical Graph

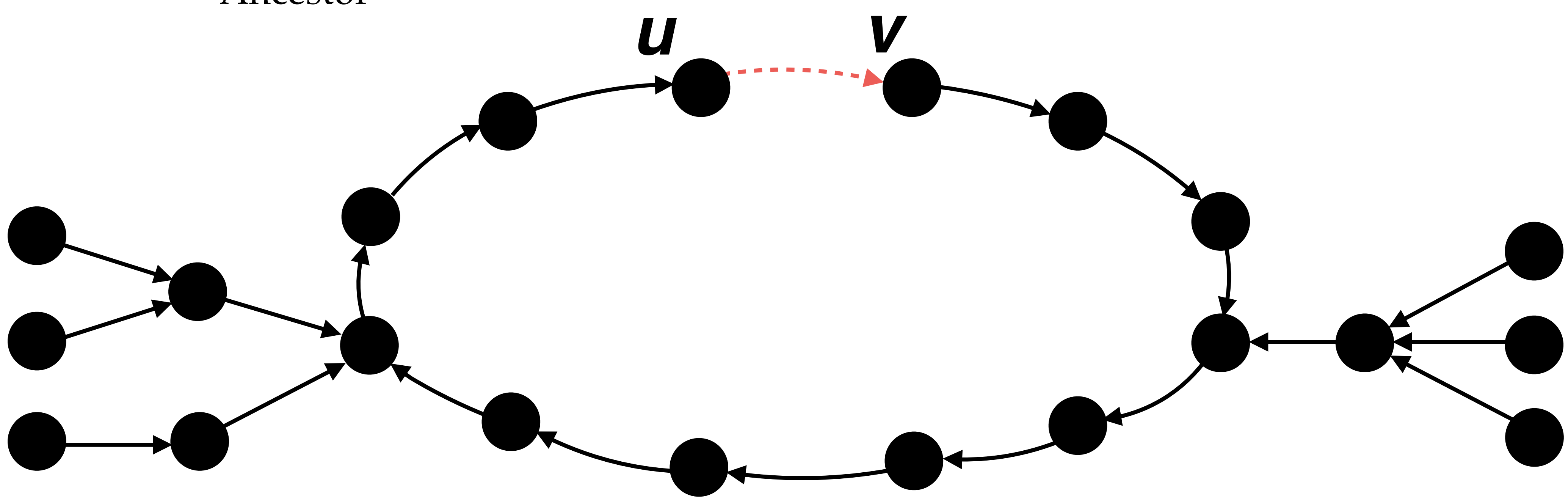
Universal Tree $\Leftarrow$ Level Ancestor

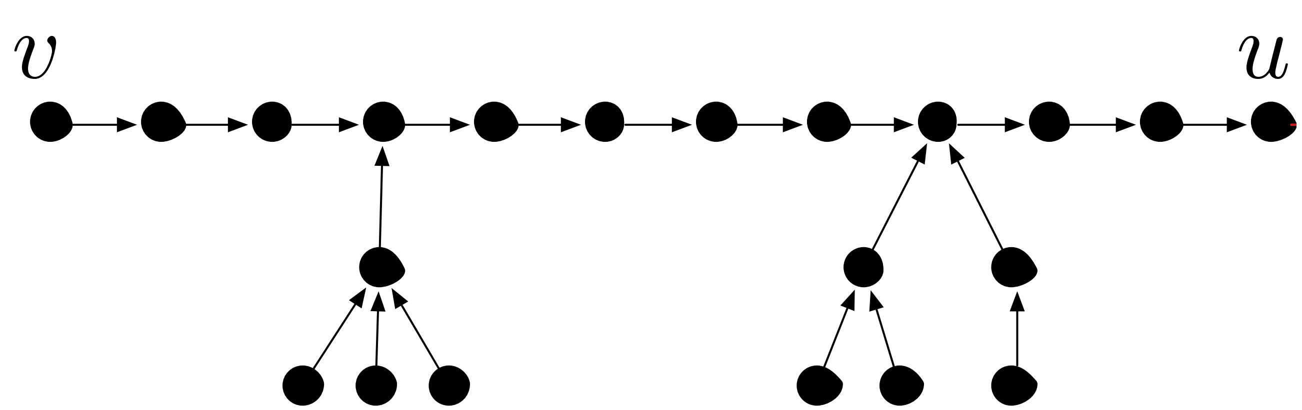


Universal Tree ⇐ Level Ancestor

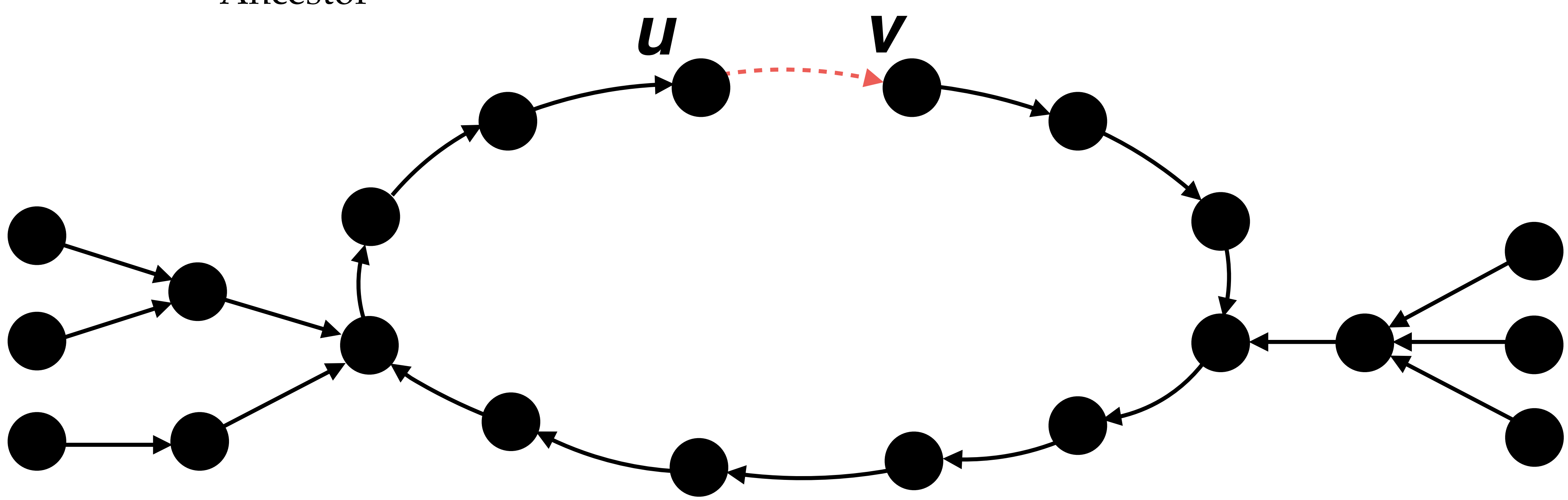


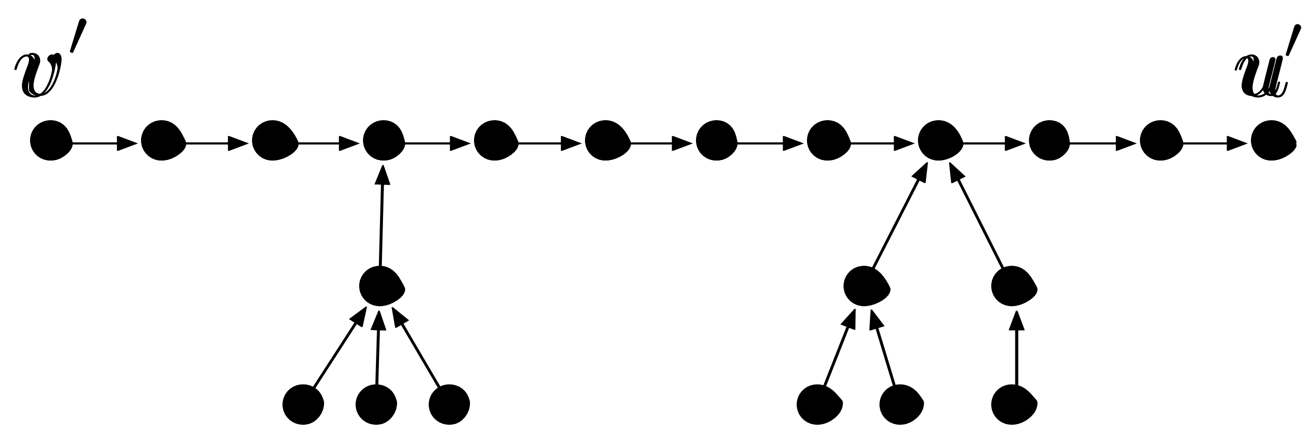
Universal Tree ⇐ Level Ancestor



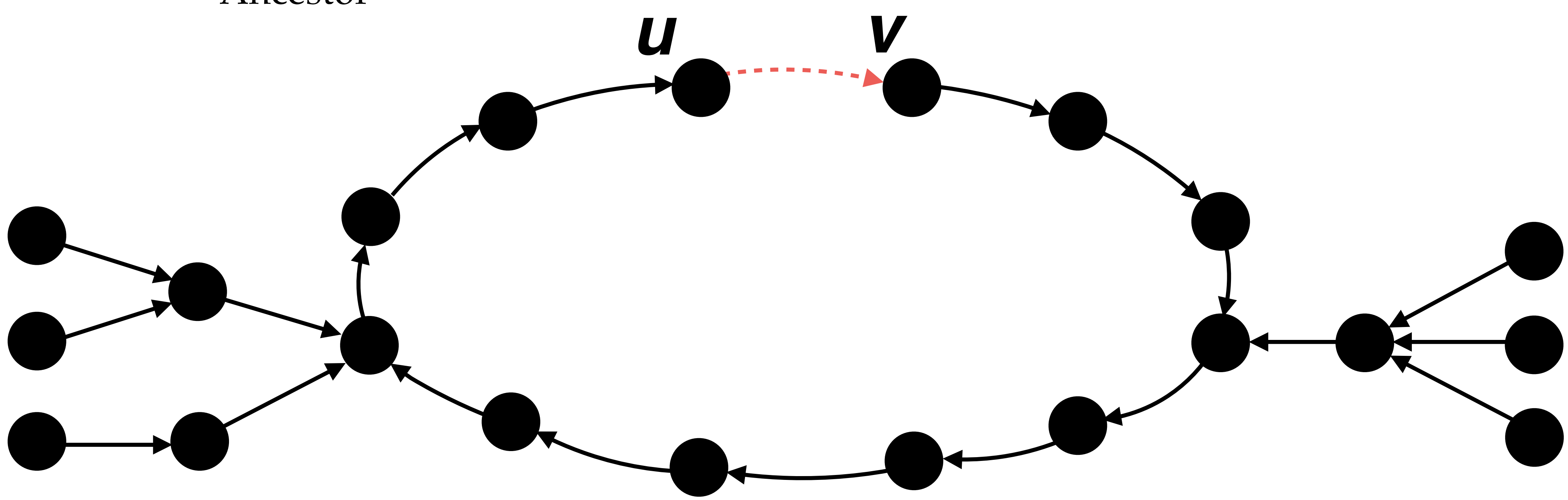


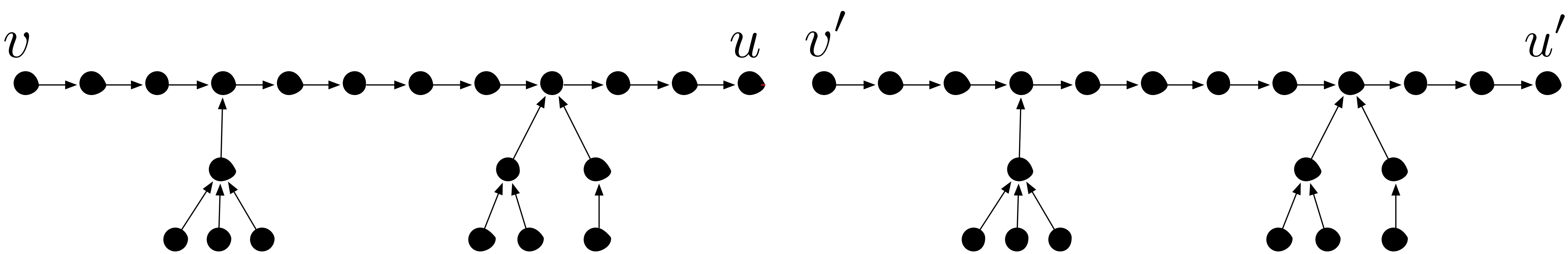
Universal Tree $\Leftarrow$? Level Ancestor

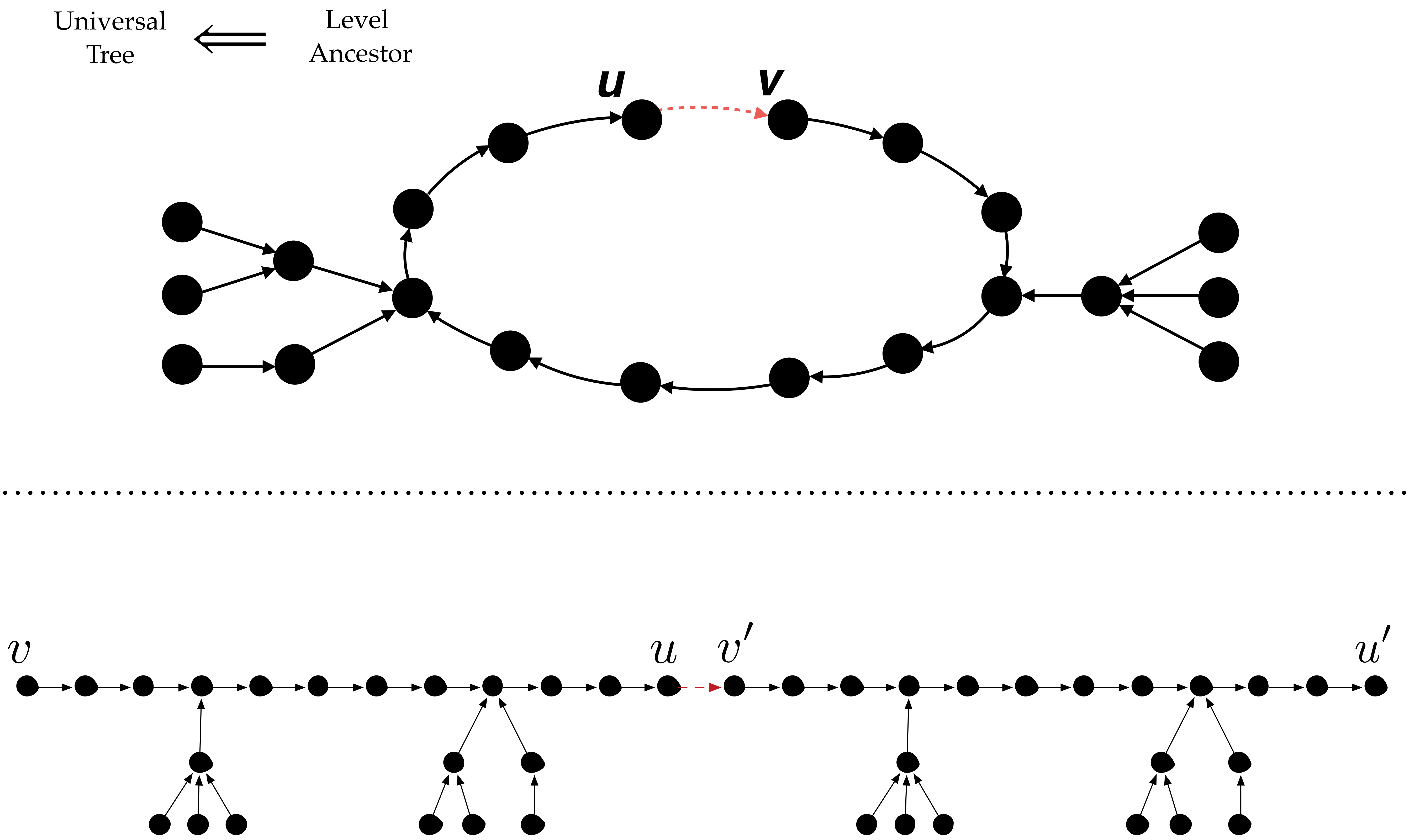




Universal Tree ⇐ Level Ancestor

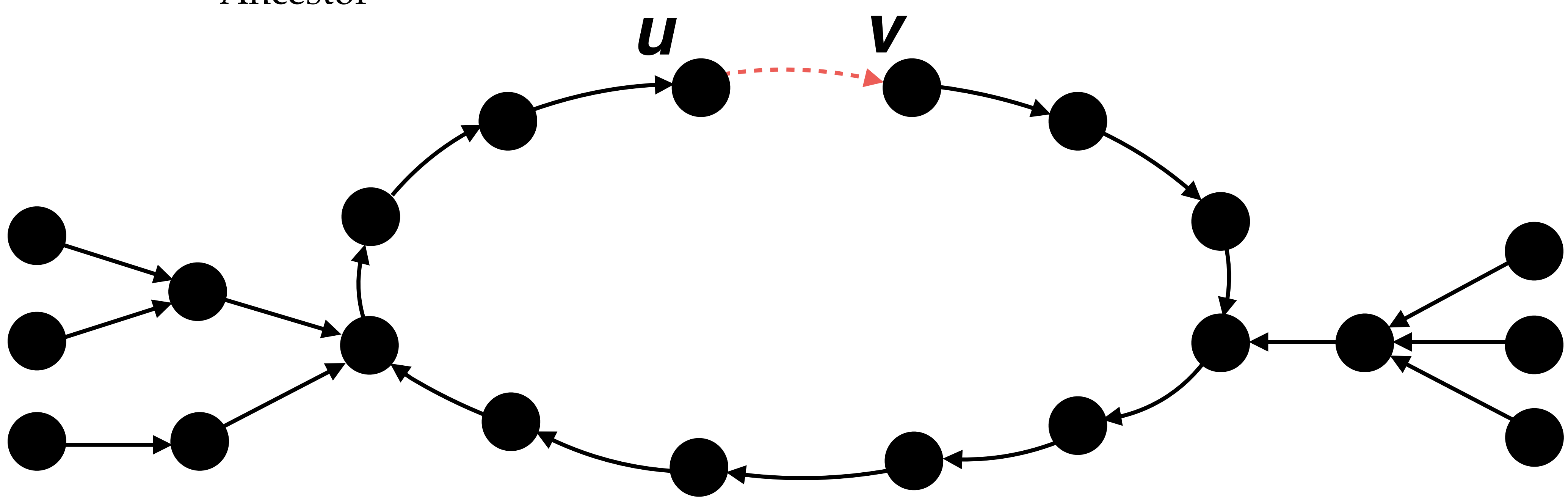


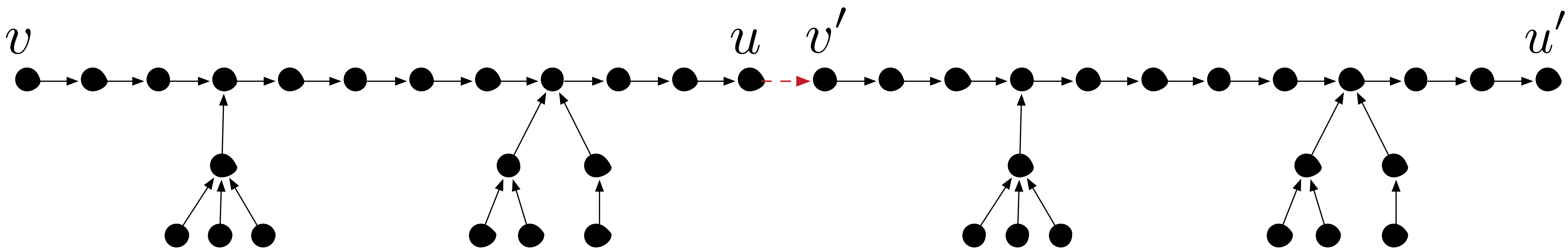




Universal Tree
Level Ancestor
u
v
v
u
v′
u′

Universal Tree ⟸ Level Ancestor





Universal Tree $\Longleftarrow$ ✓ Level Ancestor

Universal Tree $\Leftrightarrow$ ✓ Level Ancestor

Universal Tree ⇔ ✓ Level Ancestor

Exact Distance

Universal Tree $\overset{\checkmark}{\Leftrightarrow}$ Level Ancestor $\overset{?}{\Leftarrow}$ Exact Distance

Universal Tree $\Leftrightarrow$ ✓ Level Ancestor $\Leftarrow$? Exact Distance (prior results)

Universal Tree $\Leftrightarrow$ ✓ Level Ancestor $\Longleftarrow$ Exact Distance (prior results)

Universal Tree $\Leftrightarrow$ Level Ancestor $\Leftarrow$ Exact Distance (prior results)

Universal Tree $\Leftrightarrow$ Level Ancestor $\Longleftarrow$ Exact Distance

Universal Tree $\Longleftrightarrow$ Level Ancestor $\Longleftarrow$ Exact Distance


Universal Tree
Level Ancestor
Exact Distance

Universal Tree ✓ $\Leftrightarrow$ Level Ancestor ✗ $\Leftarrow$ Exact Distance

	Universal Tree	Exact Distance
Upper:	$n^{\frac{1}{2}} \log n$ [Chung et al. 1981]	$\frac{1}{2} \log^2 n$ [Alstrup et al. 2016]
Lower:	$n^{\frac{1}{2}} \log n$ [Chung et al. 1981]	$\frac{1}{4} \log^2 n$ [Alstrup et al. 2016]

Universal Tree $\Leftrightarrow$ ✓ Level Ancestor $\not\Leftarrow$ Exact Distance

	Universal Tree / Level Ancestor	Exact Distance
Upper:	$n^{\frac{1}{2}} \log n$ [Chung et al. 1981]	$\frac{1}{4} \log^2 n$
Lower:	$n^{\frac{1}{2}} \log n$ [Chung et al. 1981]	$\frac{1}{4} \log^2 n$ [Alstrup et al. 2016]

Universal Tree $\Leftrightarrow$ ✓ Level Ancestor

Upper: $n^{\frac{1}{2}} \log n$ [Chung et al. 1981]

Lower: $n^{\frac{1}{2}} \log n$ [Chung et al. 1981]

Exact Distance

$\frac{1}{4} \log^2 n$

$\frac{1}{4} \log^2 n$ [Alstrup et al. 2016]

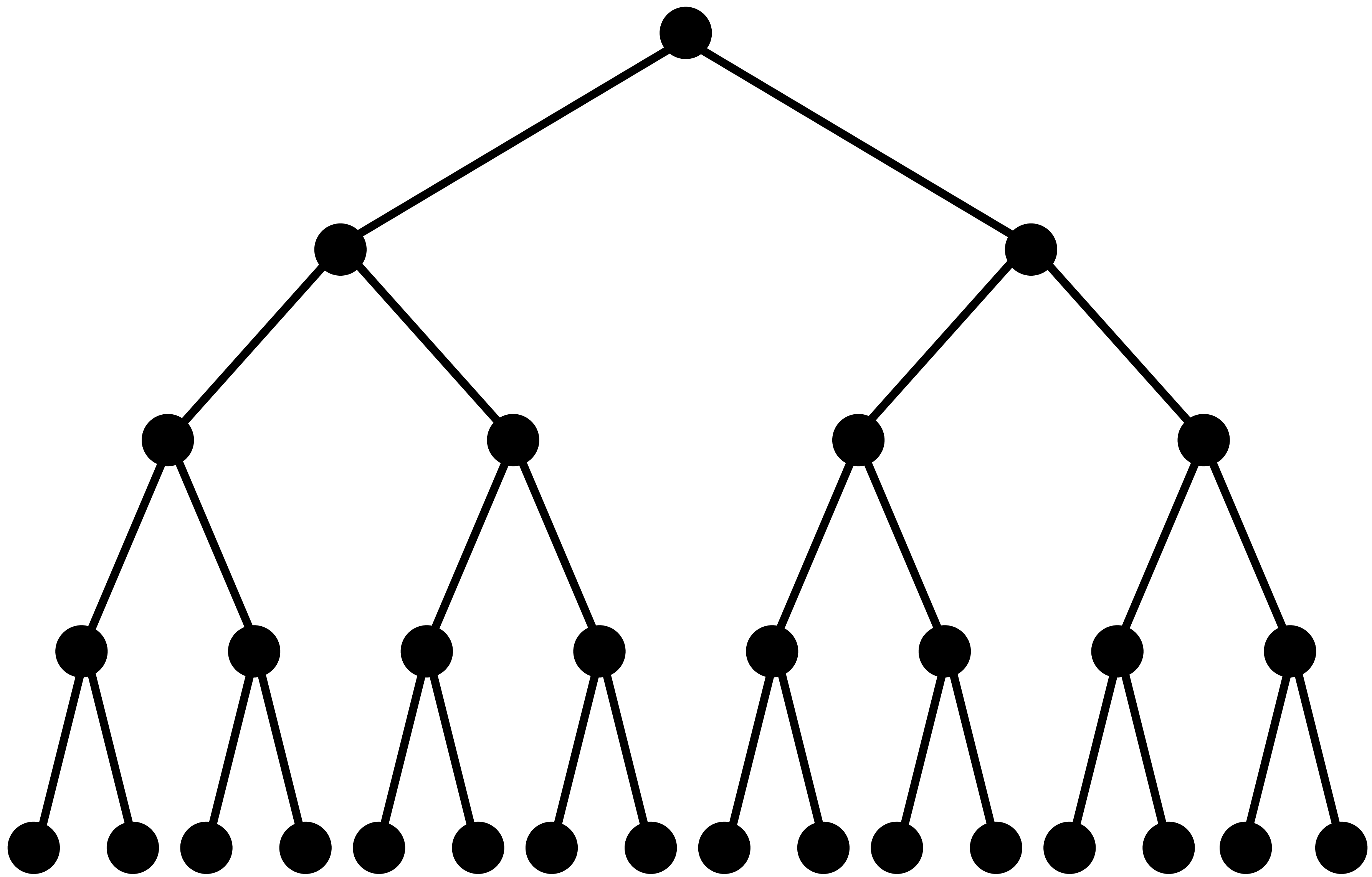
Exist

$$\frac{1}{4}\log^2 n\text{-bits}$$

Distance labeling

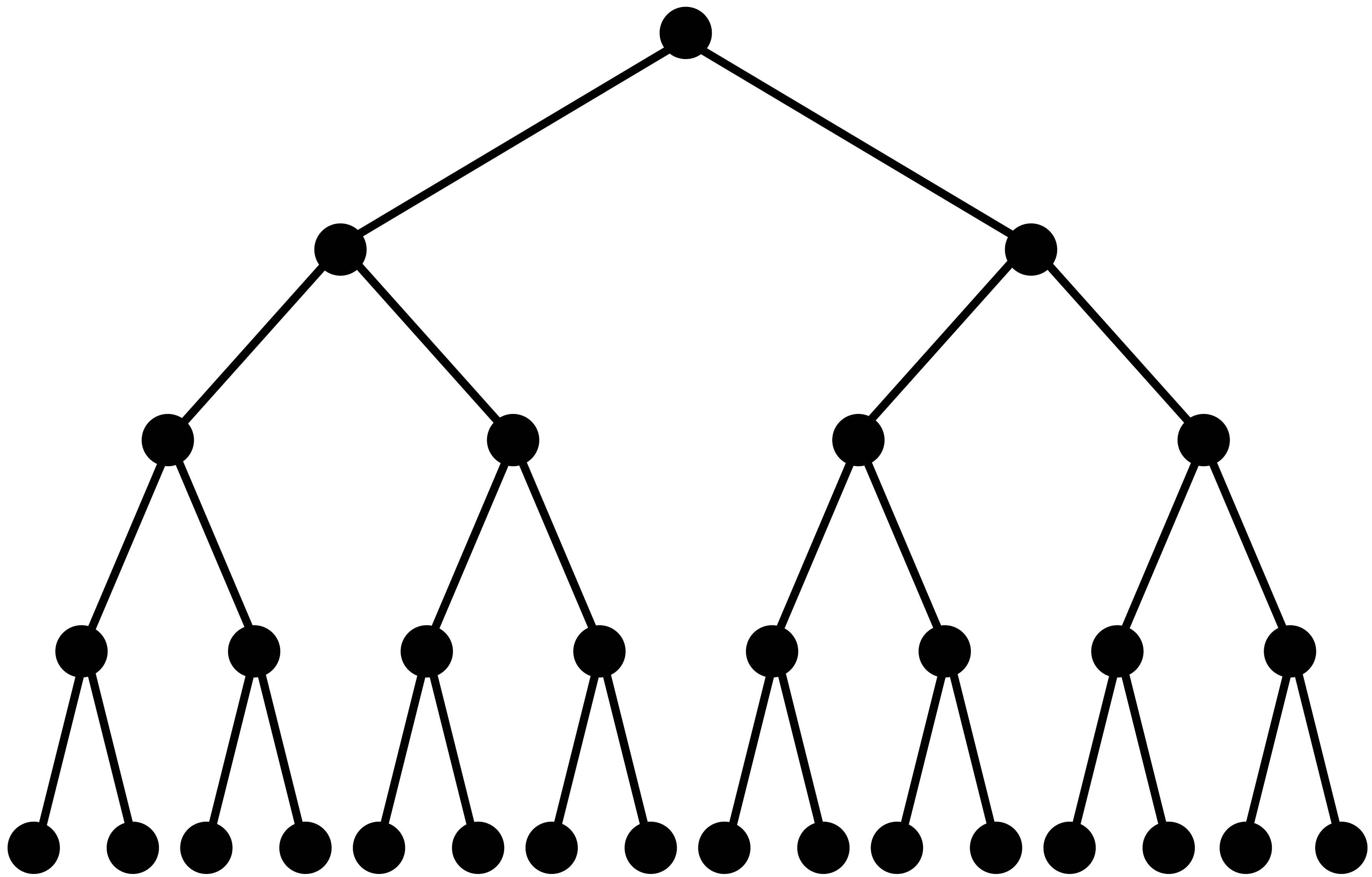
Sketch Idea

Sketch Idea



Sketch Idea

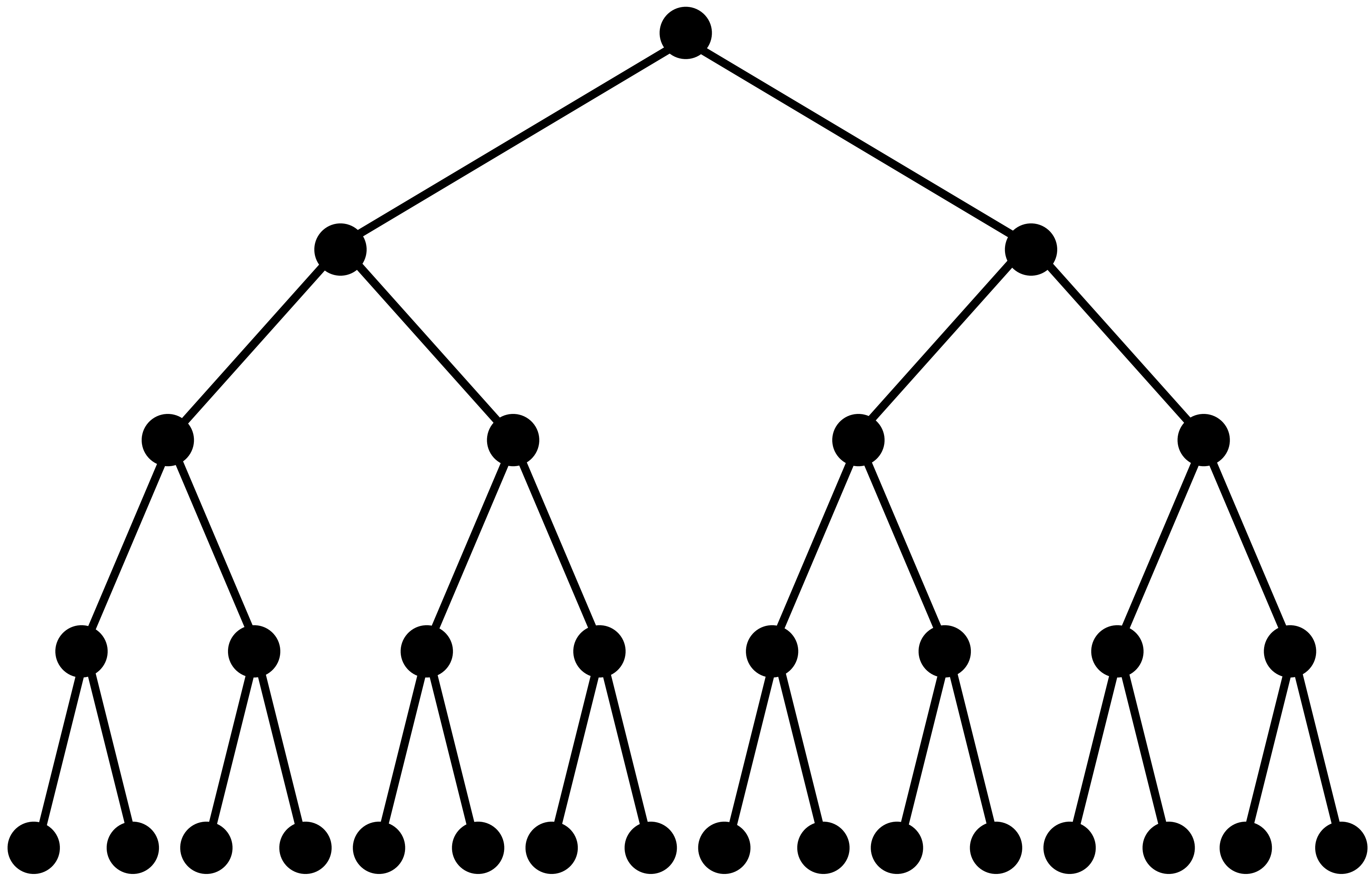
[n/16]



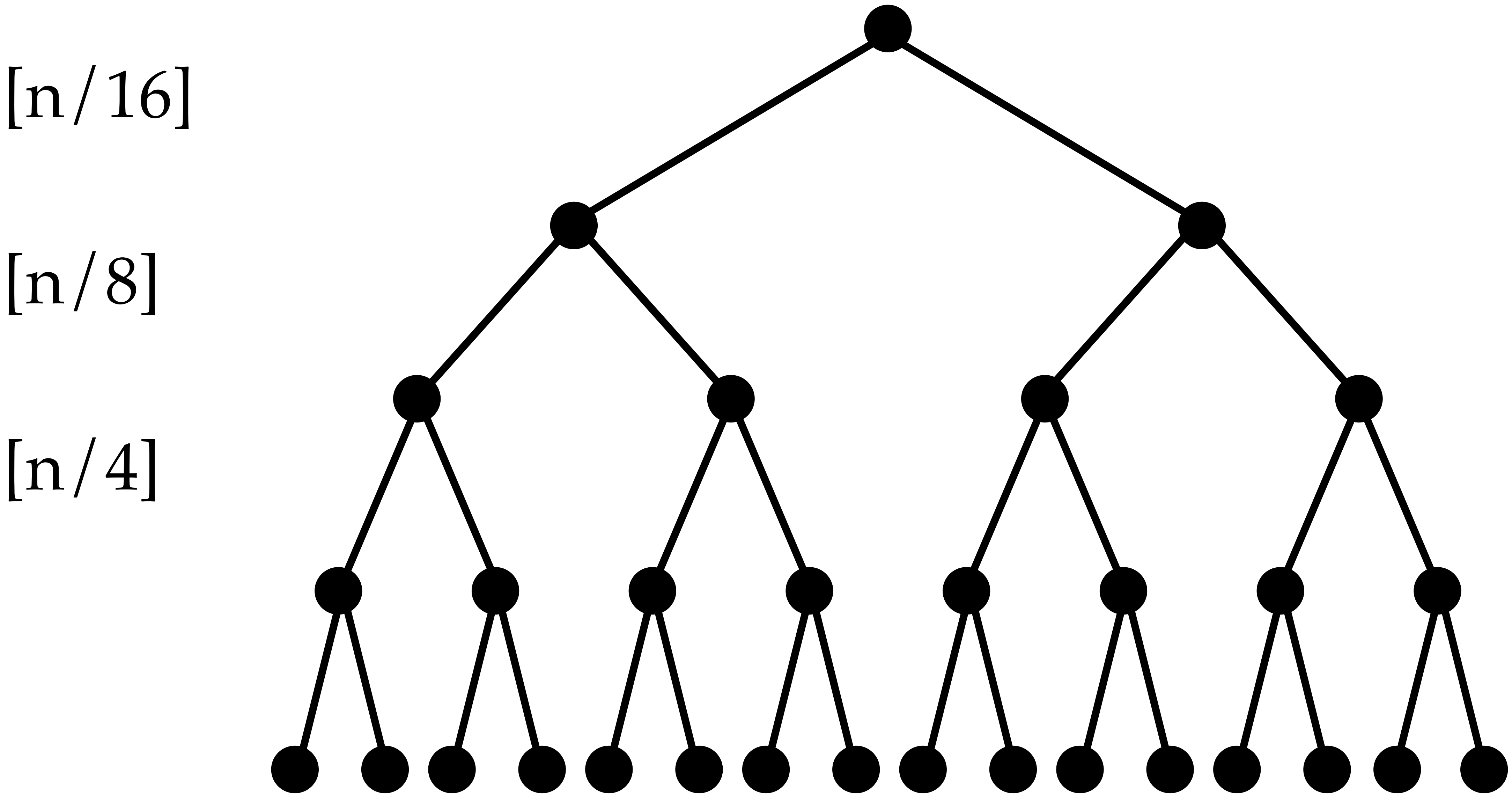
Sketch Idea

[n/16]

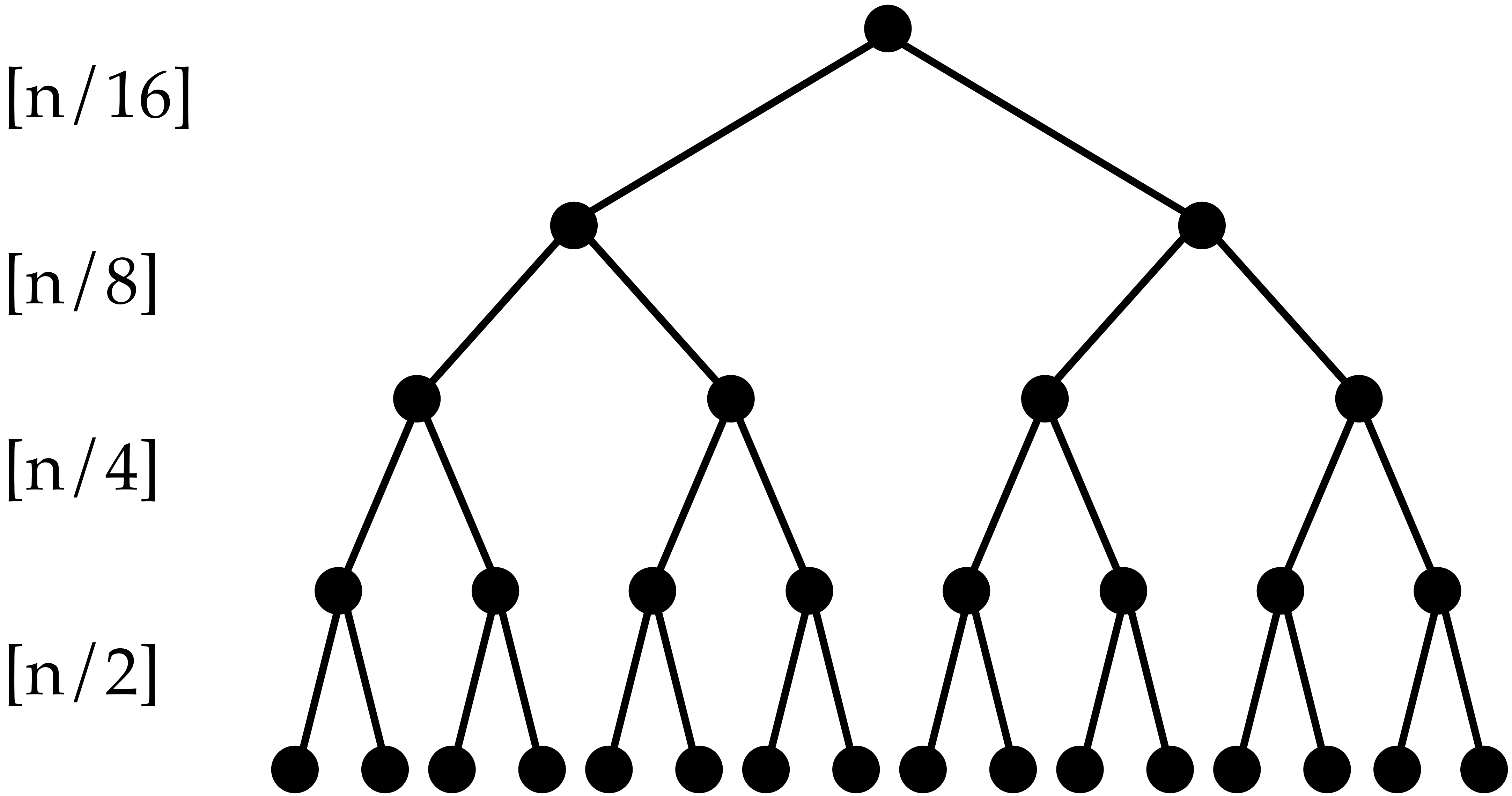
[n/8]



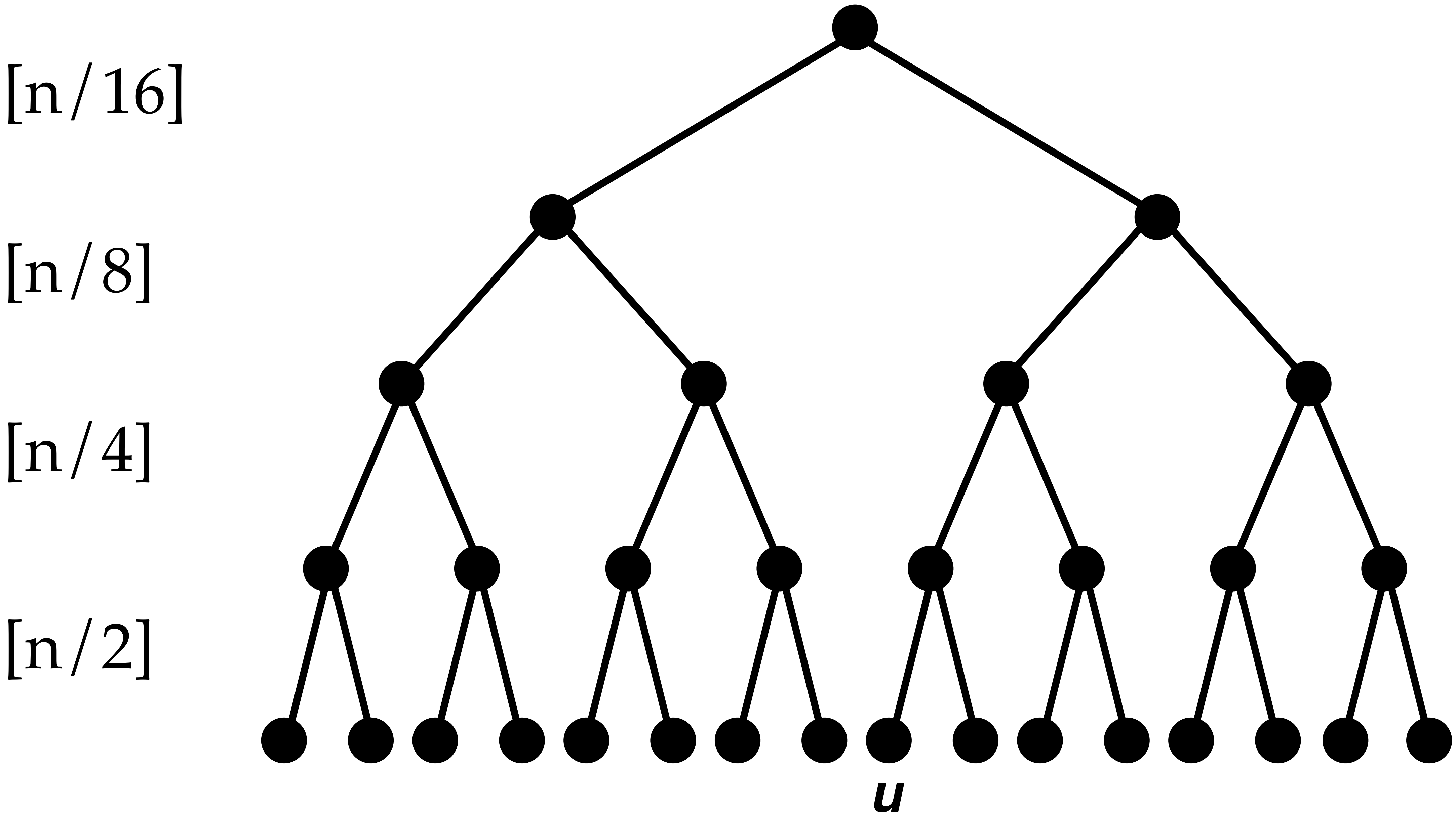
Sketch Idea



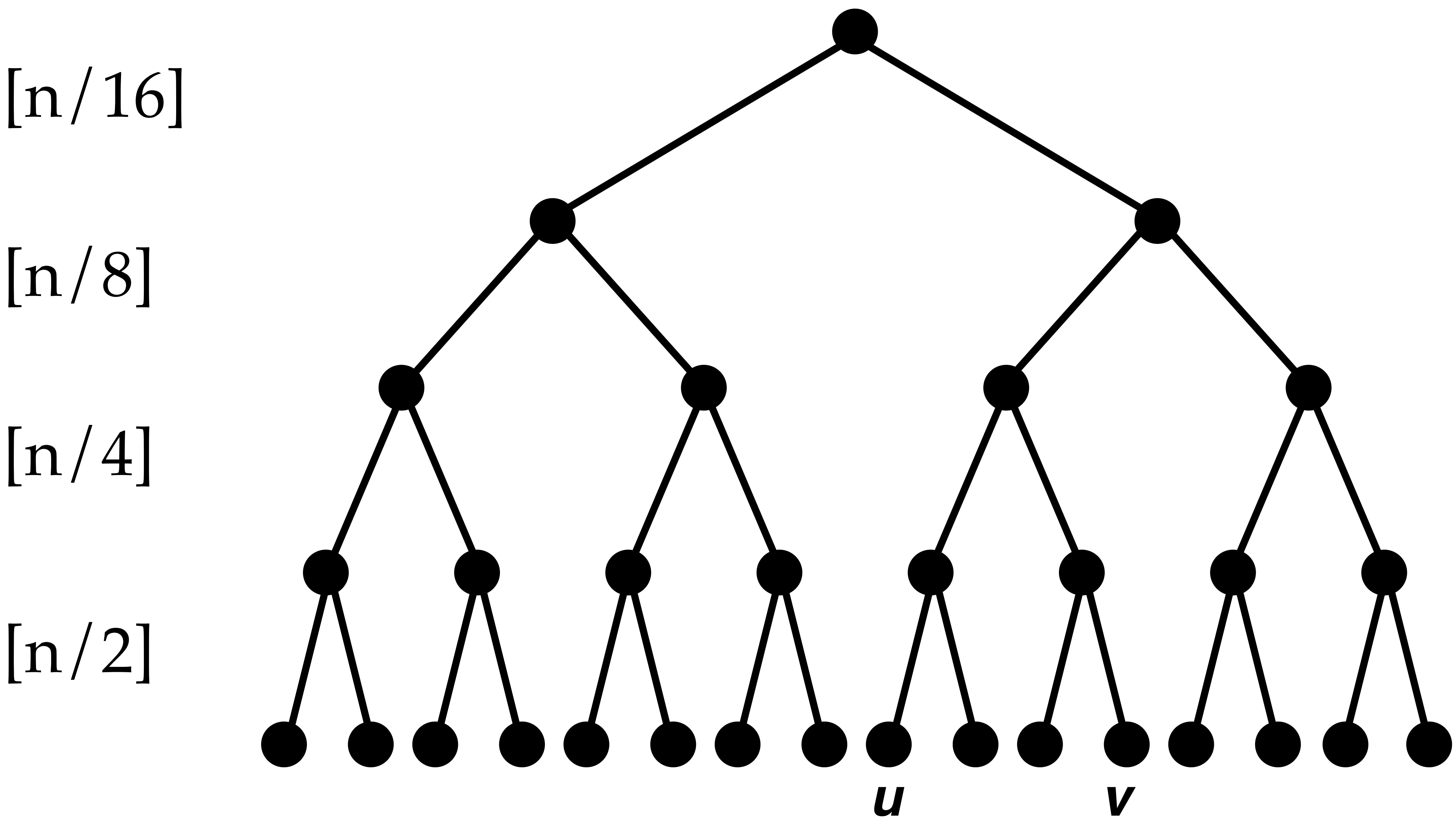
Sketch Idea



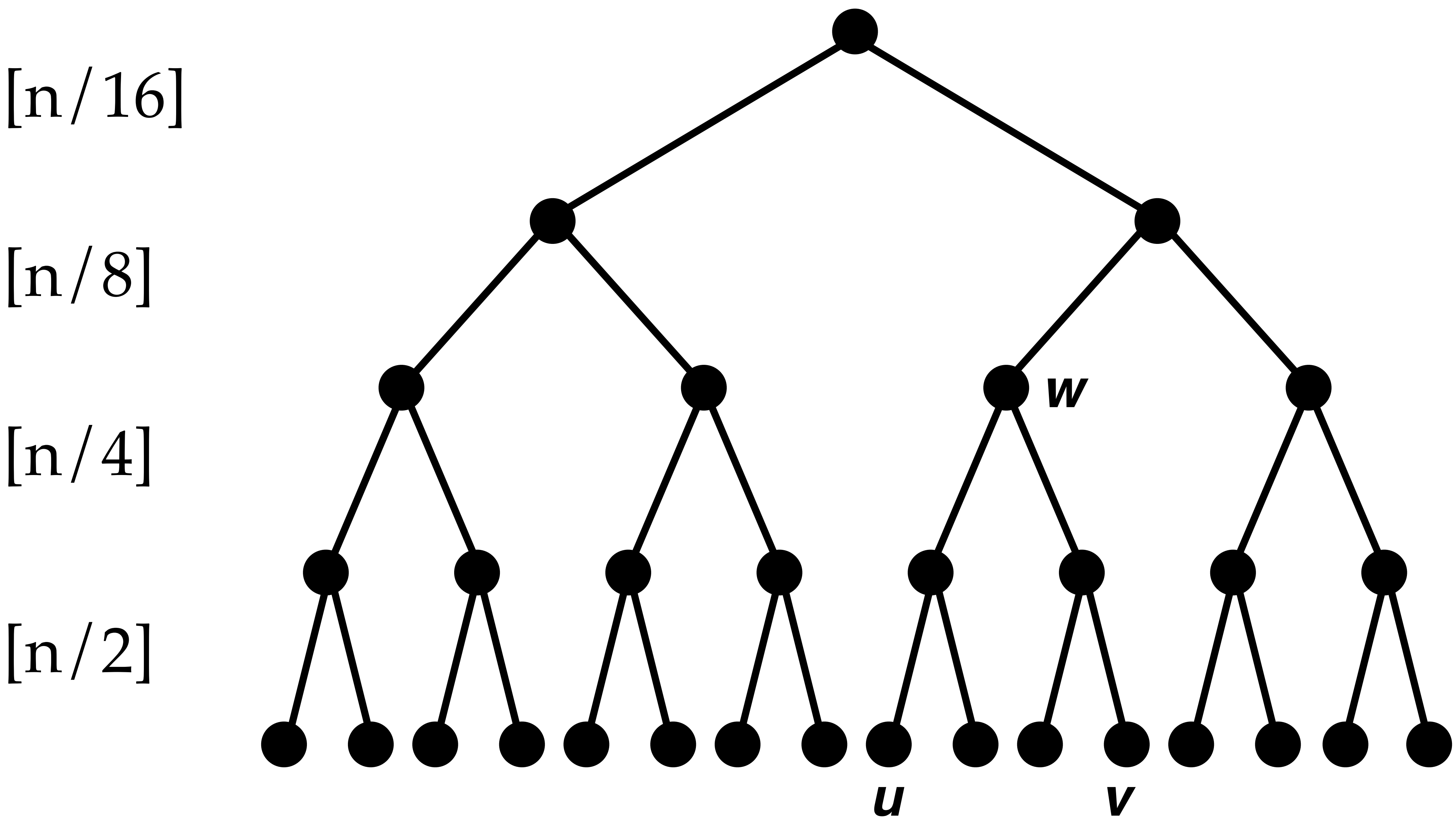
Sketch Idea



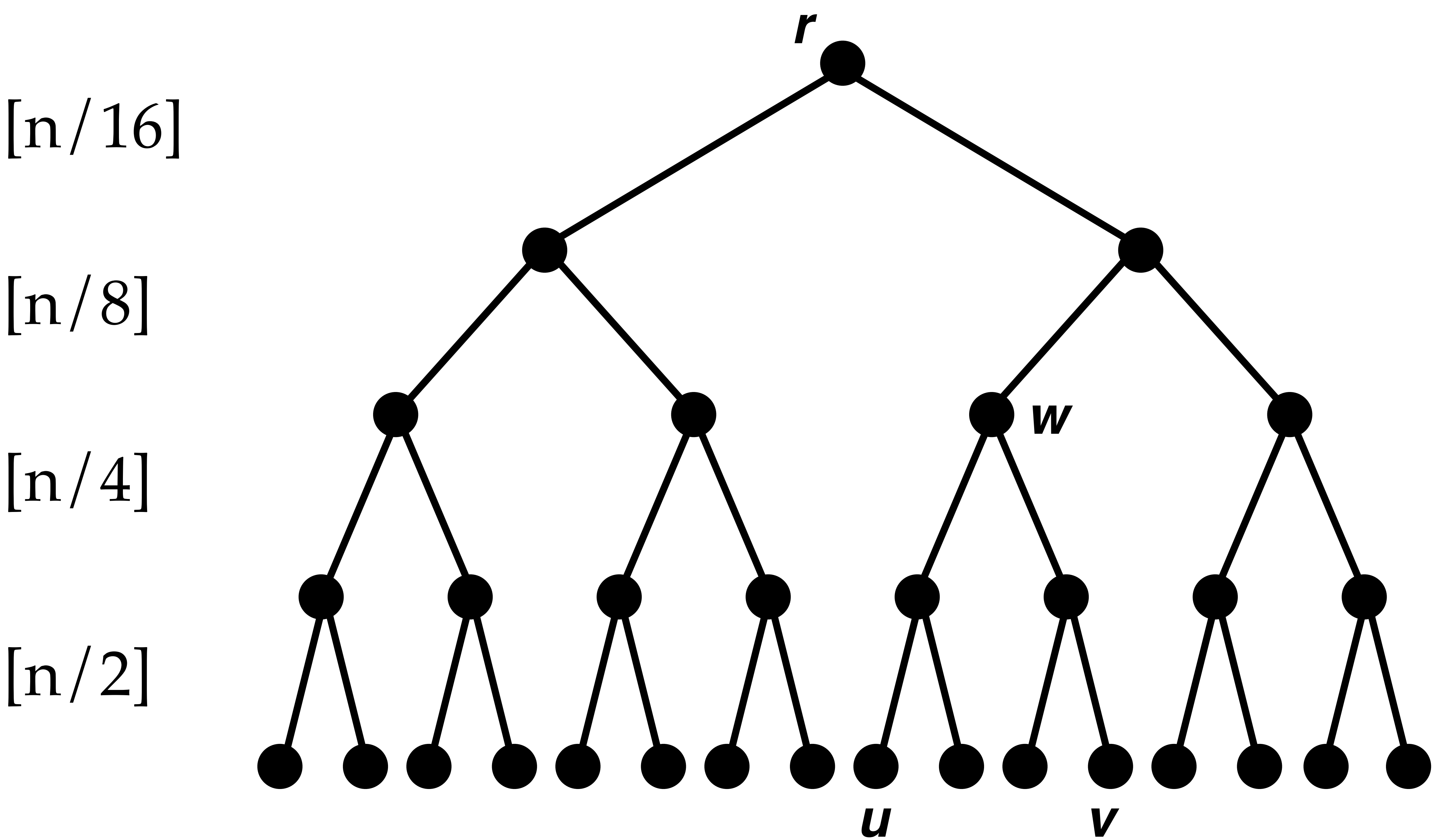
Sketch Idea



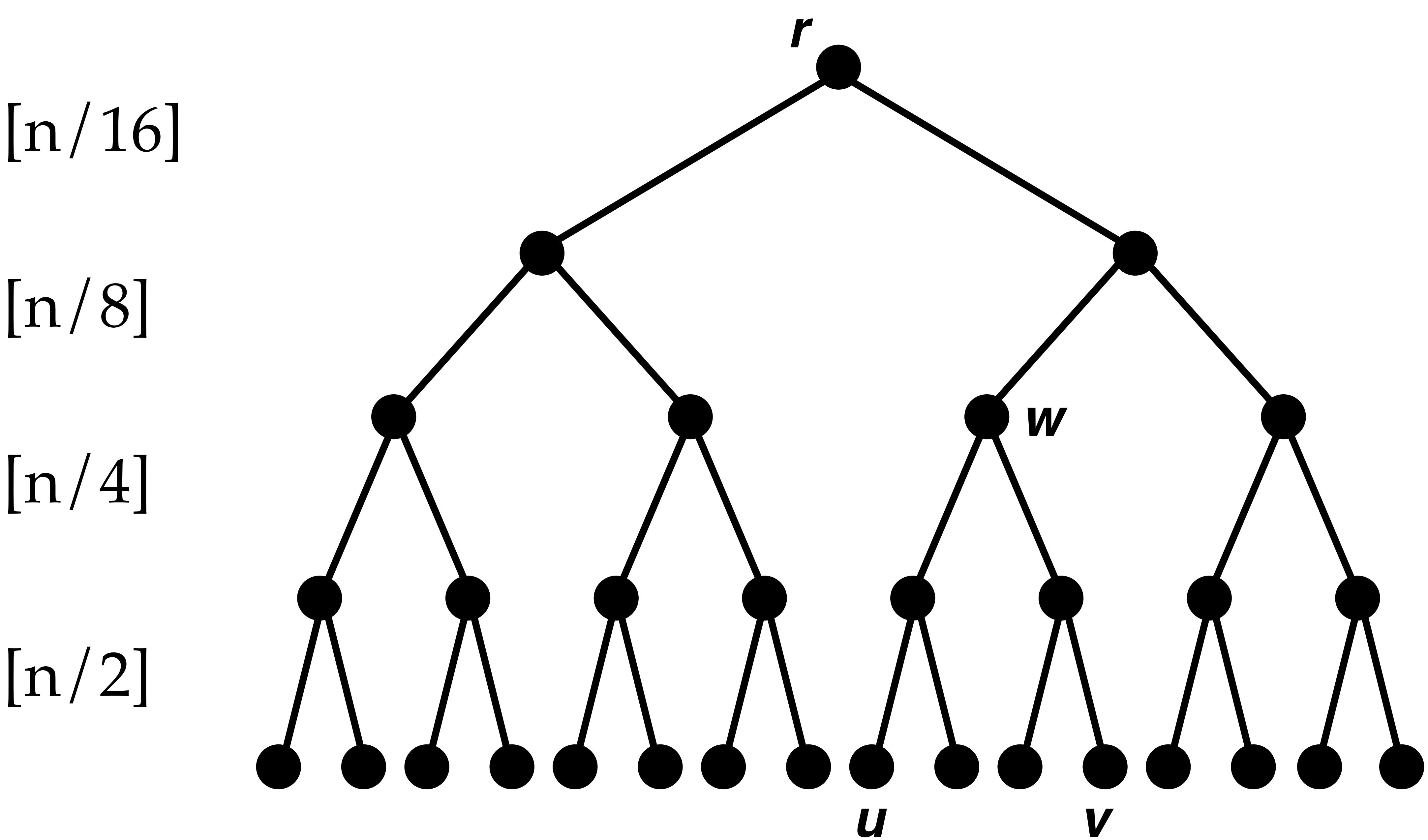
Sketch Idea



Sketch Idea

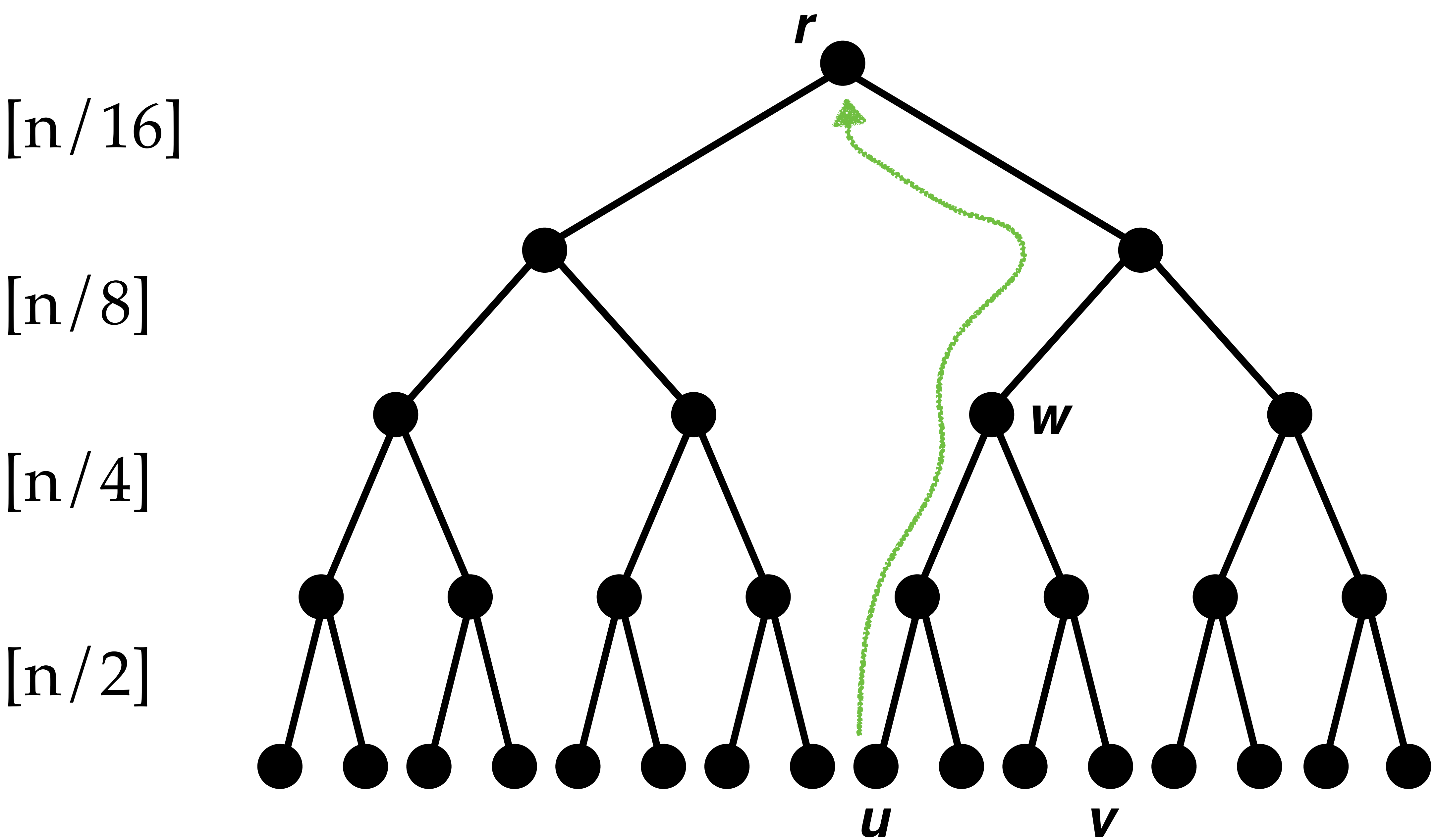


Sketch Idea



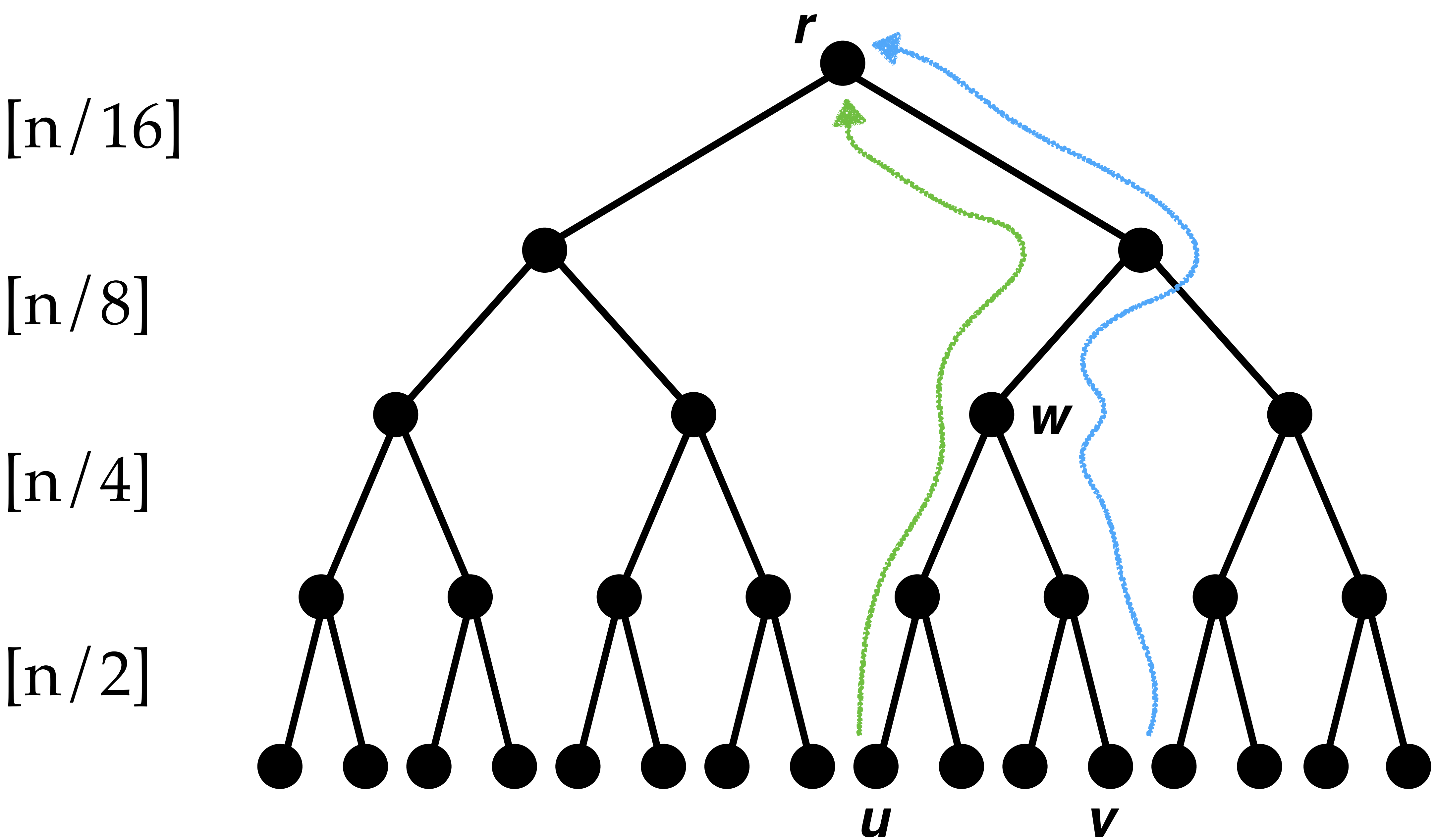
$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea



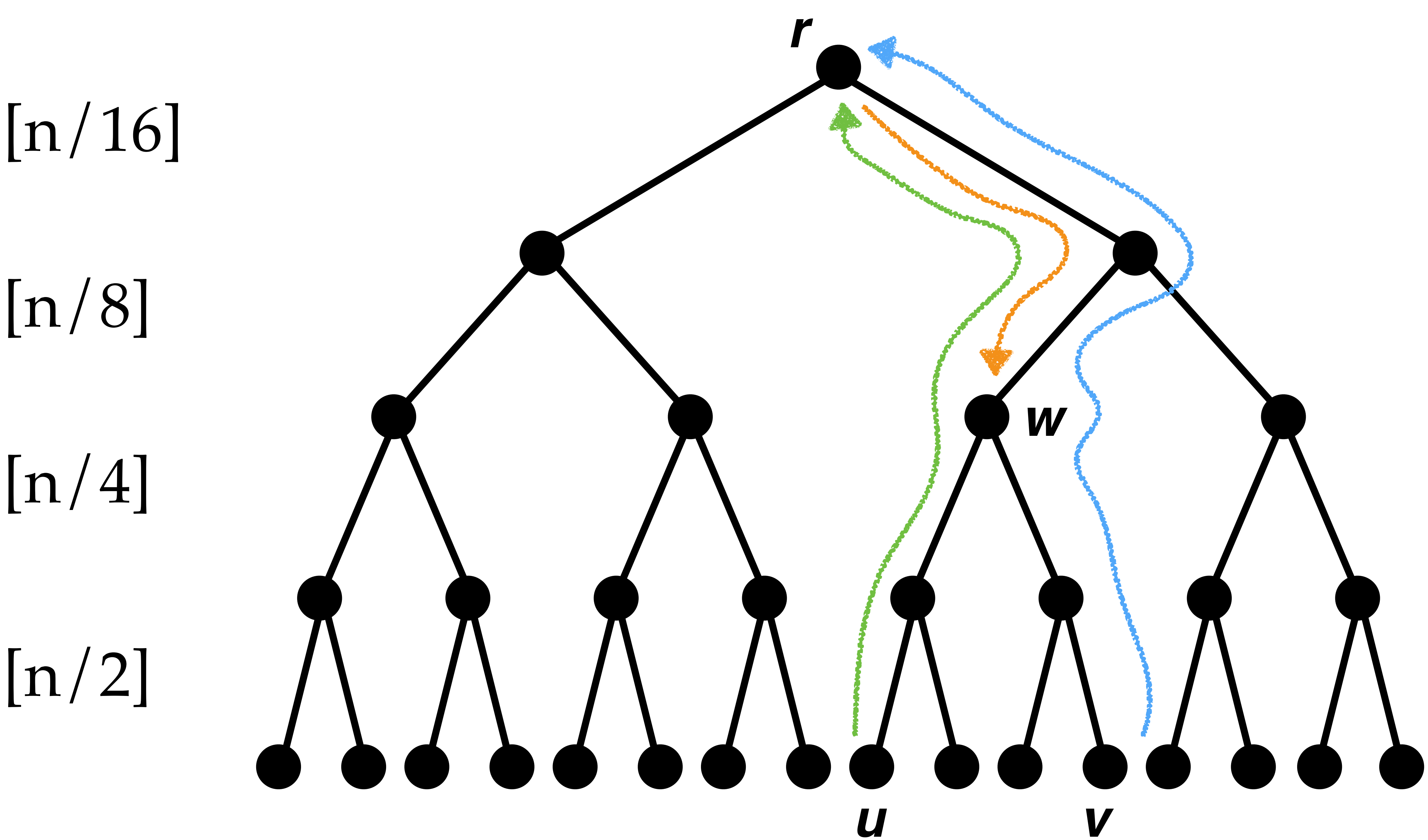
$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea



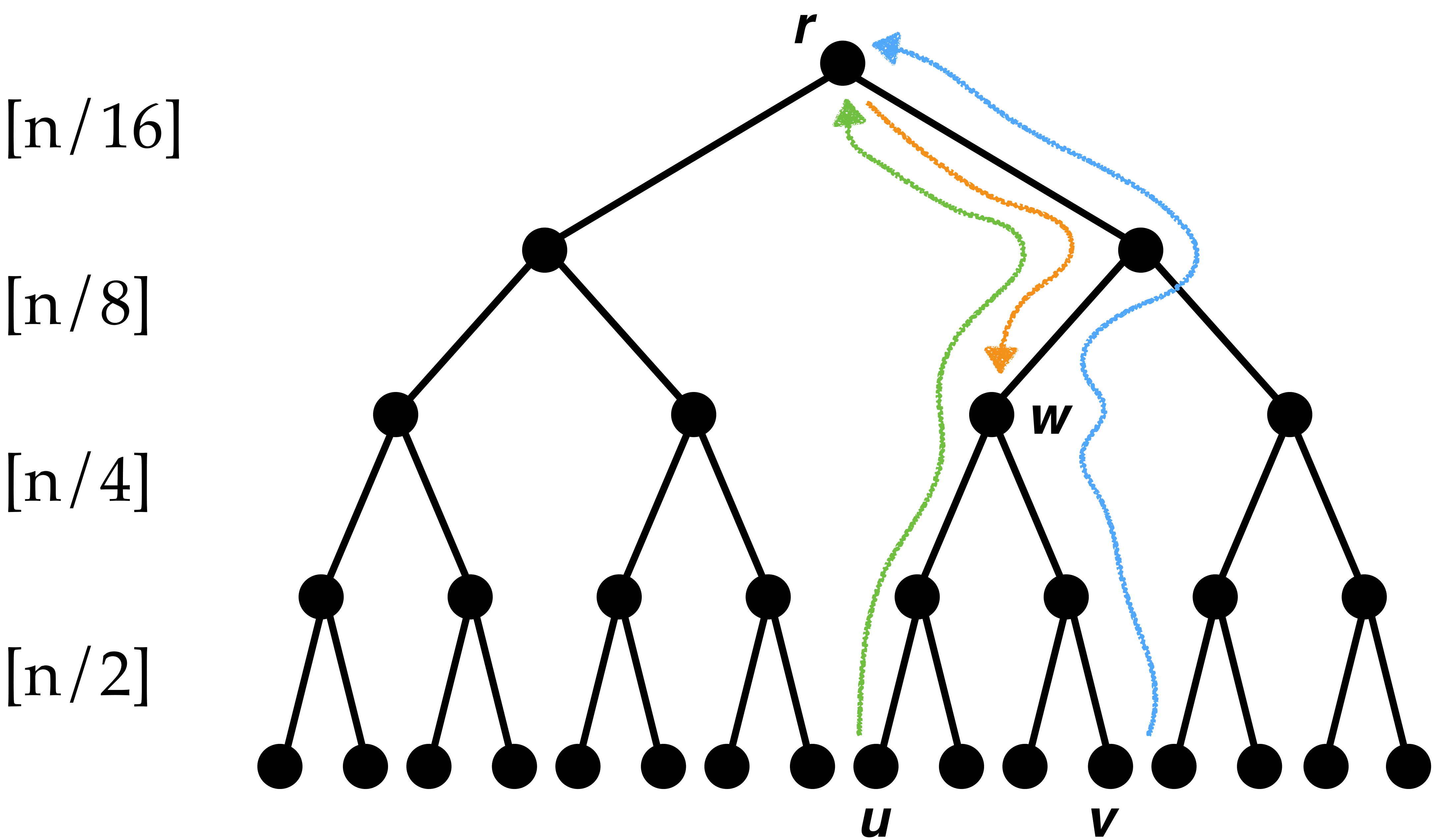
$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea



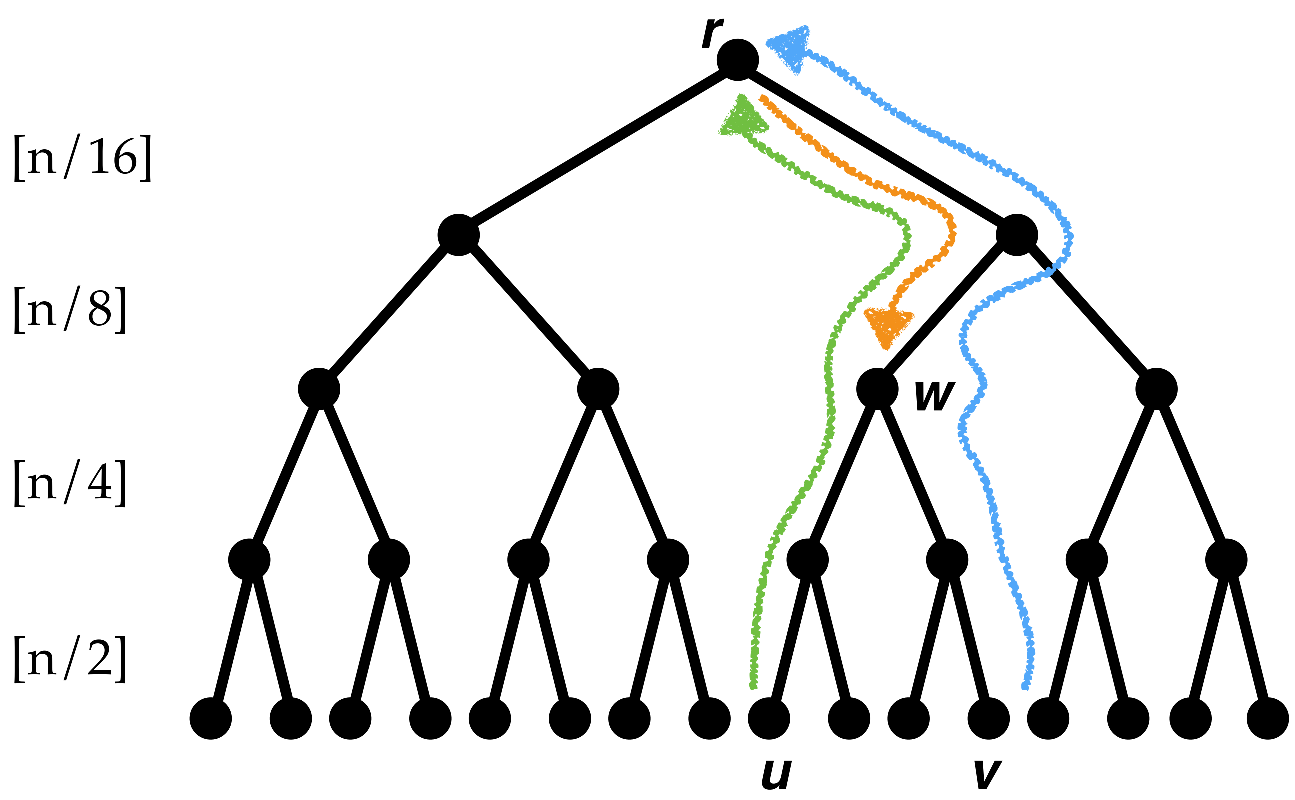
$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea



$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

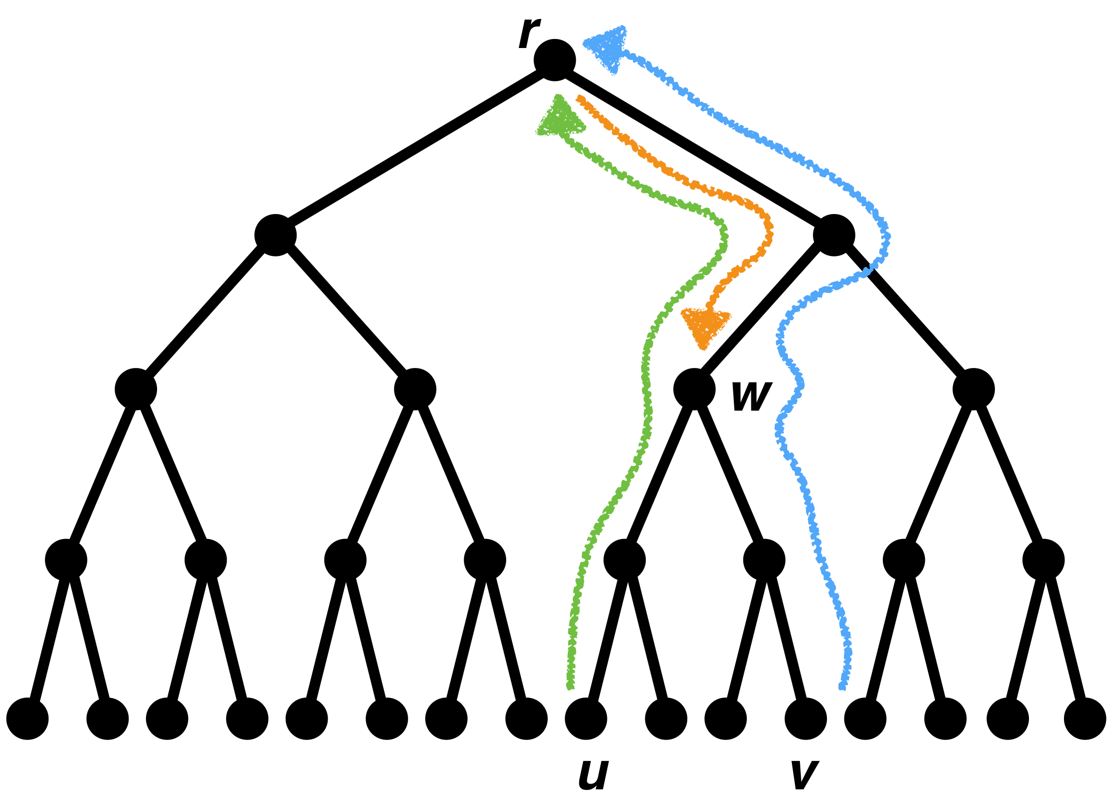
Sketch Idea

[n/16]

[n/8]

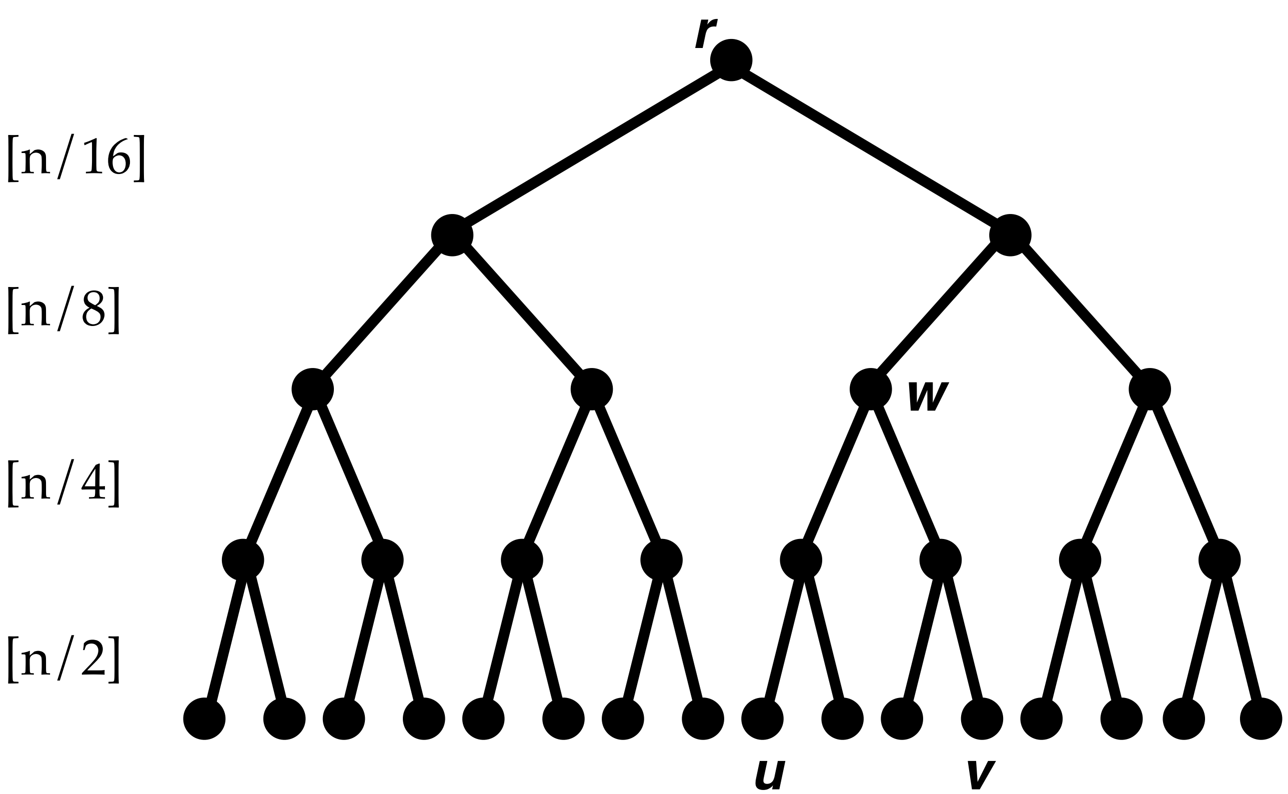
[n/4]

[n/2]



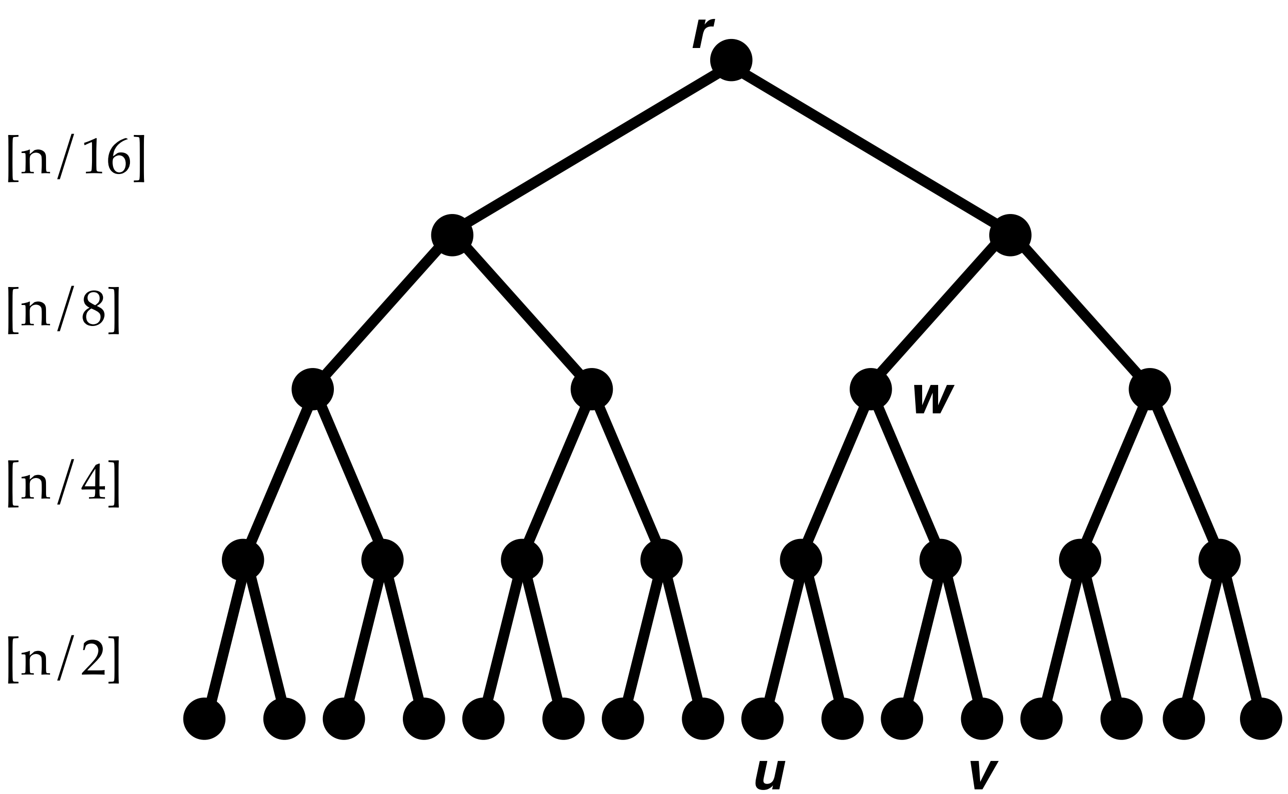
$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

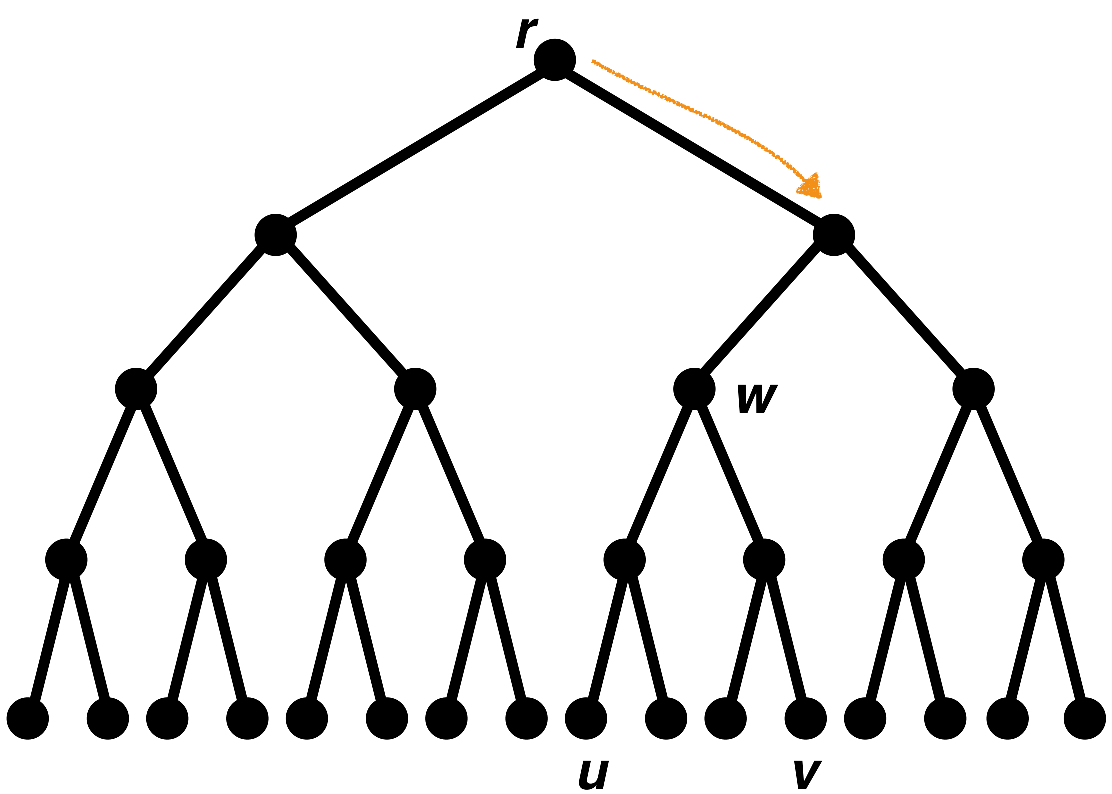
Sketch Idea

[n/16]

[n/8]

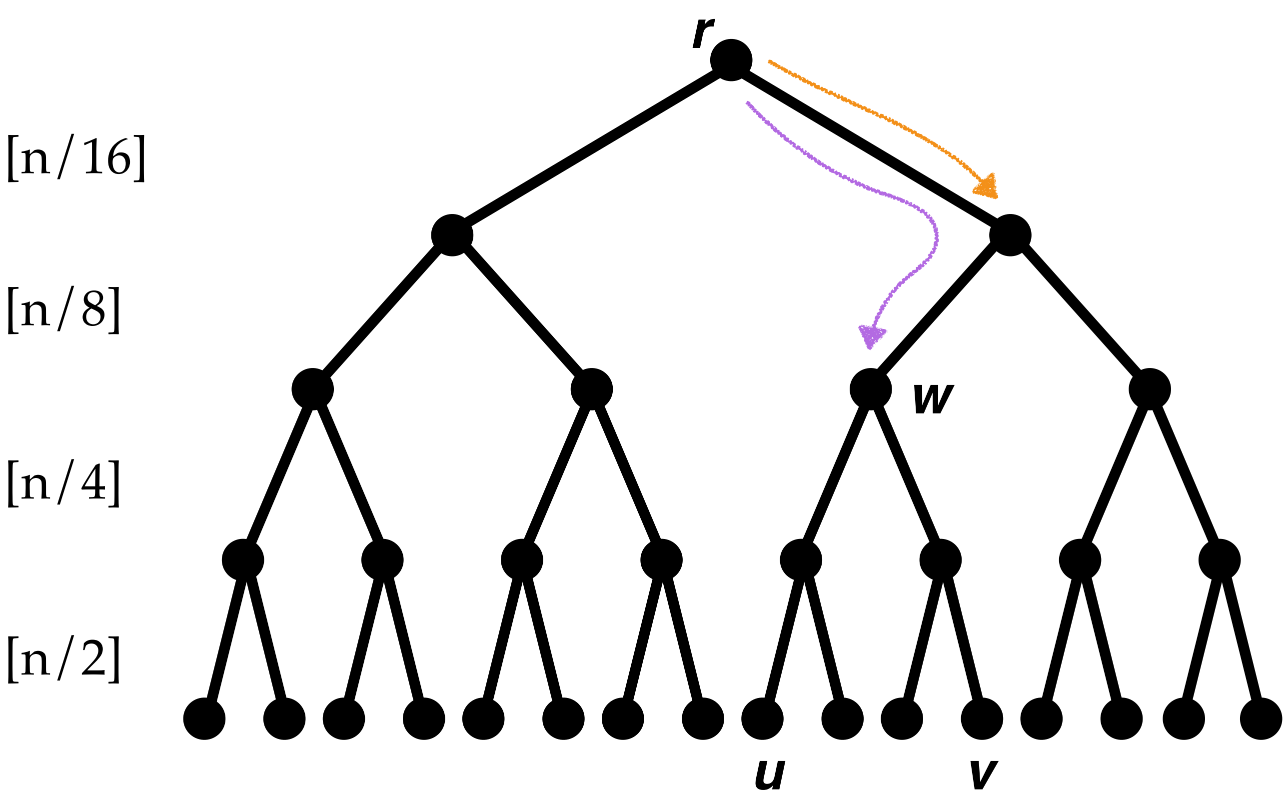
[n/4]

[n/2]



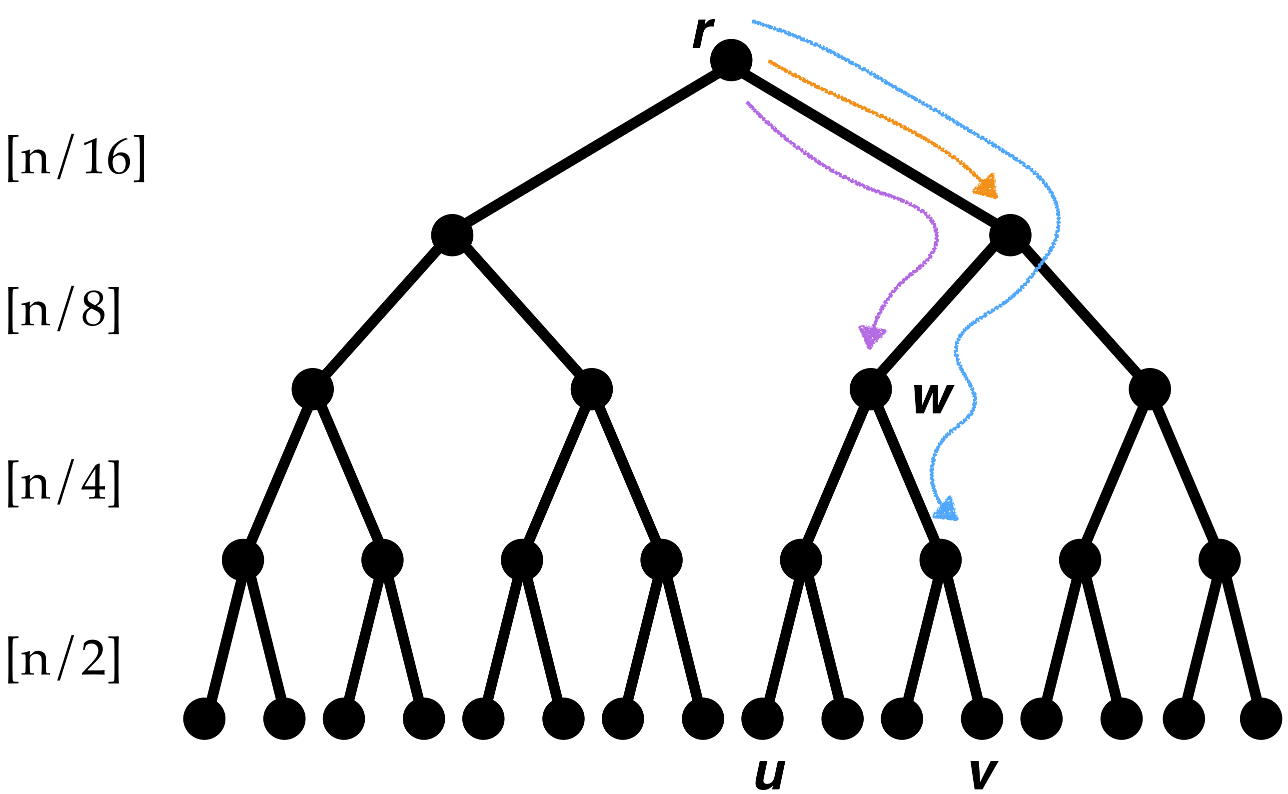
$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

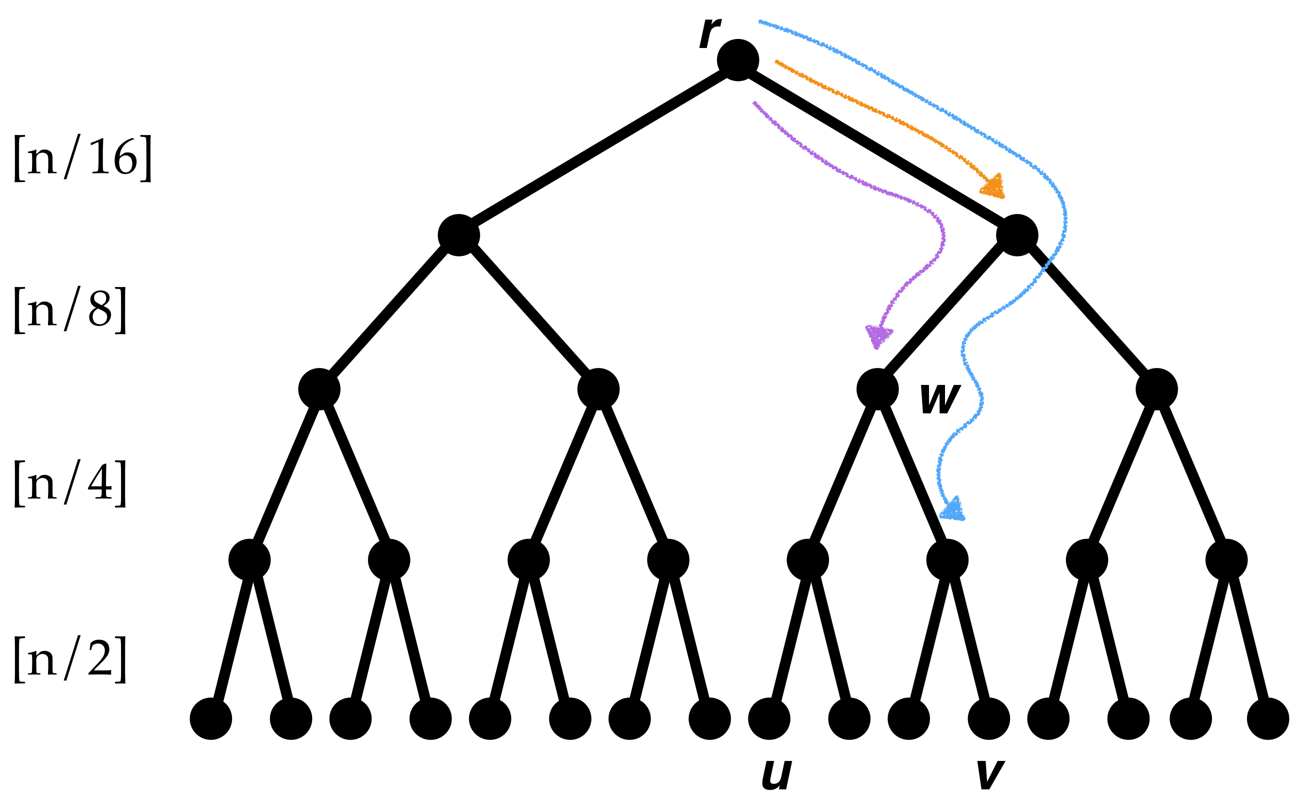
Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

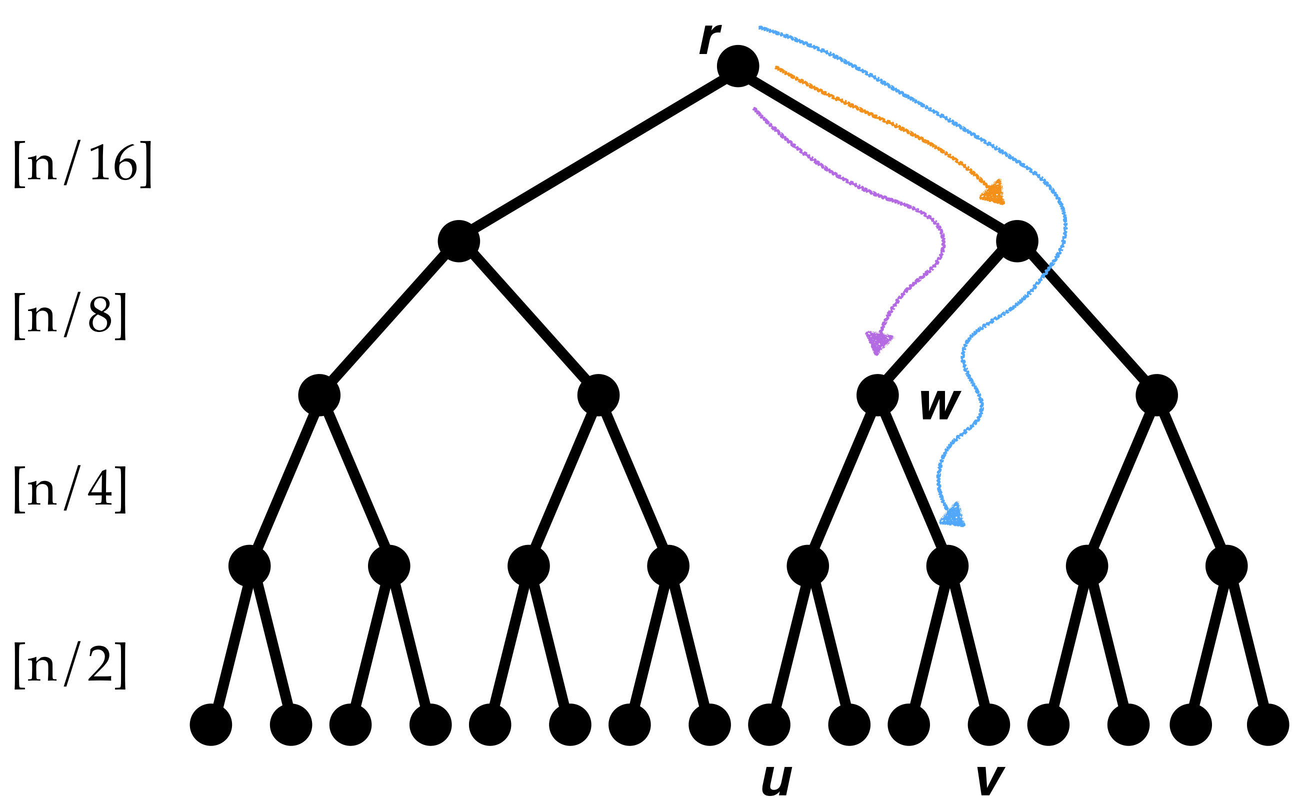
Sketch Idea

***label*(*v*):**



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

Sketch Idea

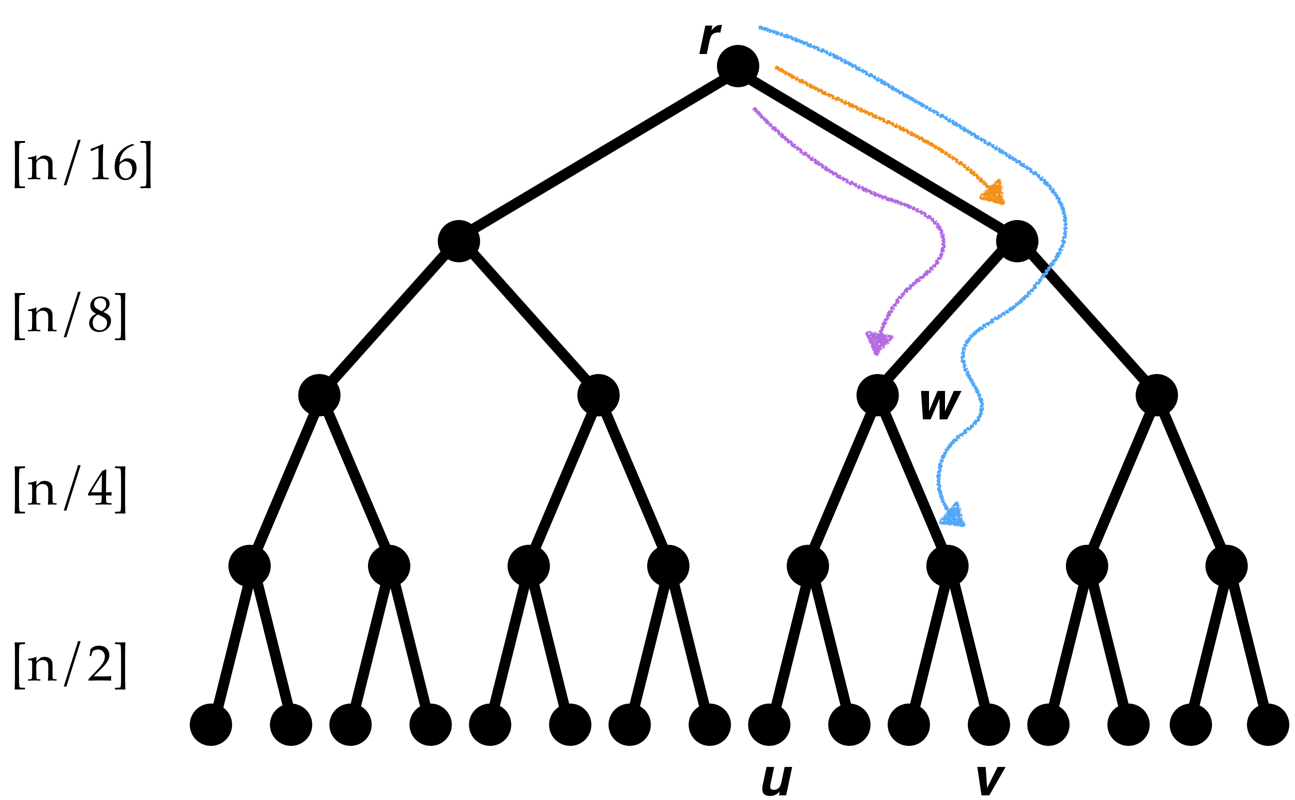


$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

(1)

Sketch Idea

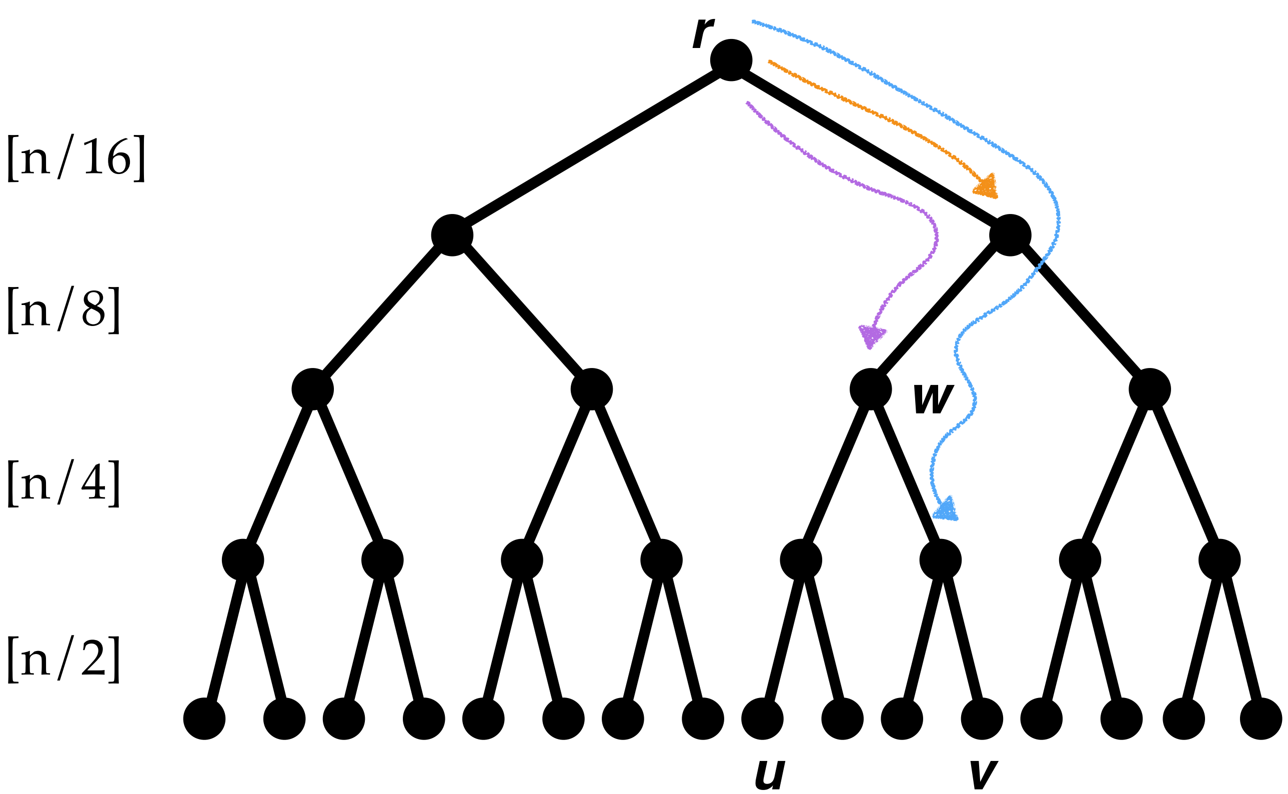


$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

(1) $d(v,r)$

Sketch Idea



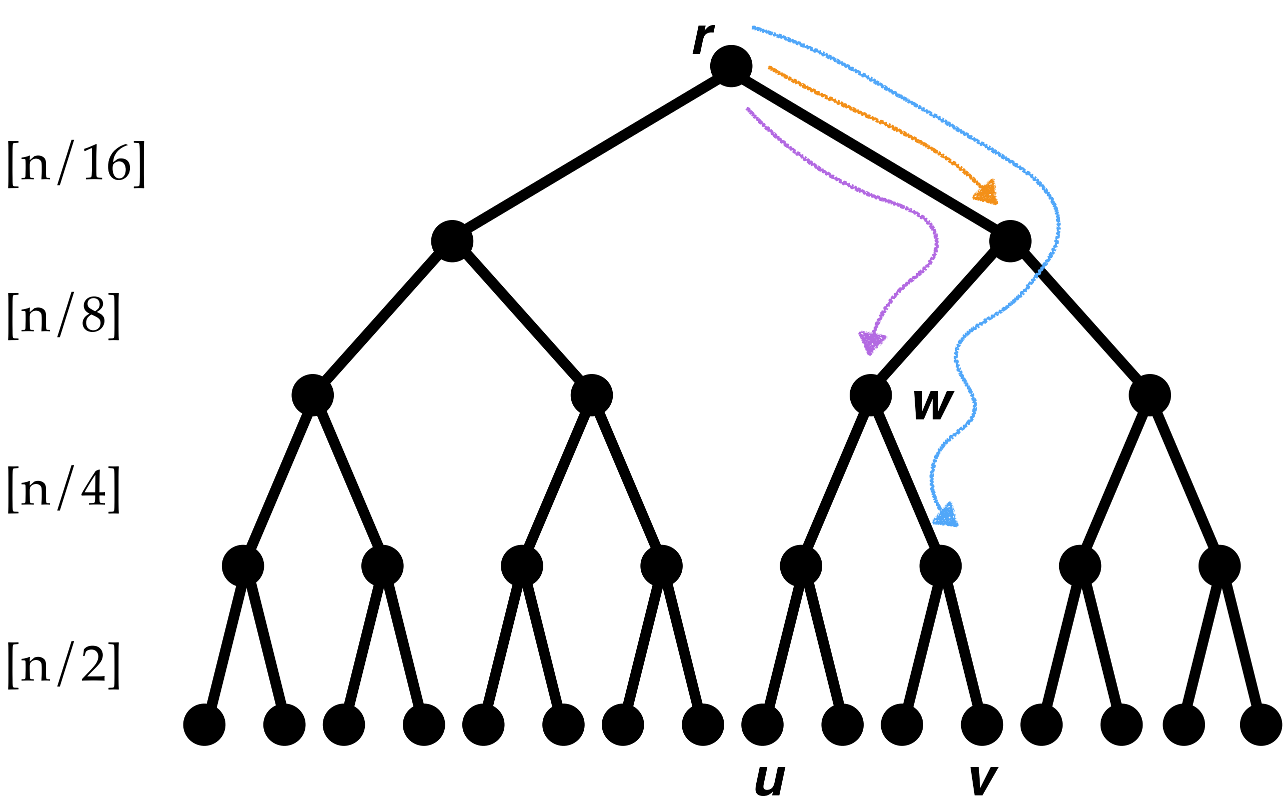
$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

(1) $d(v,r)$

(2)  ,...

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

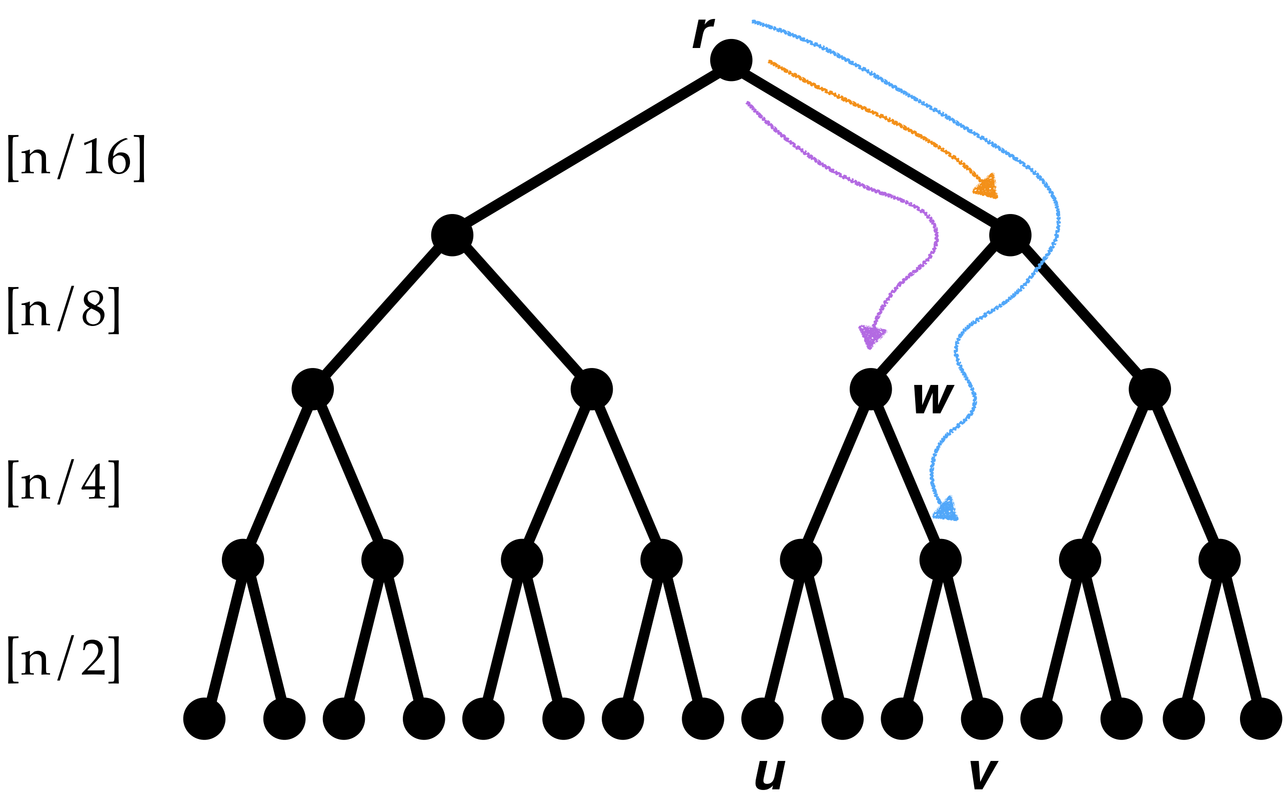
***label*(*v*):**

log(n)

(1) $d(v,r)$

(2) 

Sketch Idea



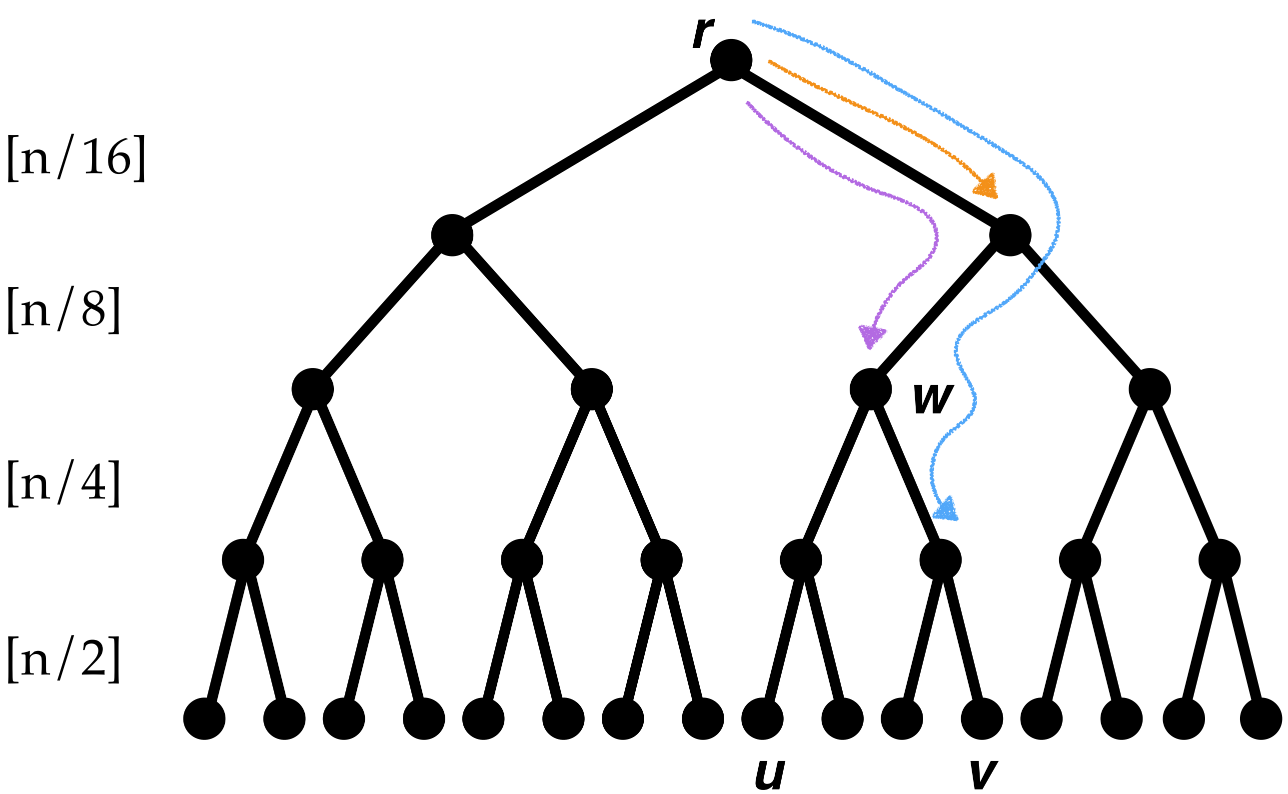
$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

***label*(*v*):**

(1) $d(v,r)$ — $\log(n)$

(2) ↘ , ↘ , ↘ ,… — $1/2\log^2(n)$

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

log(n)

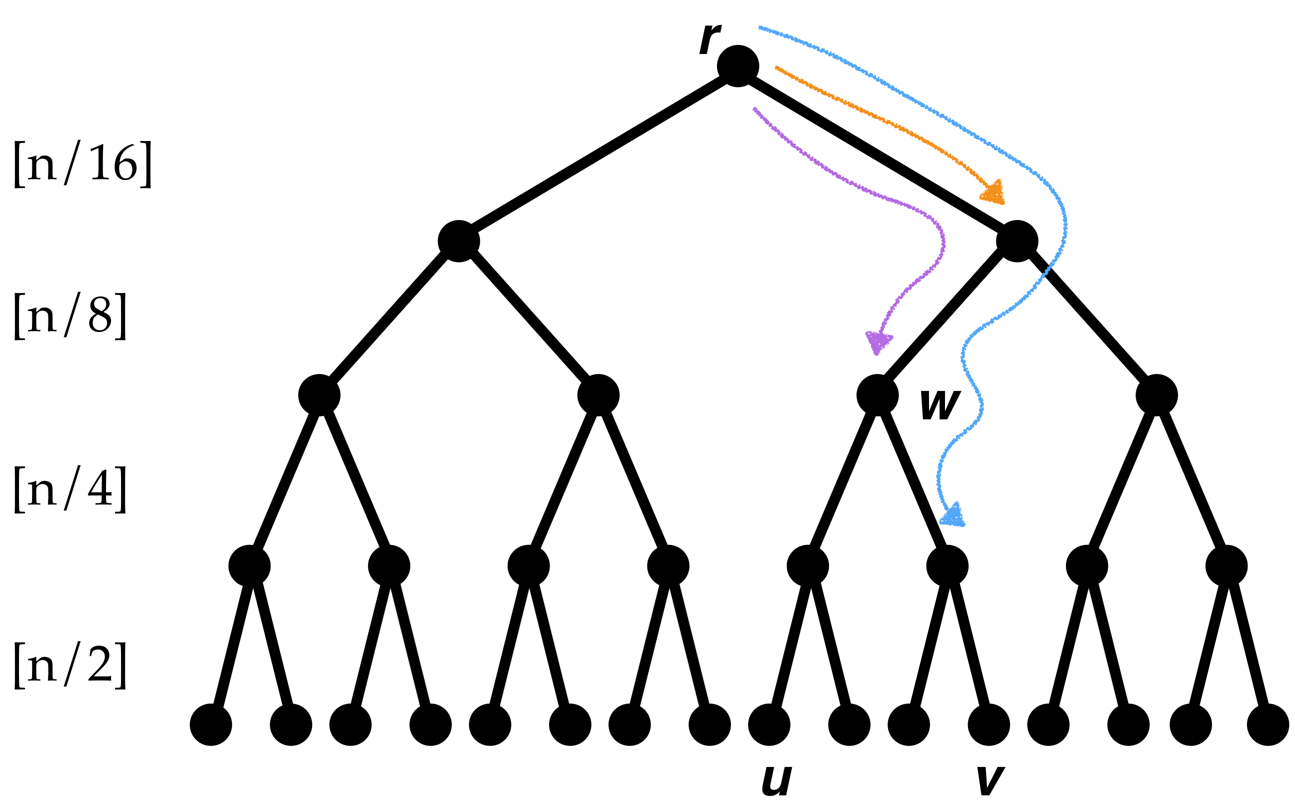
(1) $d(v,r)$

(2) , , ,...

$1/2\log^2(n)$

***label*(*u*):**

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

log(n)

(1) $d(v,r)$

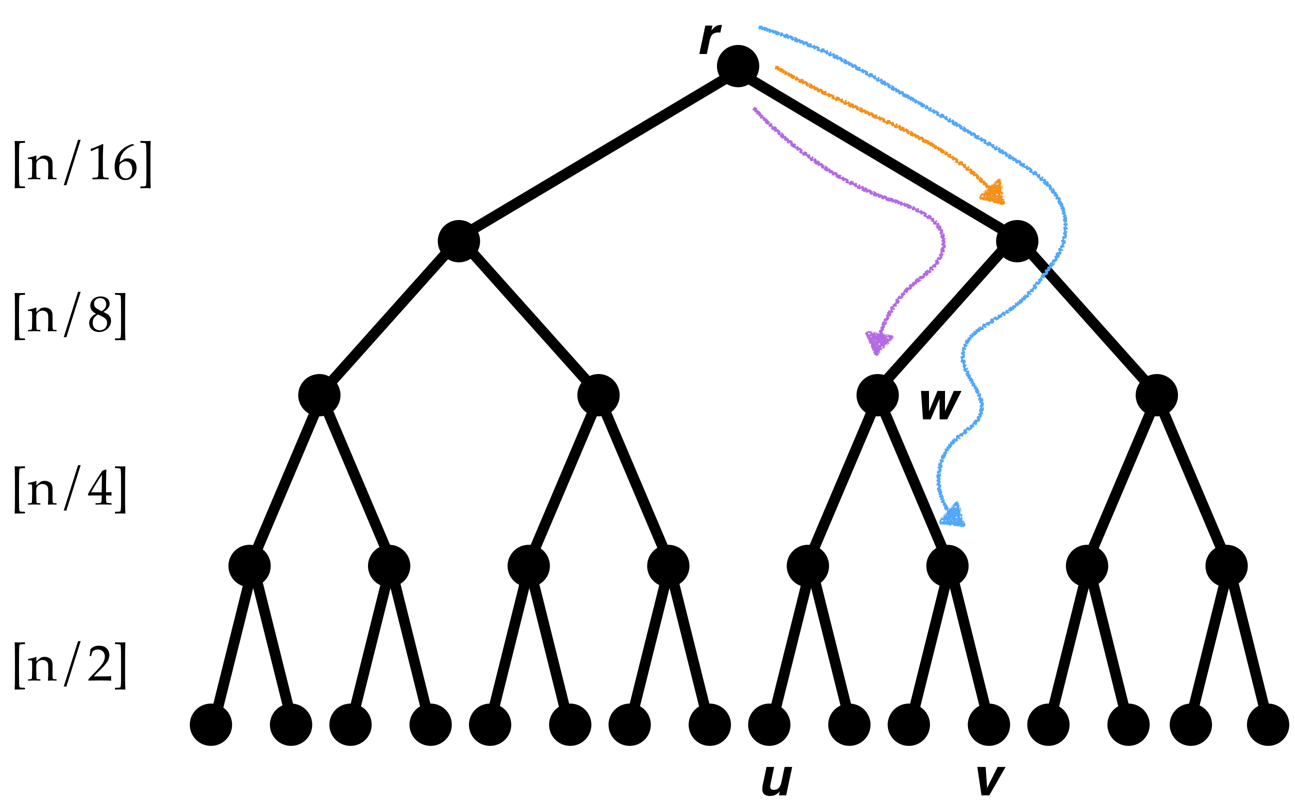
(2) , , ,...

1/2log^2(n)

***label*(*u*):**

(1)

Sketch Idea



$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

***label*(*v*):**

log(n)

(1) $d(v,r)$

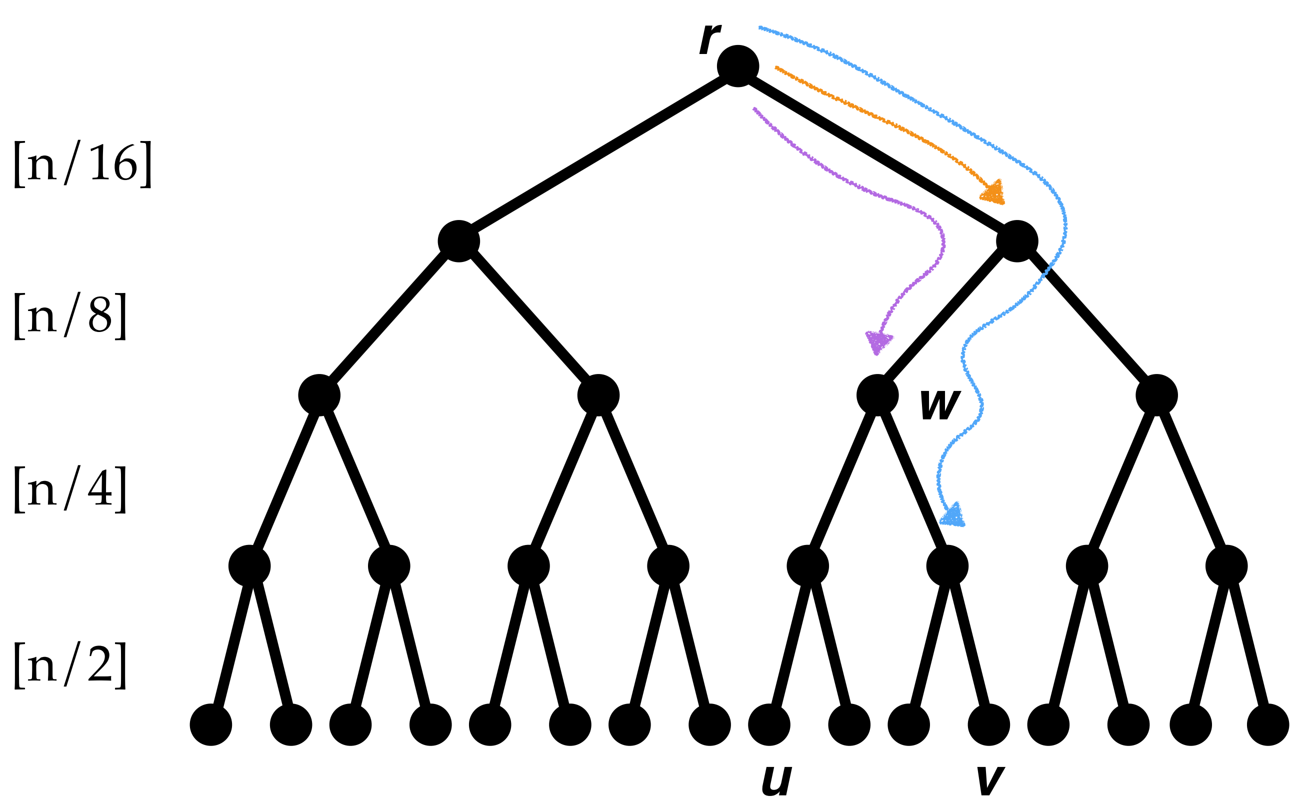
(2) , , ,...

1/2log^2(n)

***label*(*u*):**

(1) $d(u,r)$

Sketch Idea

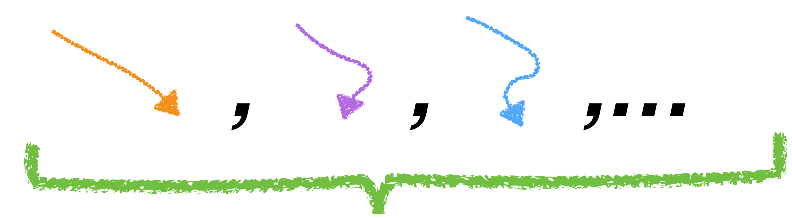


$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

***label*(*v*):**

log(n)

(1) $d(v,r)$

(2) 

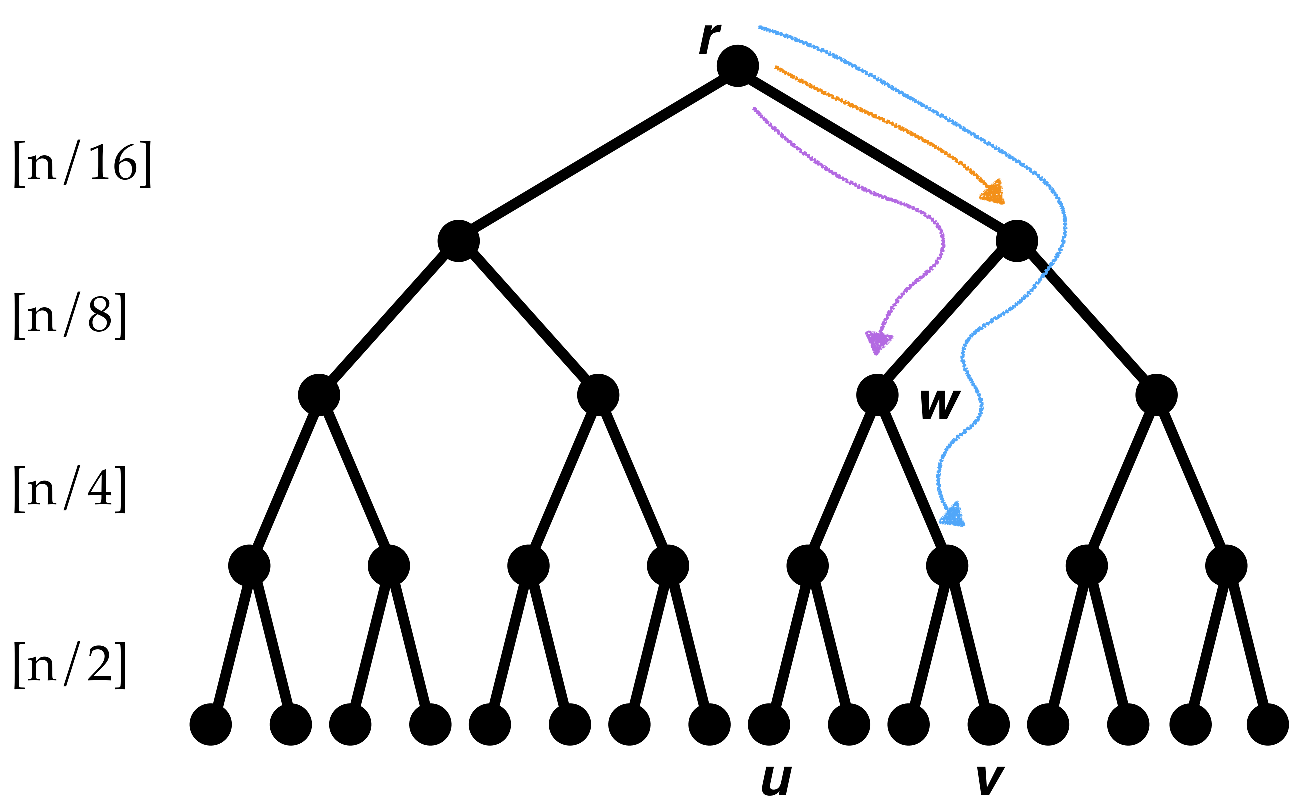
1/2log^2(n)

***label*(*u*):**

(1) $d(u,r)$

(2)

Sketch Idea



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***label*(*v*):**

(1) $d(v,r)$ — log(n)

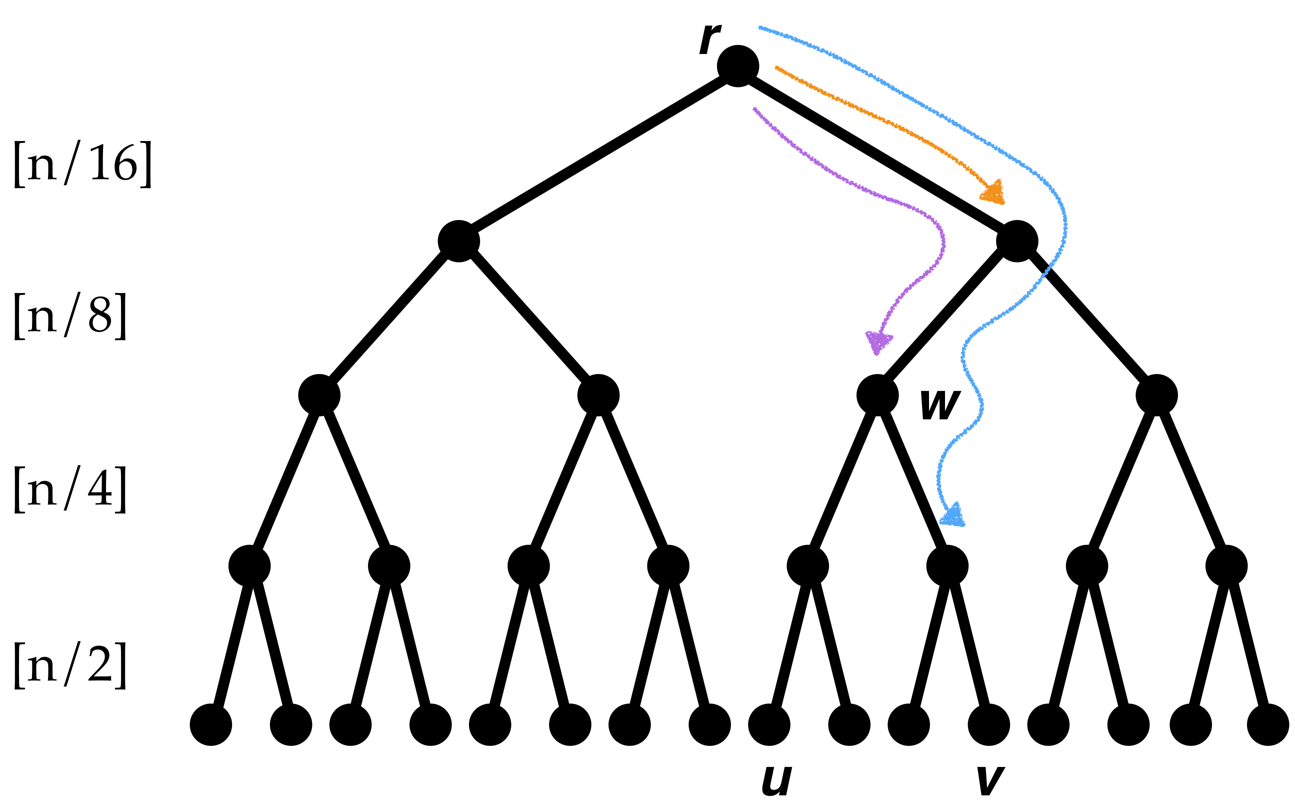
(2) , , ,… — $1/2\log^2(n)$

***label*(*u*):**

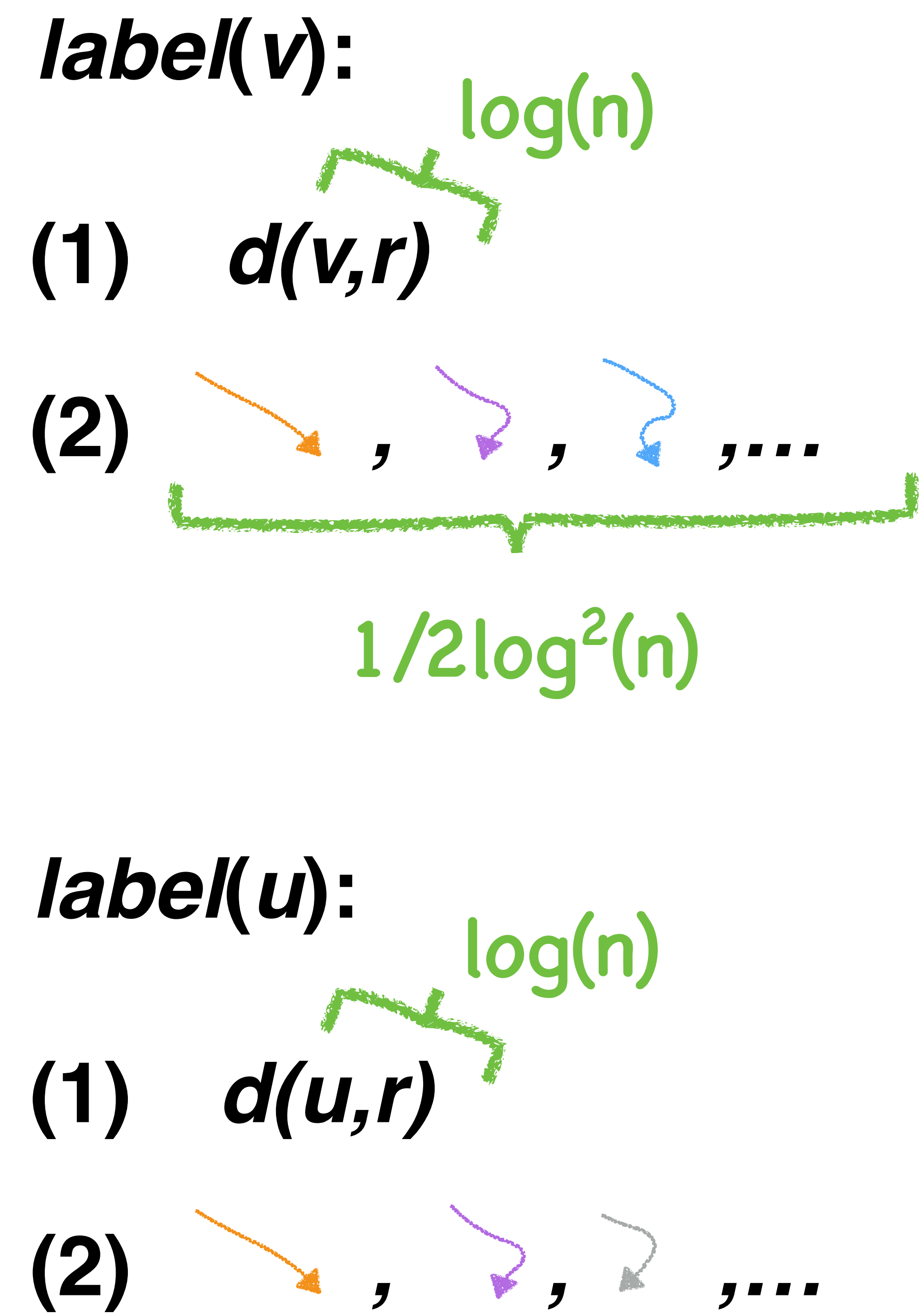
(1) $d(u,r)$

(2) , , ,…

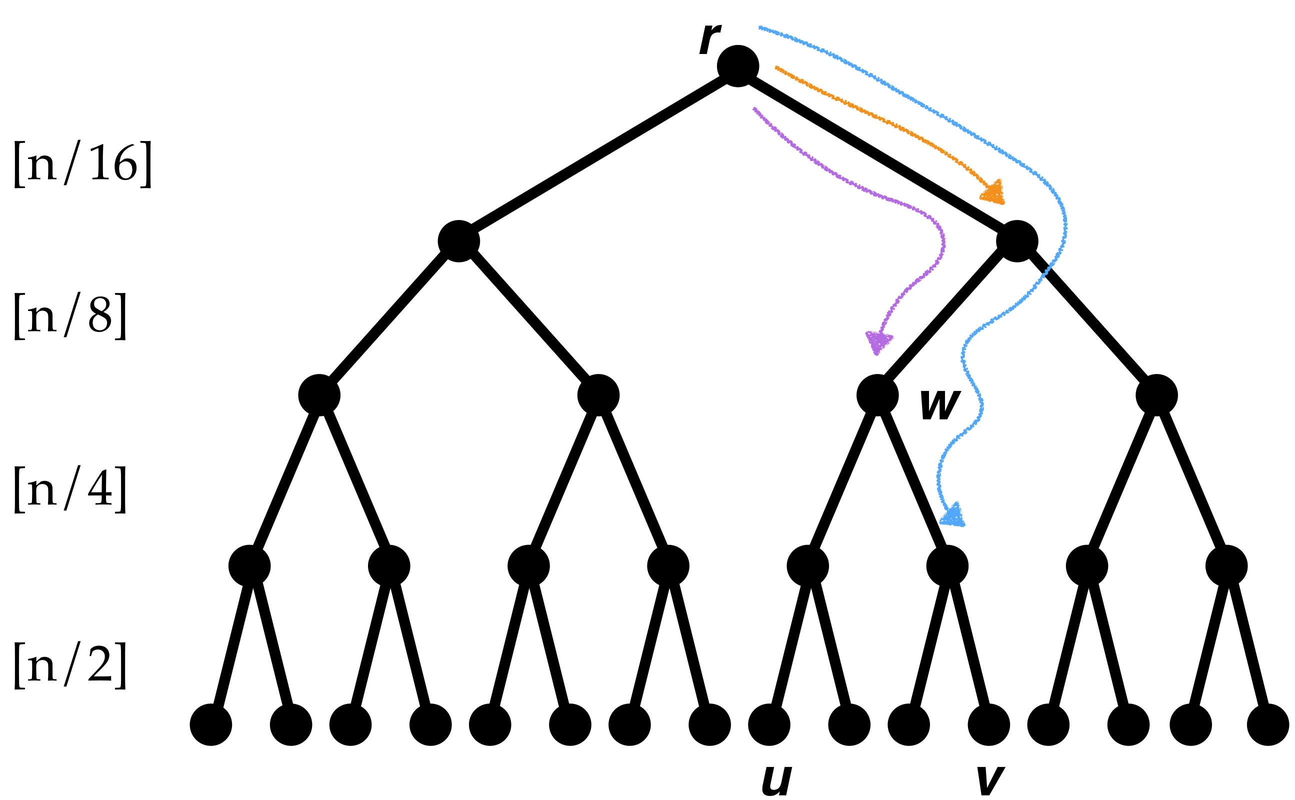
Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$



Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

log(n)

(1) $d(v,r)$

(2) , , ,...

1/2log^2(n)

***label*(*u*):**

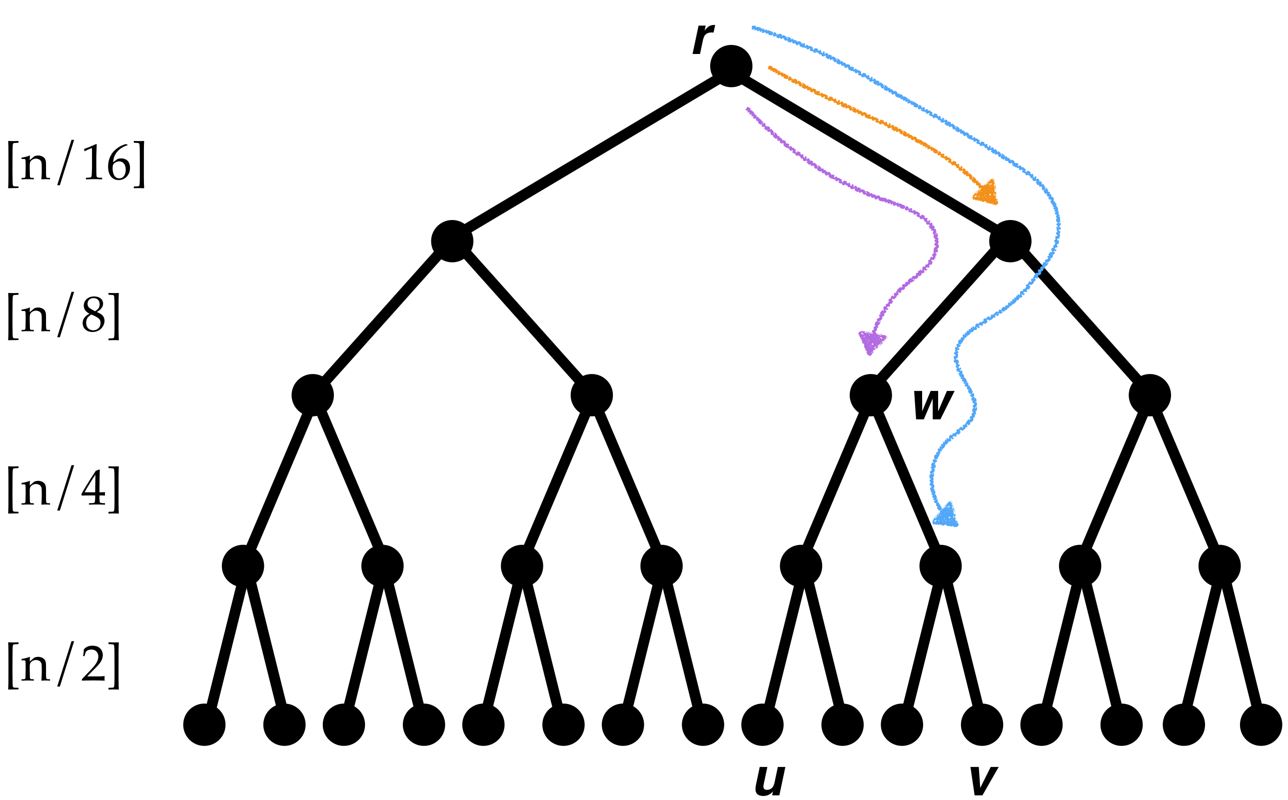
log(n)

(1) $d(u,r)$

(2) , , ,...

1/2log^2(n)

Sketch Idea



$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

***label*(*v*):**

log(n)

(1) $d(v,r)$

(2) , , ,...

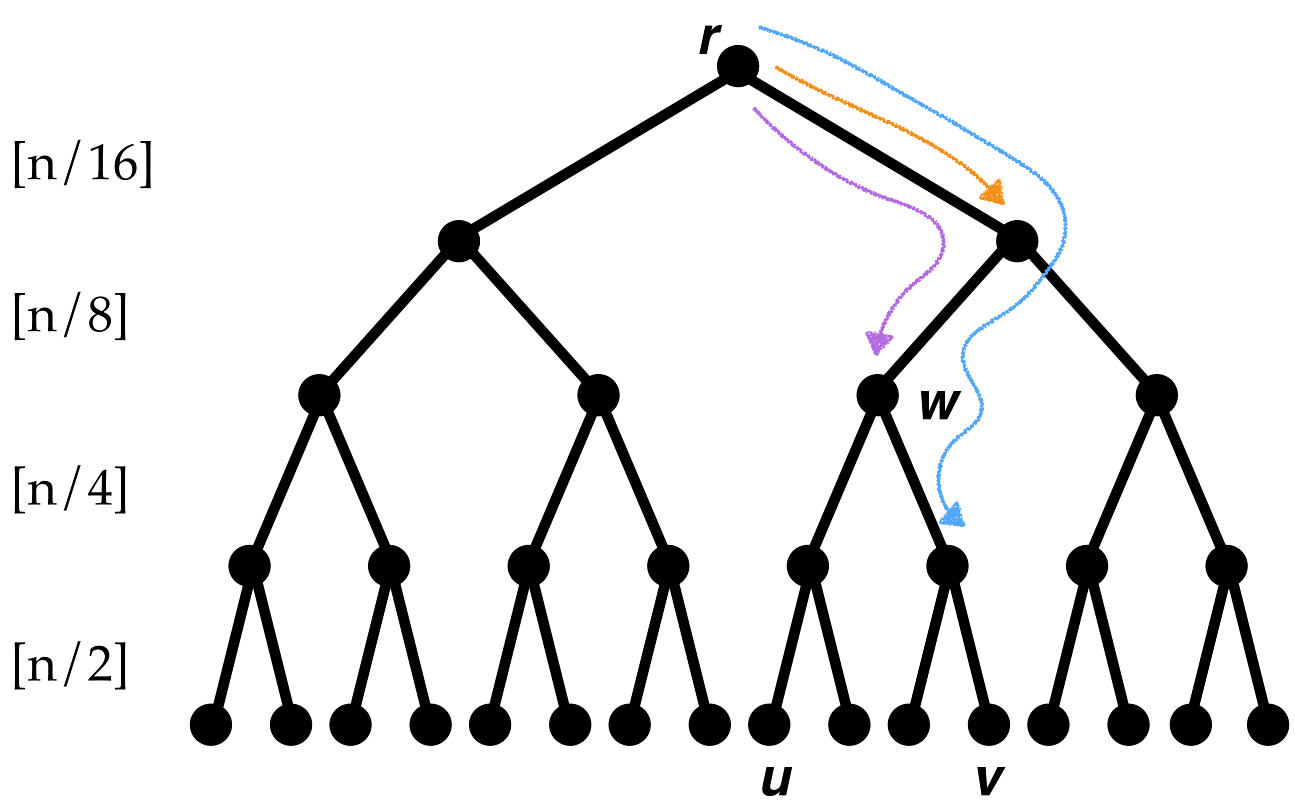
***label*(*u*):**

log(n)

(1) $d(u,r)$

(2) , , ,...

Sketch Idea



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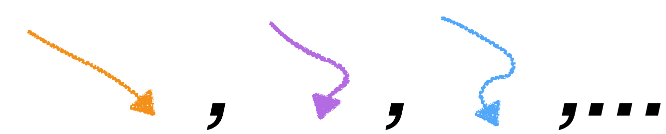
***label*(*v*):**

log(n)

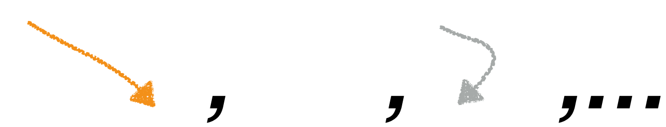
(1) $d(v,r)$

(2) 

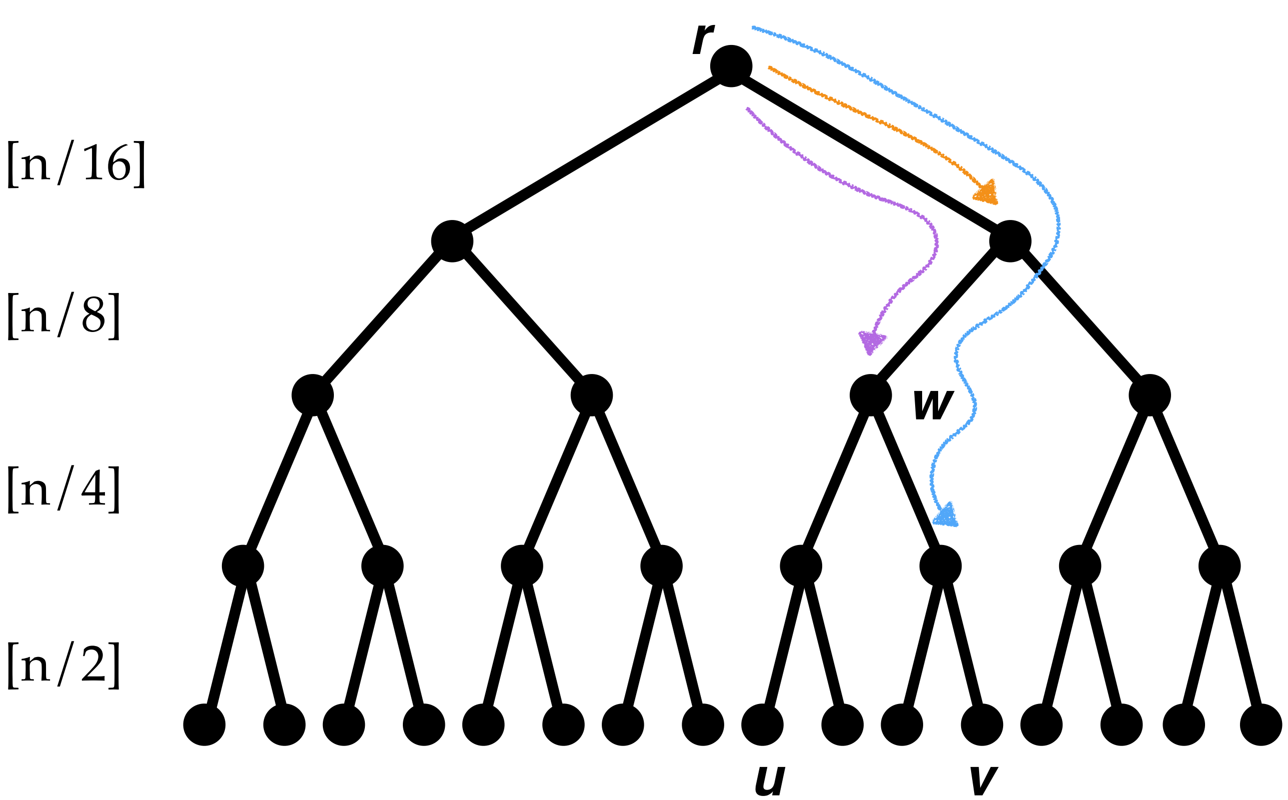
***label*(*u*):**

log(n)

(1) $d(u,r)$

(2) 

Sketch Idea



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***label*(*v*):**

log(n)

(1) $d(v,r)$

(2) , , ,...

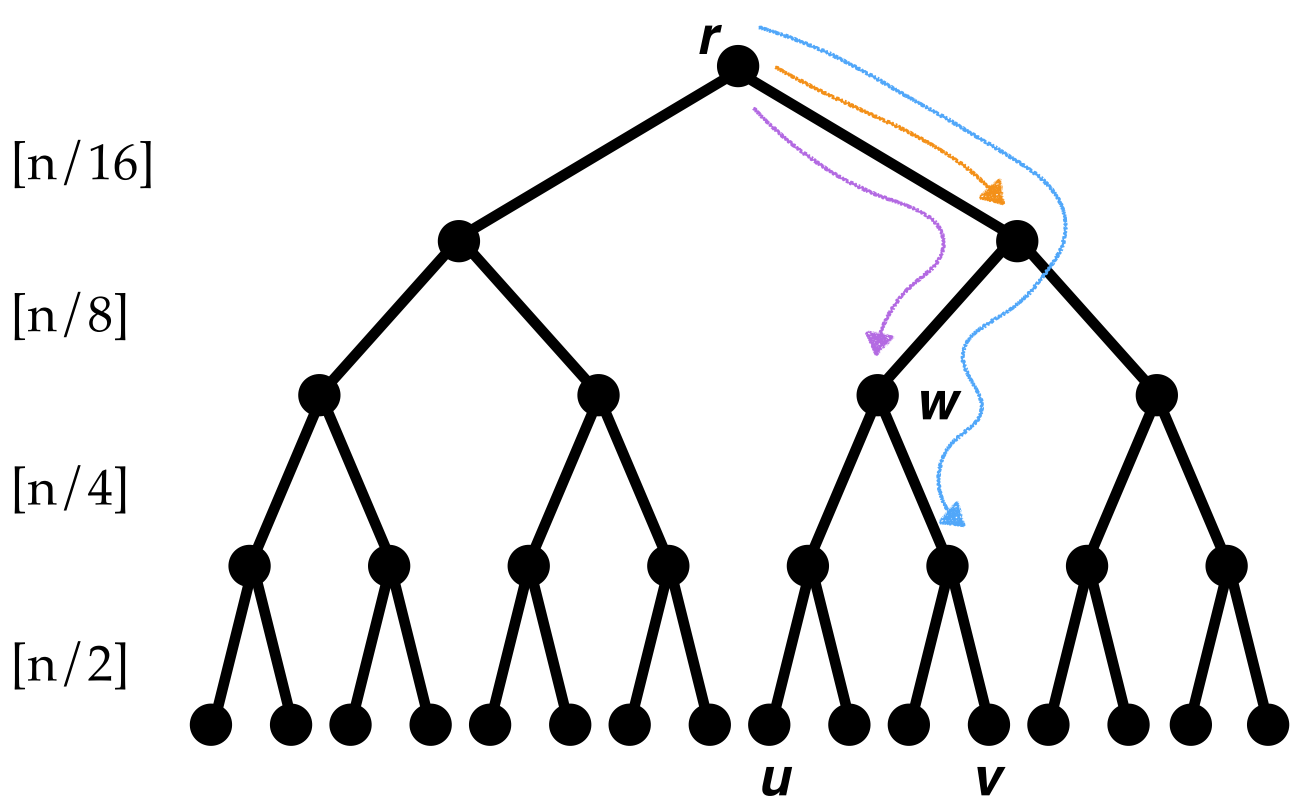
***label*(*u*):**

log(n)

(1) $d(u,r)$

(2) , , ,...

Sketch Idea



$$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$$

***label*(*v*):**

(1) $d(v,r)$ — log(n)

(2) , , ,... — $1/4\log^2(n)$

***label*(*u*):**

(1) $d(u,r)$ — log(n)

(2) , , ,... — $1/4\log^2(n)$

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
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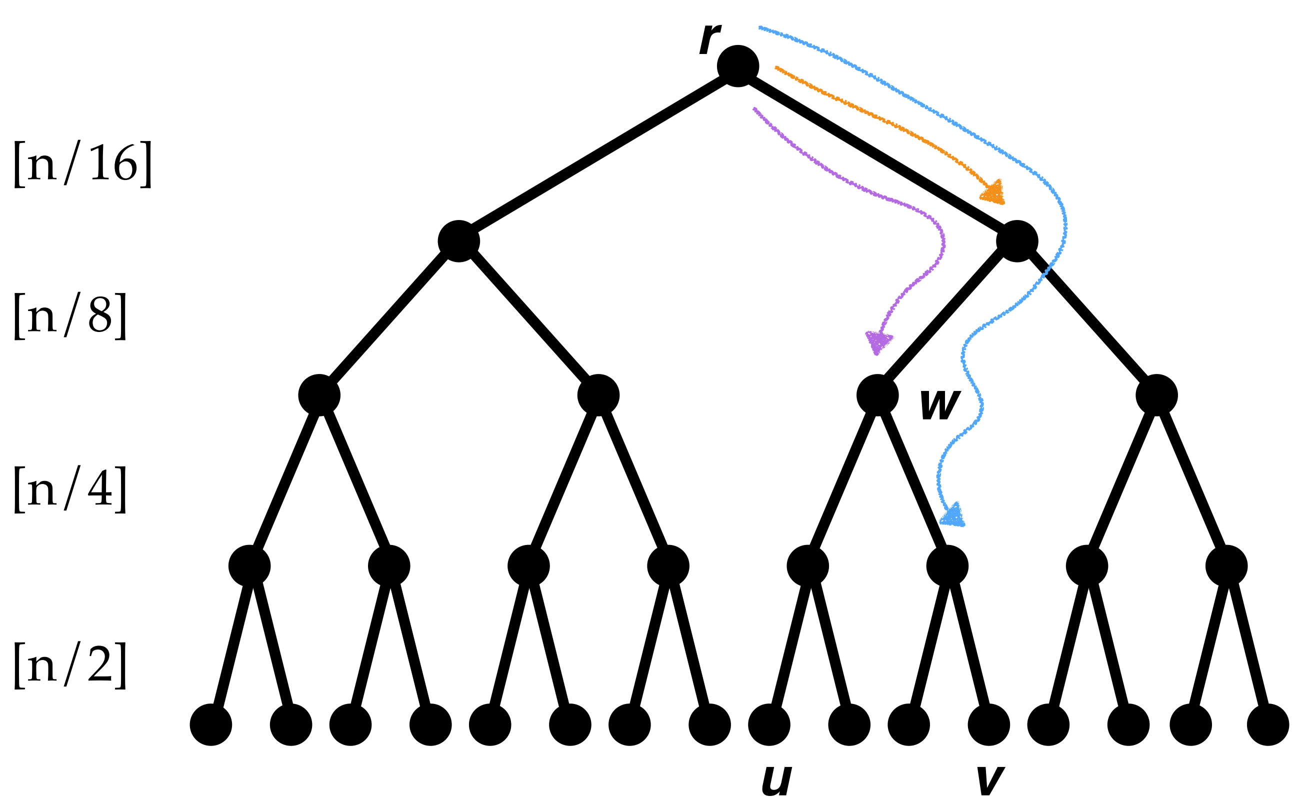
Sketch Idea



$d(u,v) = d(u,r) + d(v,r) - 2d(r,w)$

***label*(*v*):**

(1) $d(v,r)$ — log(n)

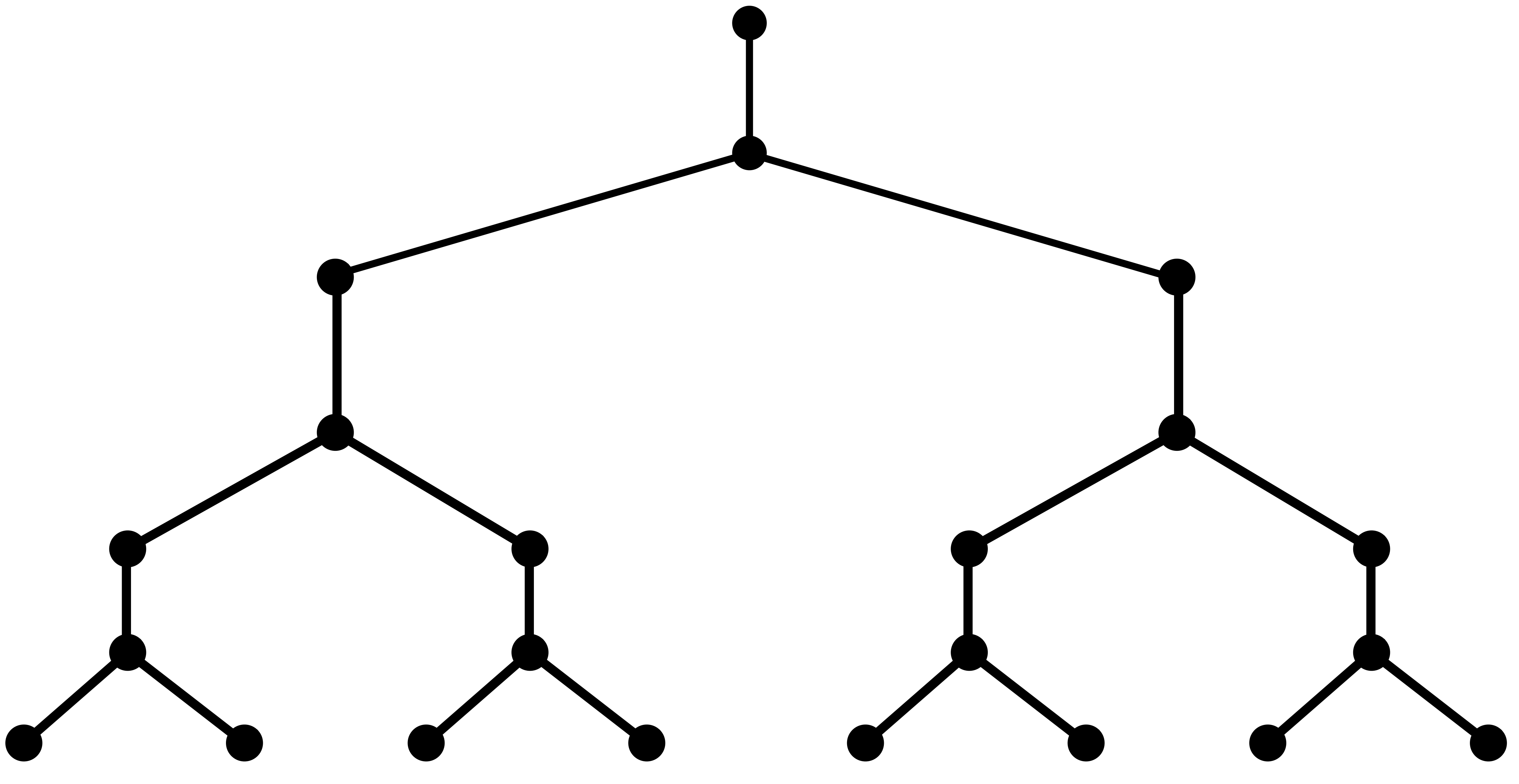
(2) , , ,... — 1/4log²(n)

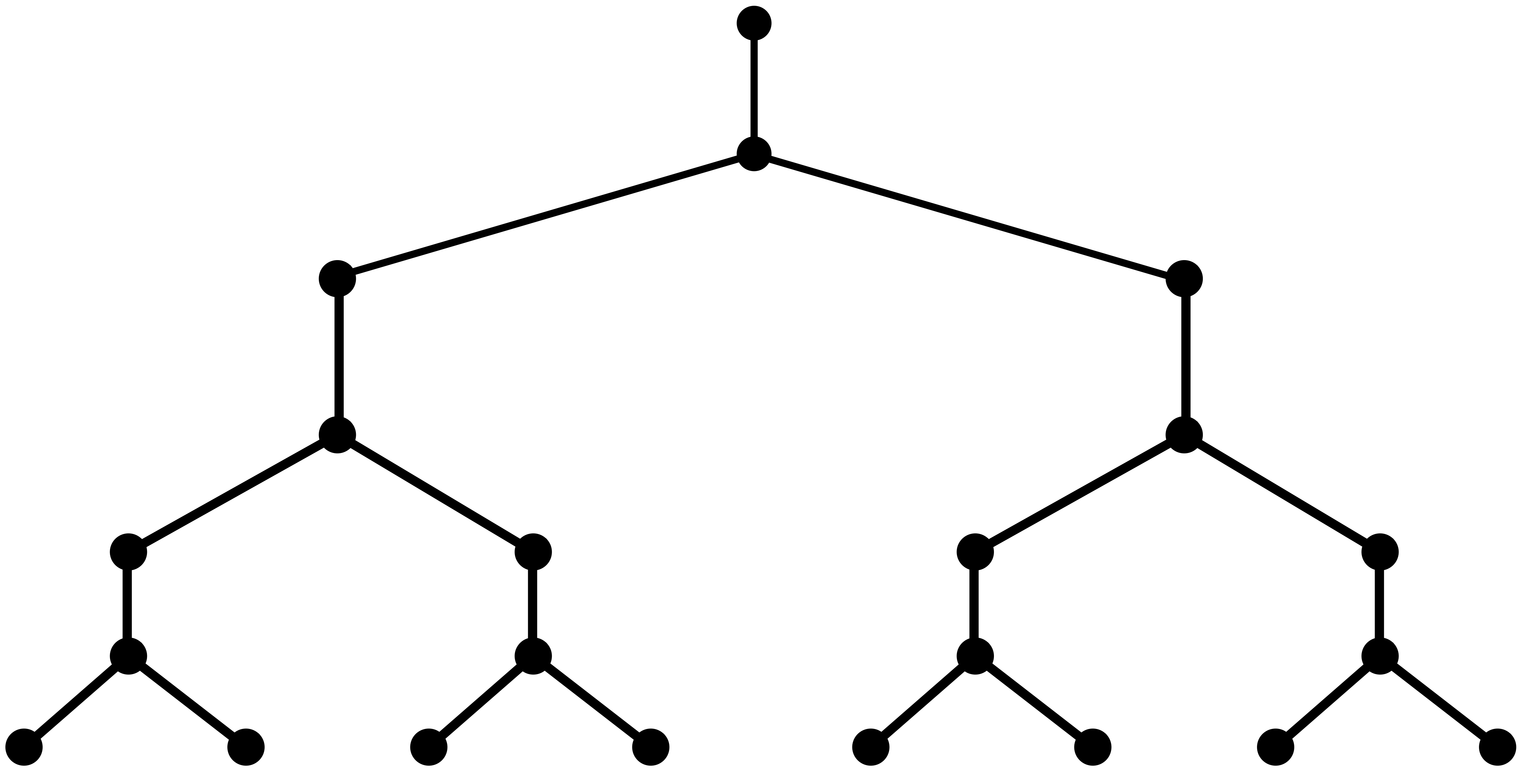
***label*(*u*):**

(1) $d(u,r)$ — log(n)

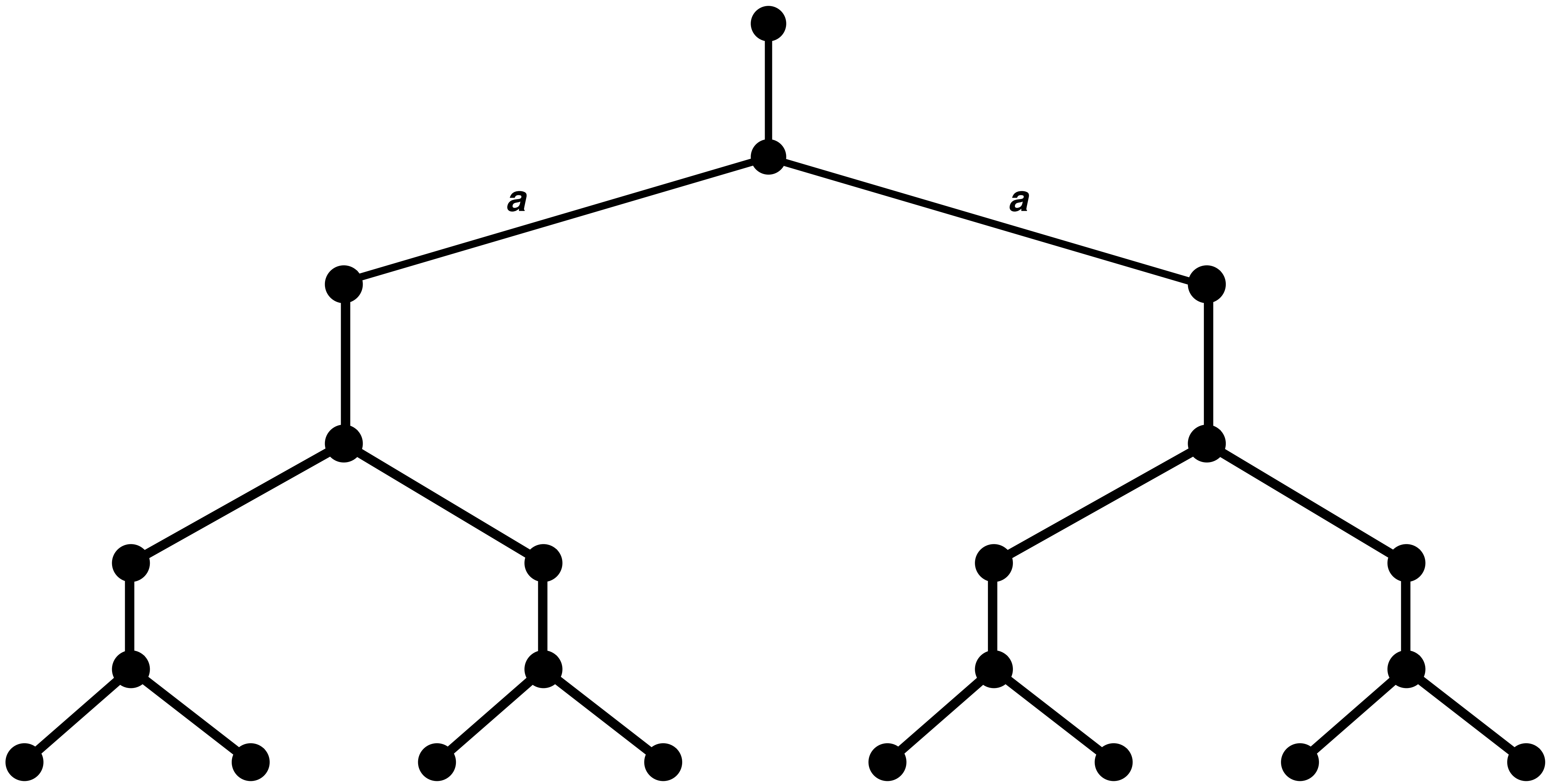
(2) , , ,... — 1/4log²(n)

Exact
Lower bound

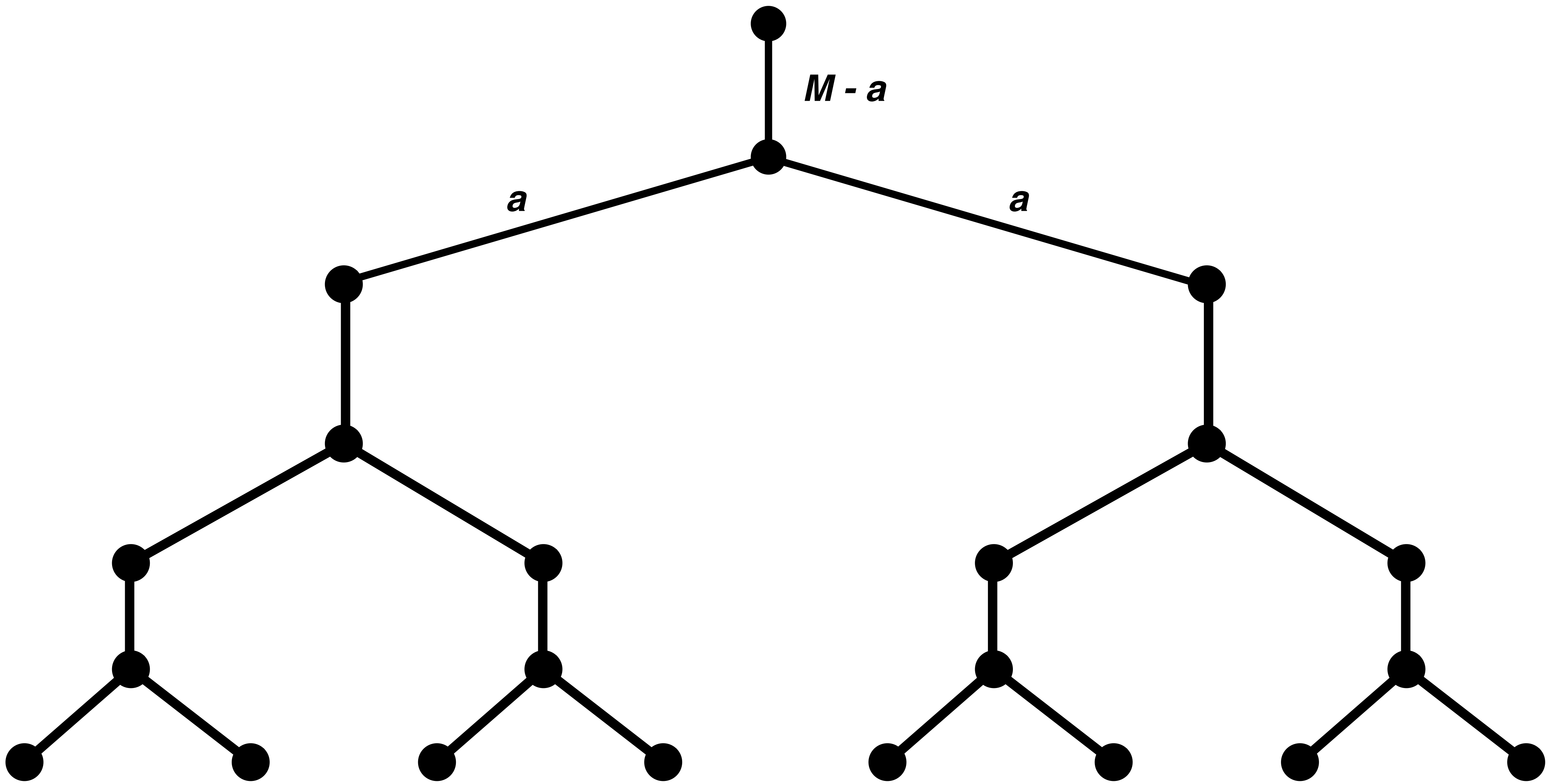




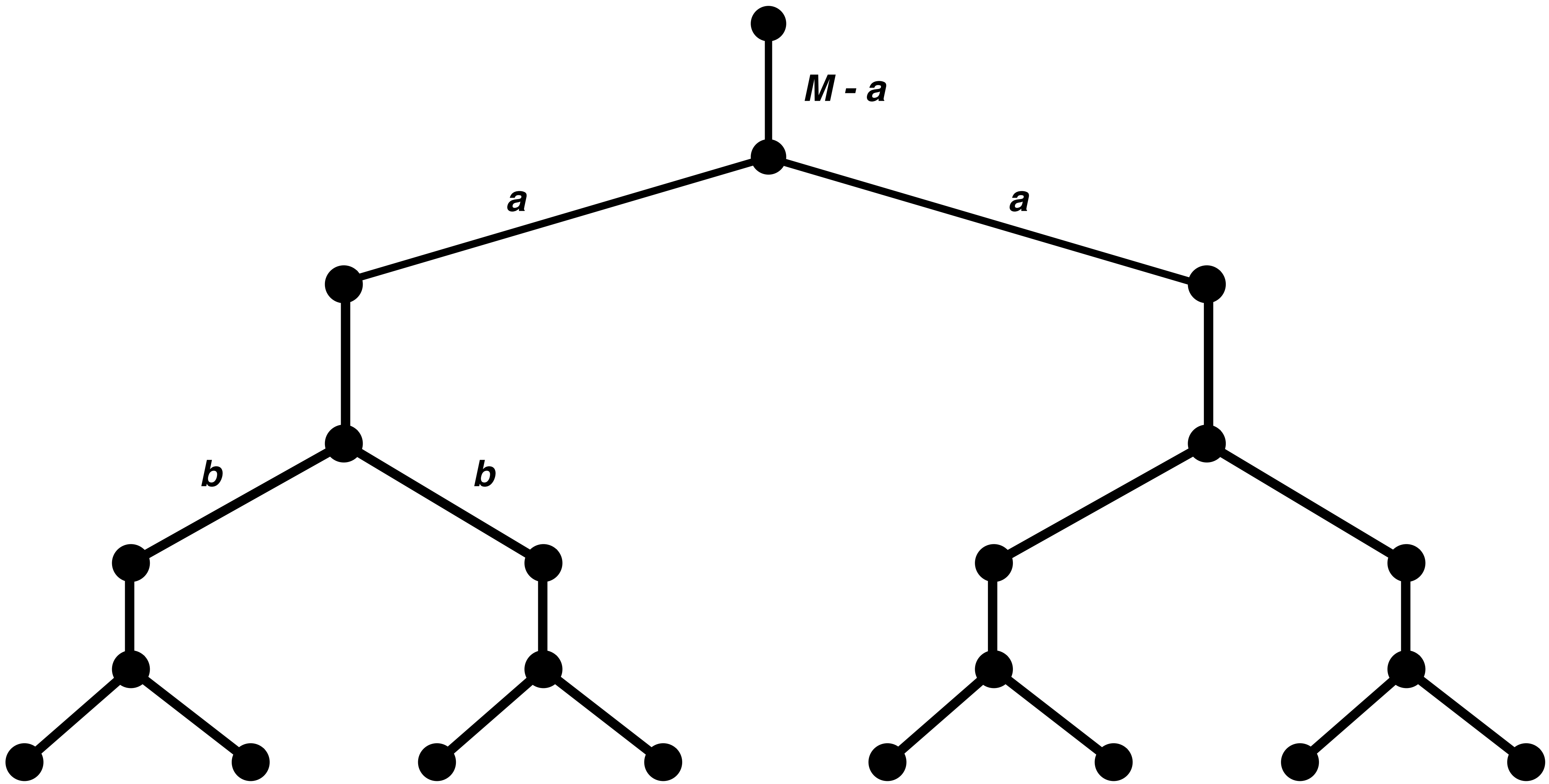
(h,M)-tree



(h,M)-tree



(h,M)-tree



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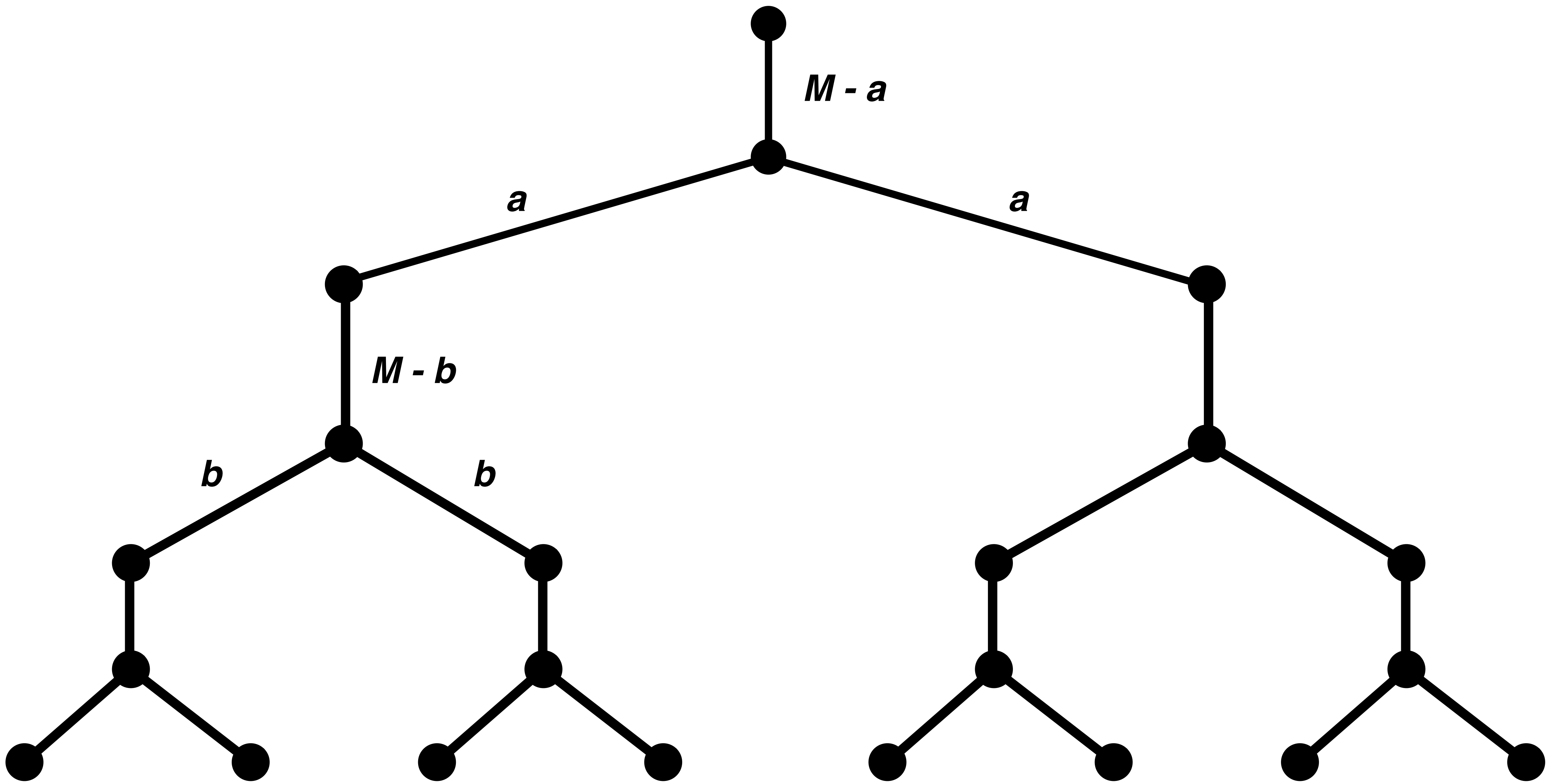
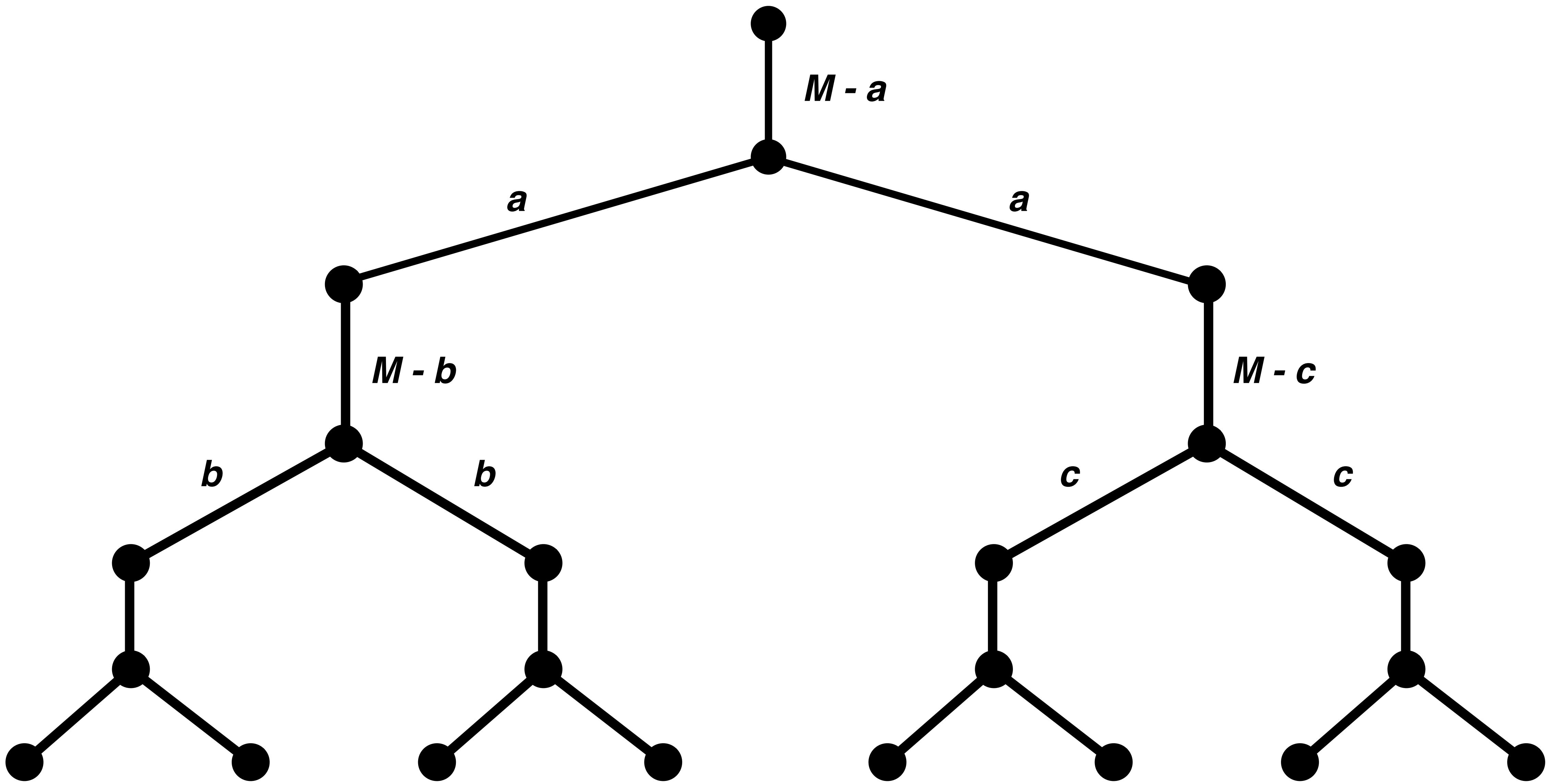


(h,M)-tree



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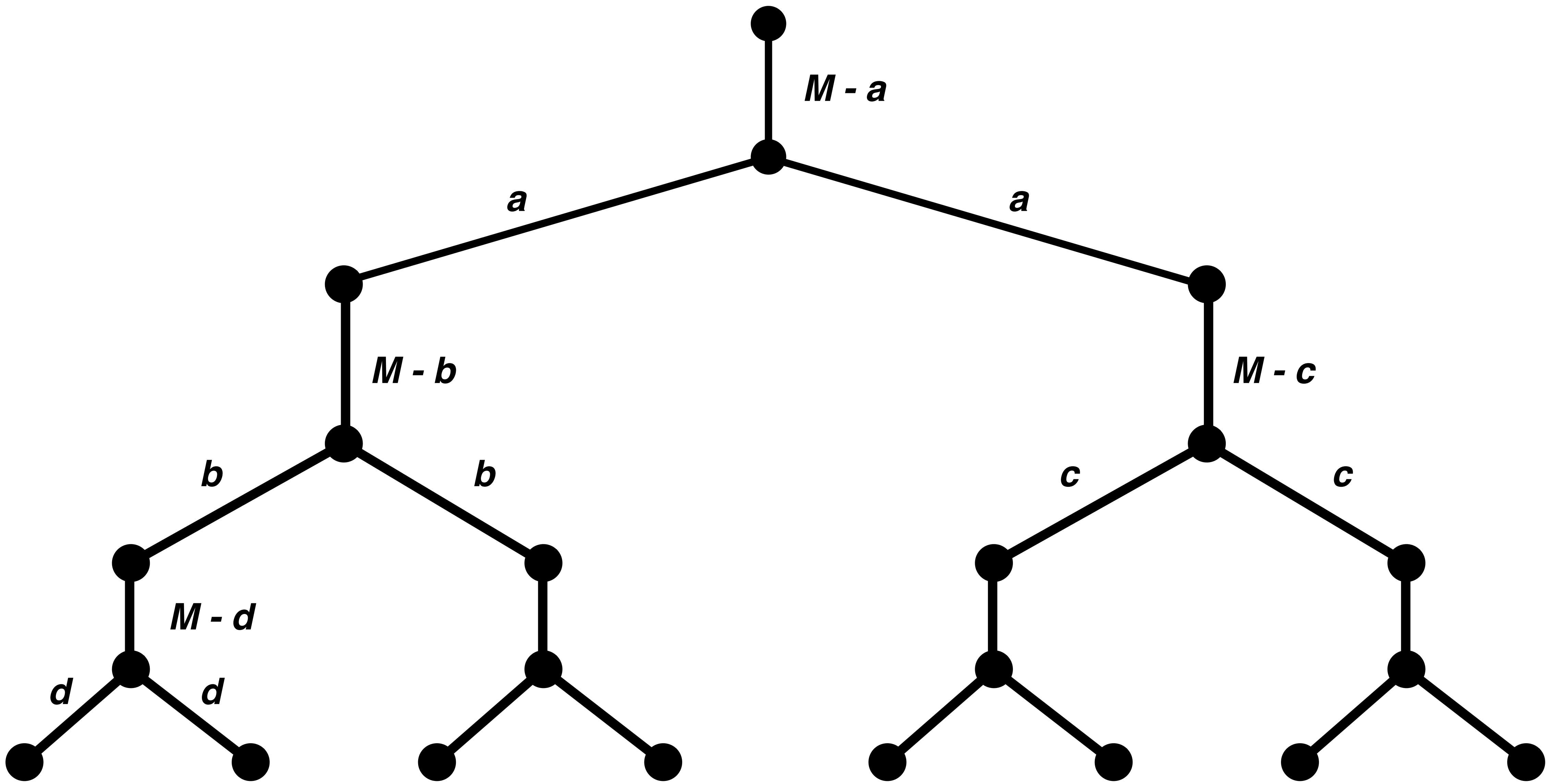
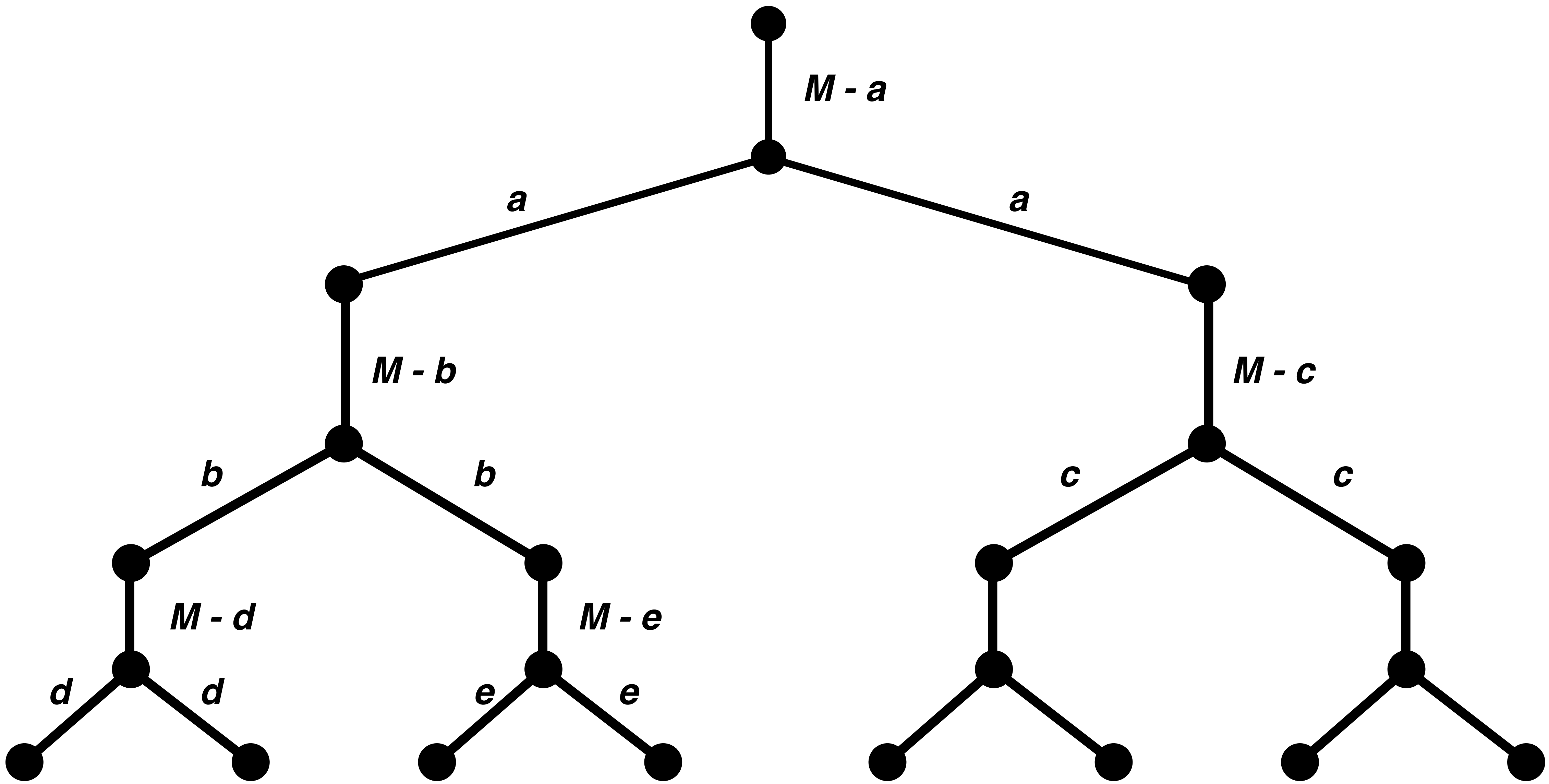


(h,M)-tree



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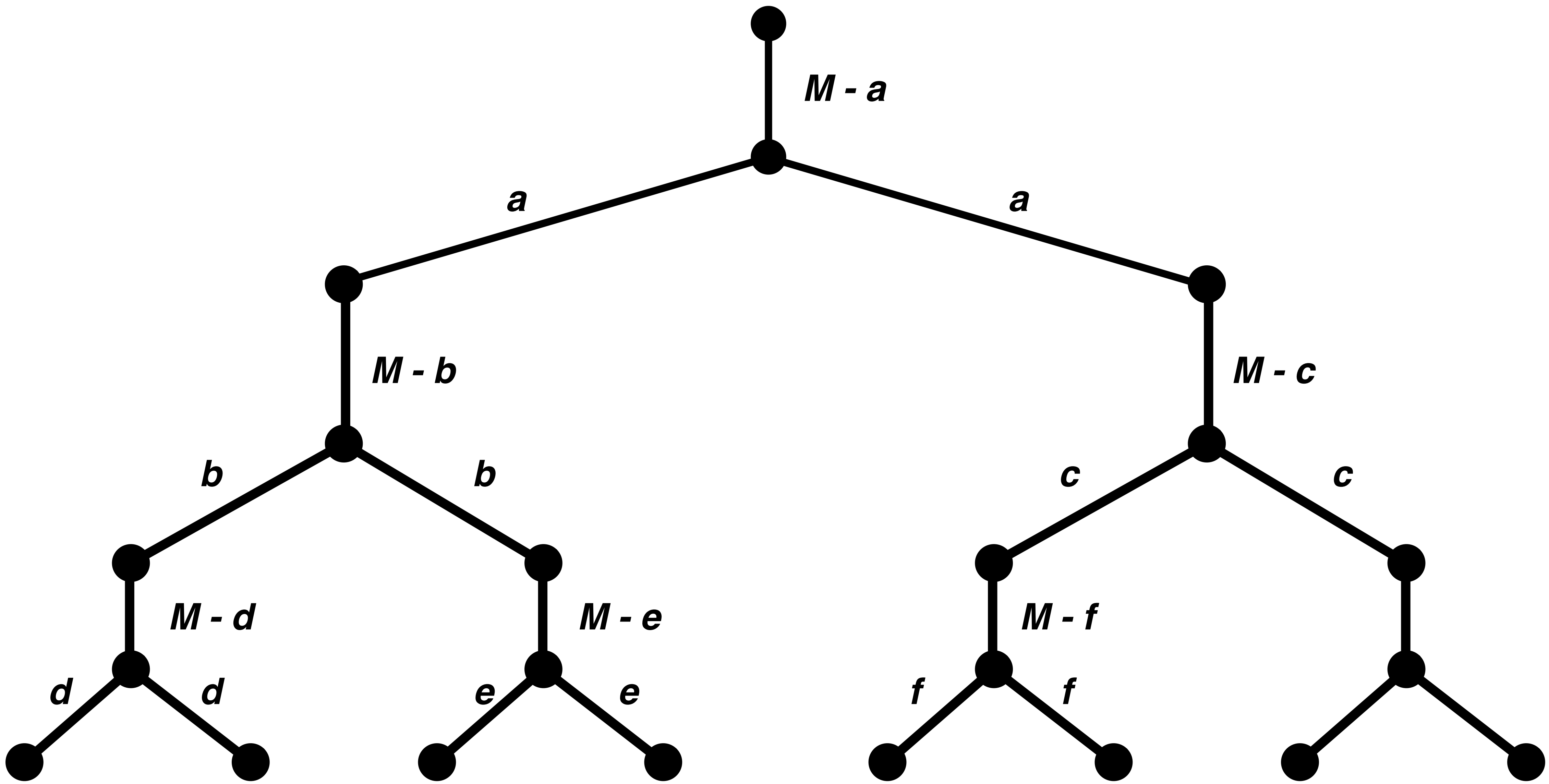
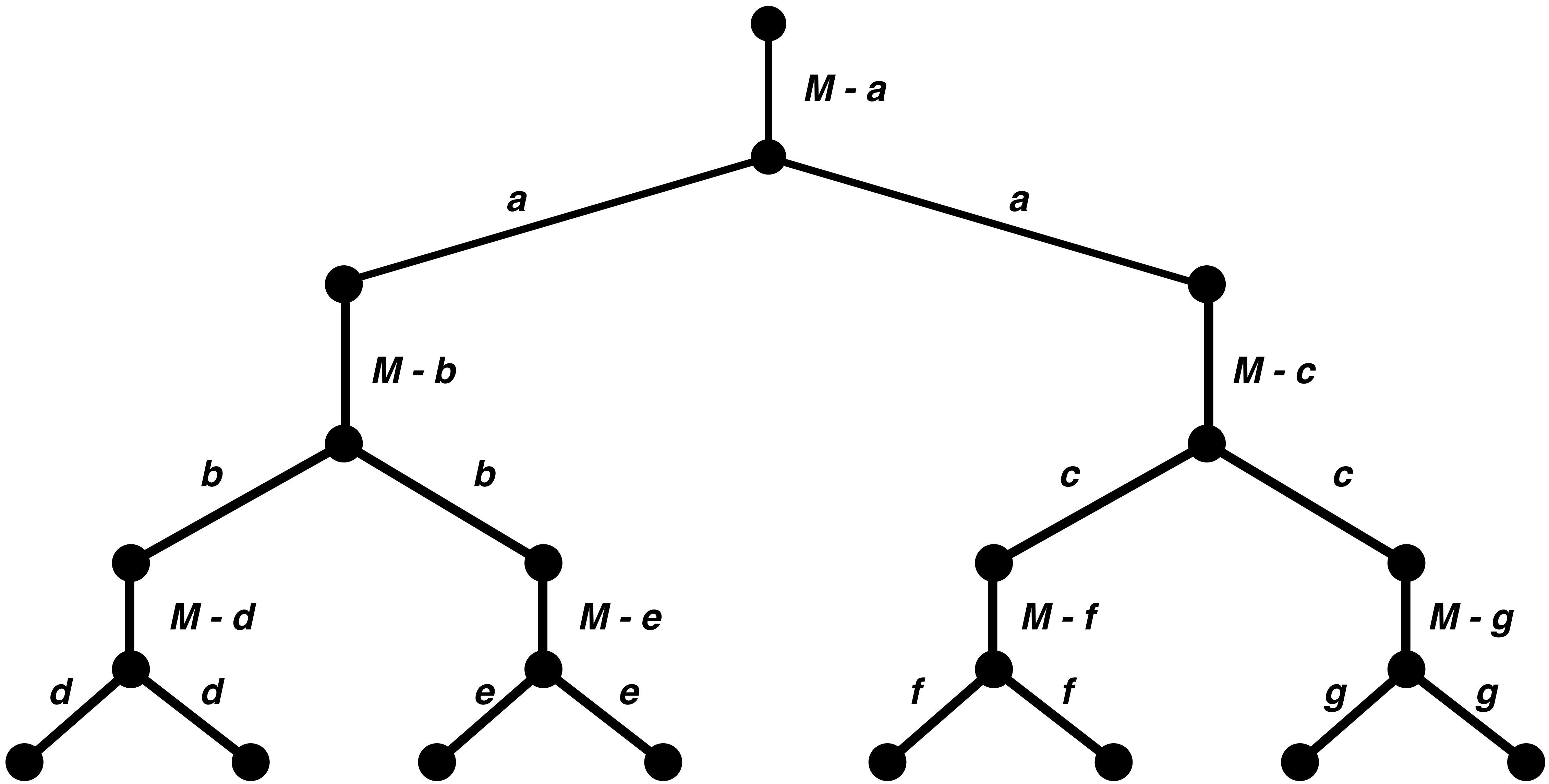


(h,M)-tree



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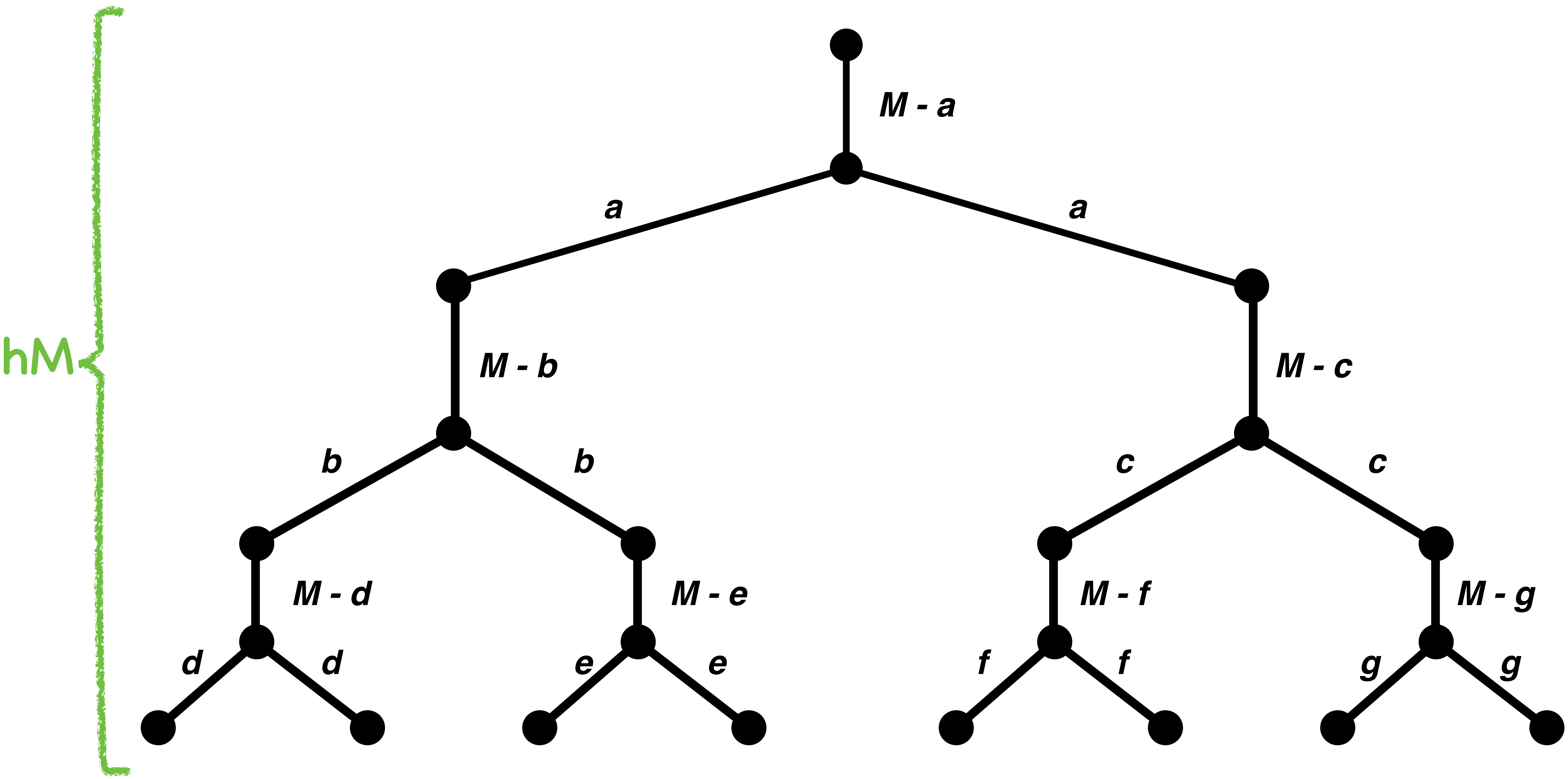

(h,M)-tree

$$g(h, M) \geq M^{\frac{h}{2}}$$

unique labels for this family

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unique labels
for this family

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maximize when:

unique labels for this family

$$g(h, M) \geq M^{\frac{h}{2}}$$

maximize when:

$$M \coloneqq \sqrt{n}$$

unique labels
for this family

$$g(h, M) \geq M^{\frac{h}{2}}$$

maximize when:

$$M \coloneqq \sqrt{n}$$
$$h \coloneqq \log \sqrt{n}$$

$$\frac{1}{8}\log^2 n$$

$$\frac{1}{8}\log^2 n$$

[Gavoille et al. 2001]

$$\frac{1}{4}\log^2 n$$

$$\frac{1}{4}\log^2 n$$

[Alstrup et al. 2016]

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate		
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

$$\frac{1}{4}\log^2 n$$

[Alstrup et al. 2016]

Approximate Lower Bound

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate		
k-Distance $k \leq \log n$		
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Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$	
k-Distance $k \leq \log n$		
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Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Approximate Lower Bound

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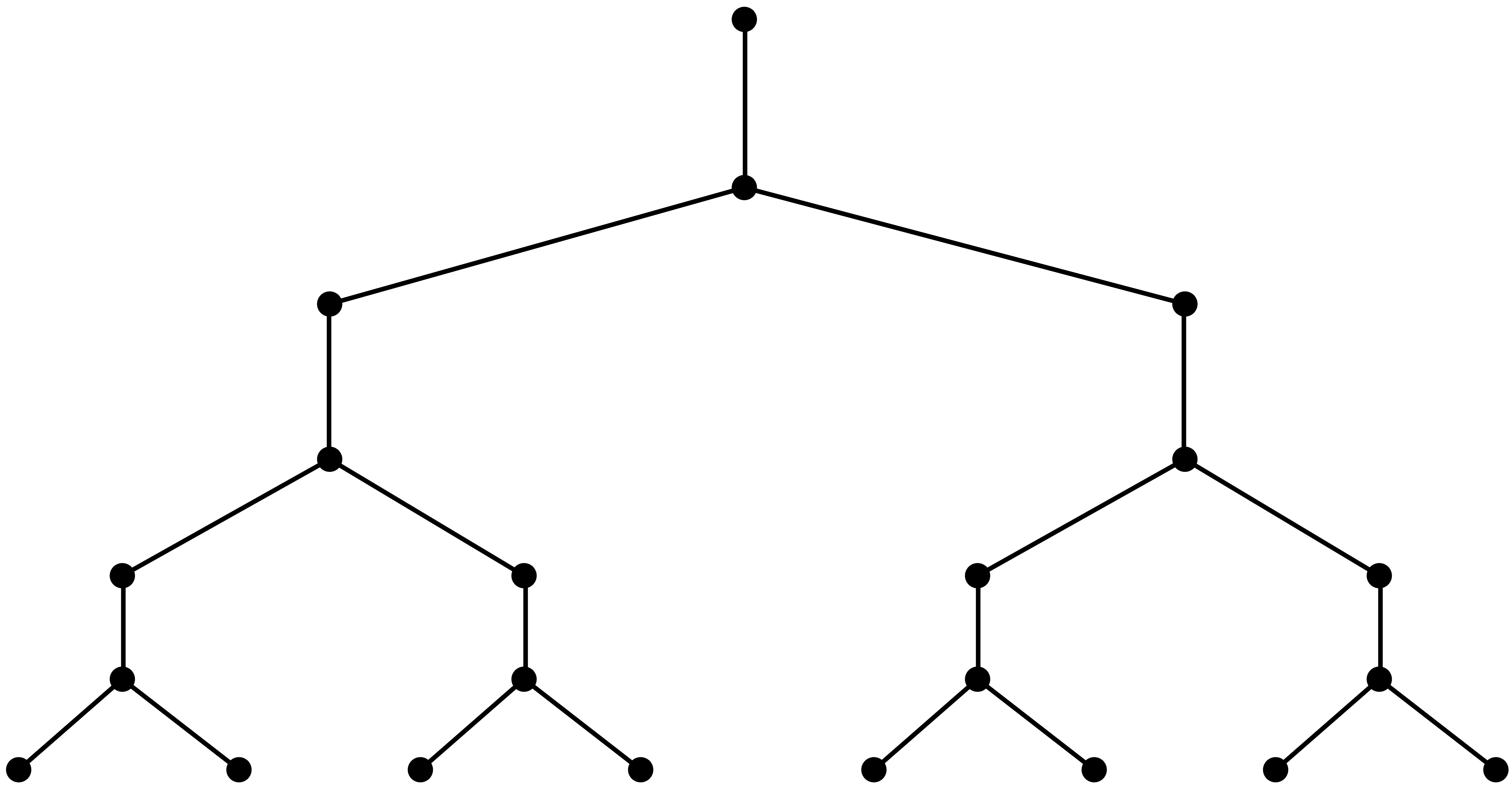
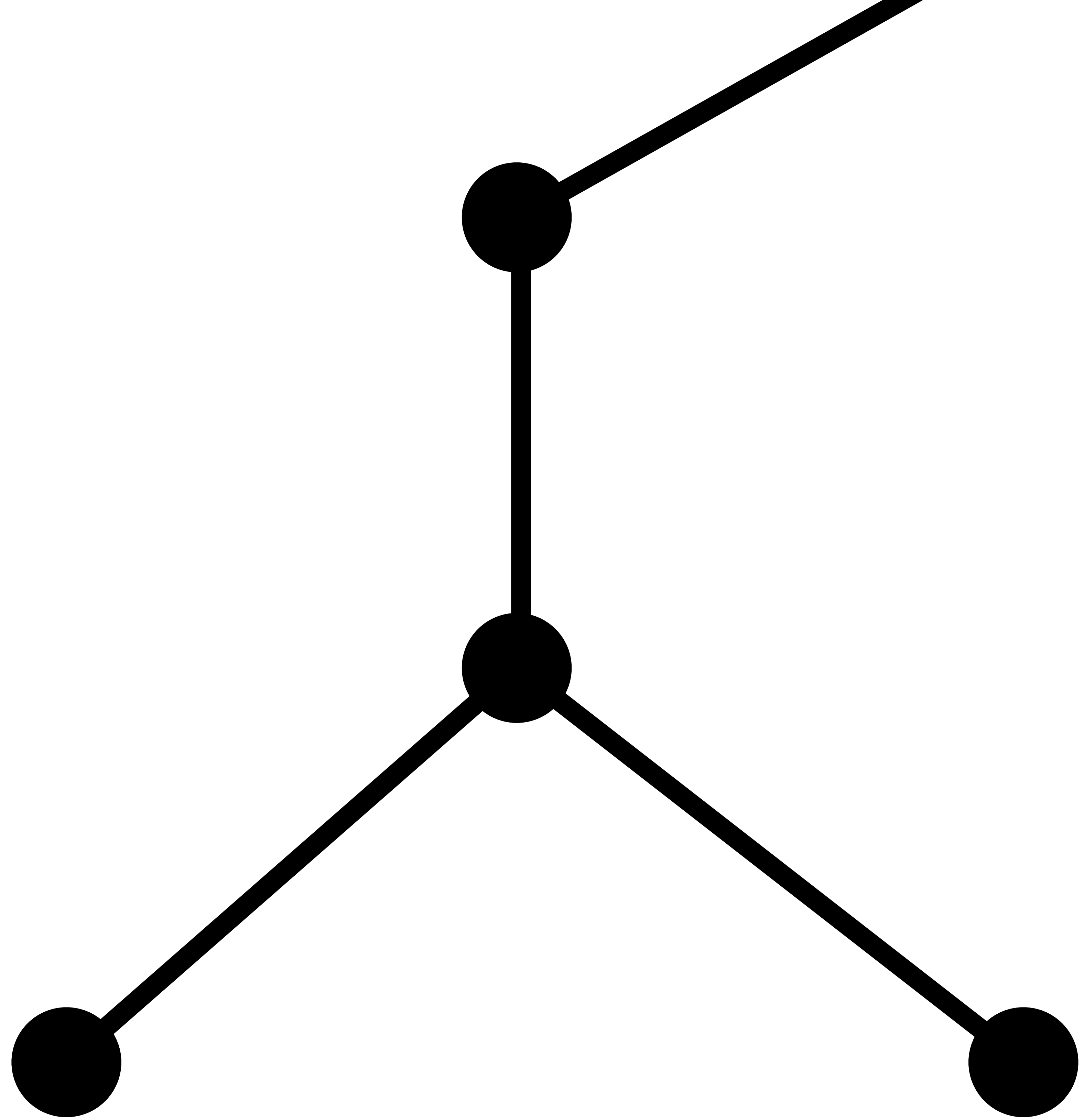
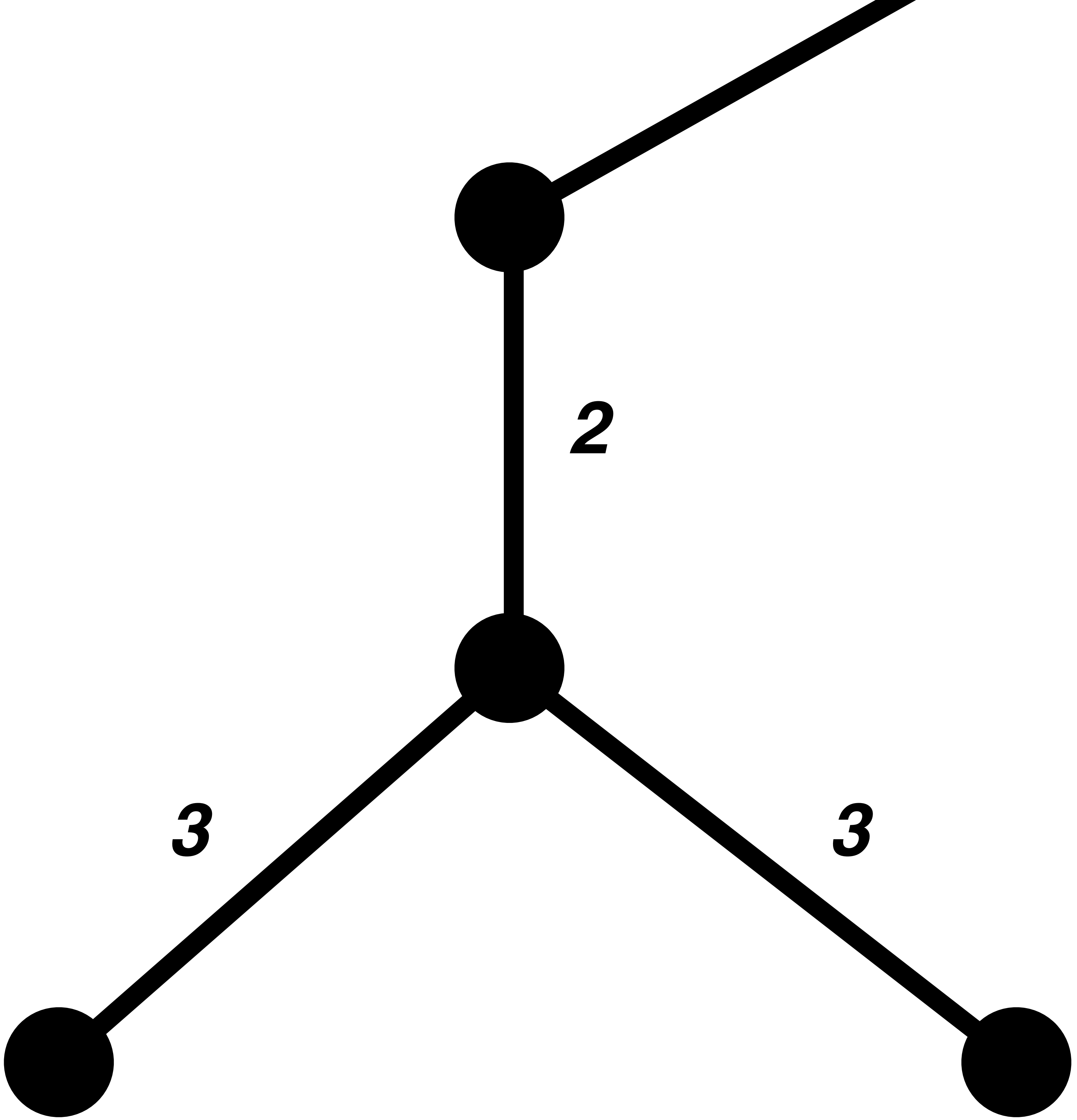
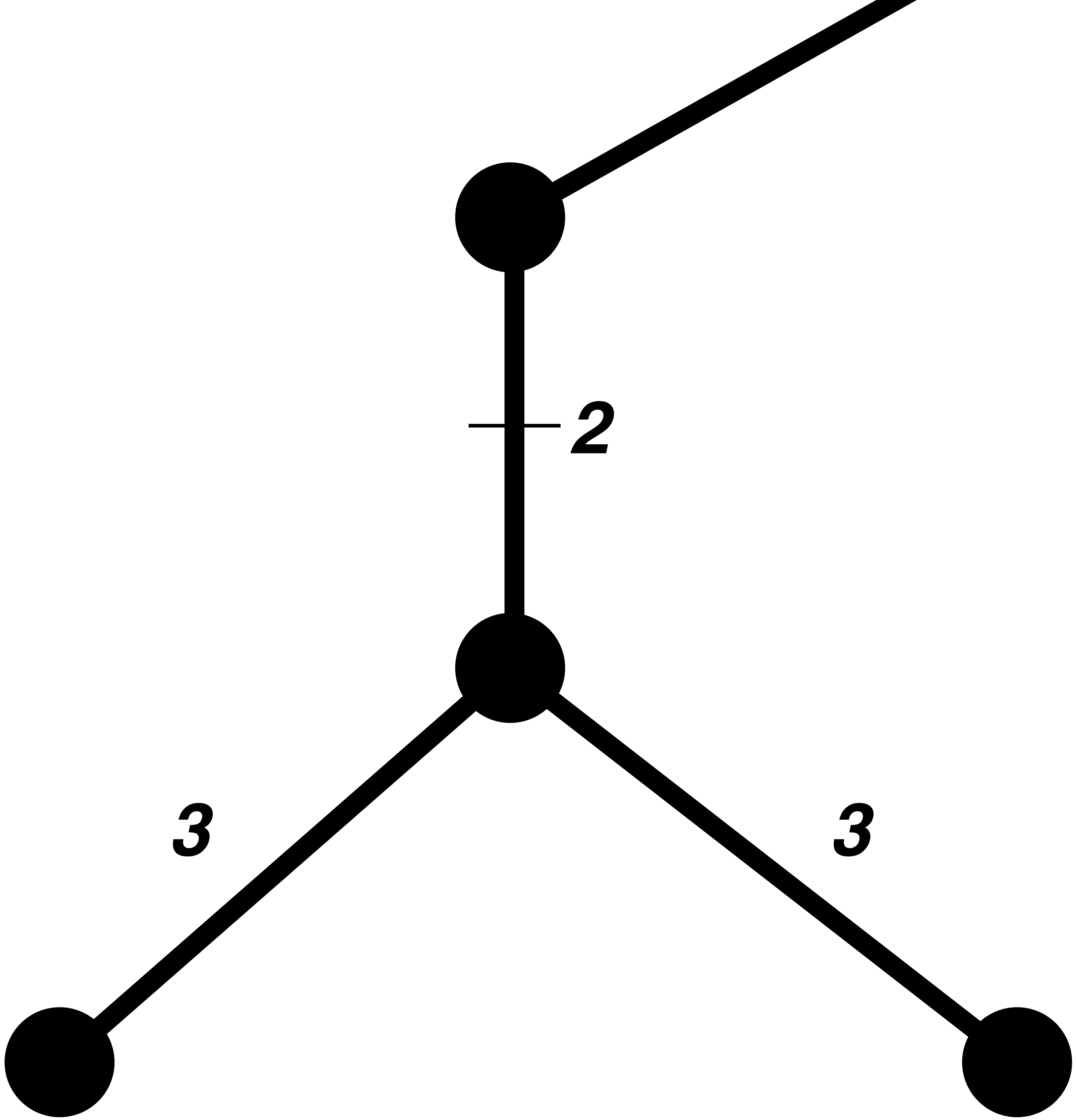





2
3
3



2
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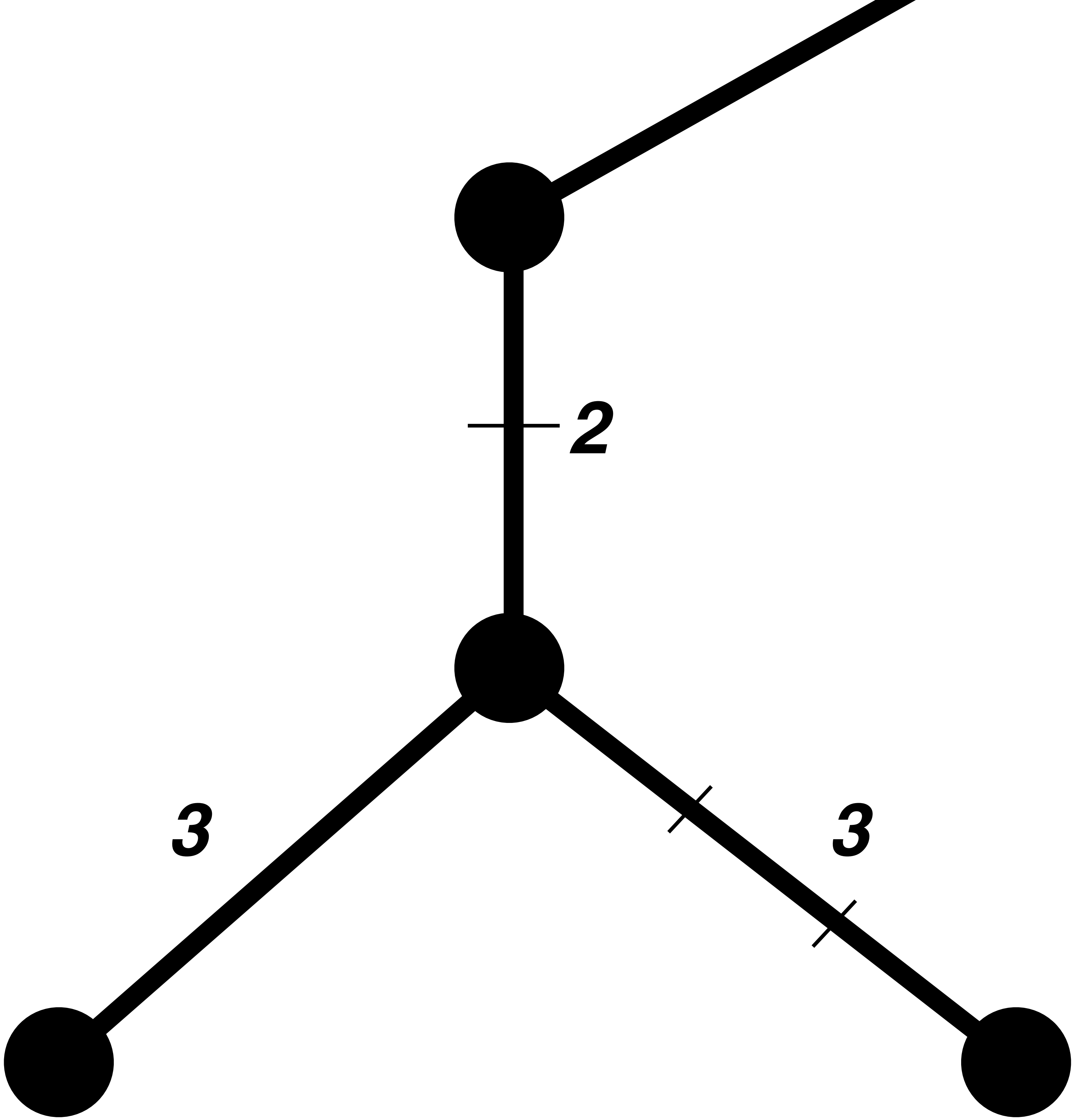
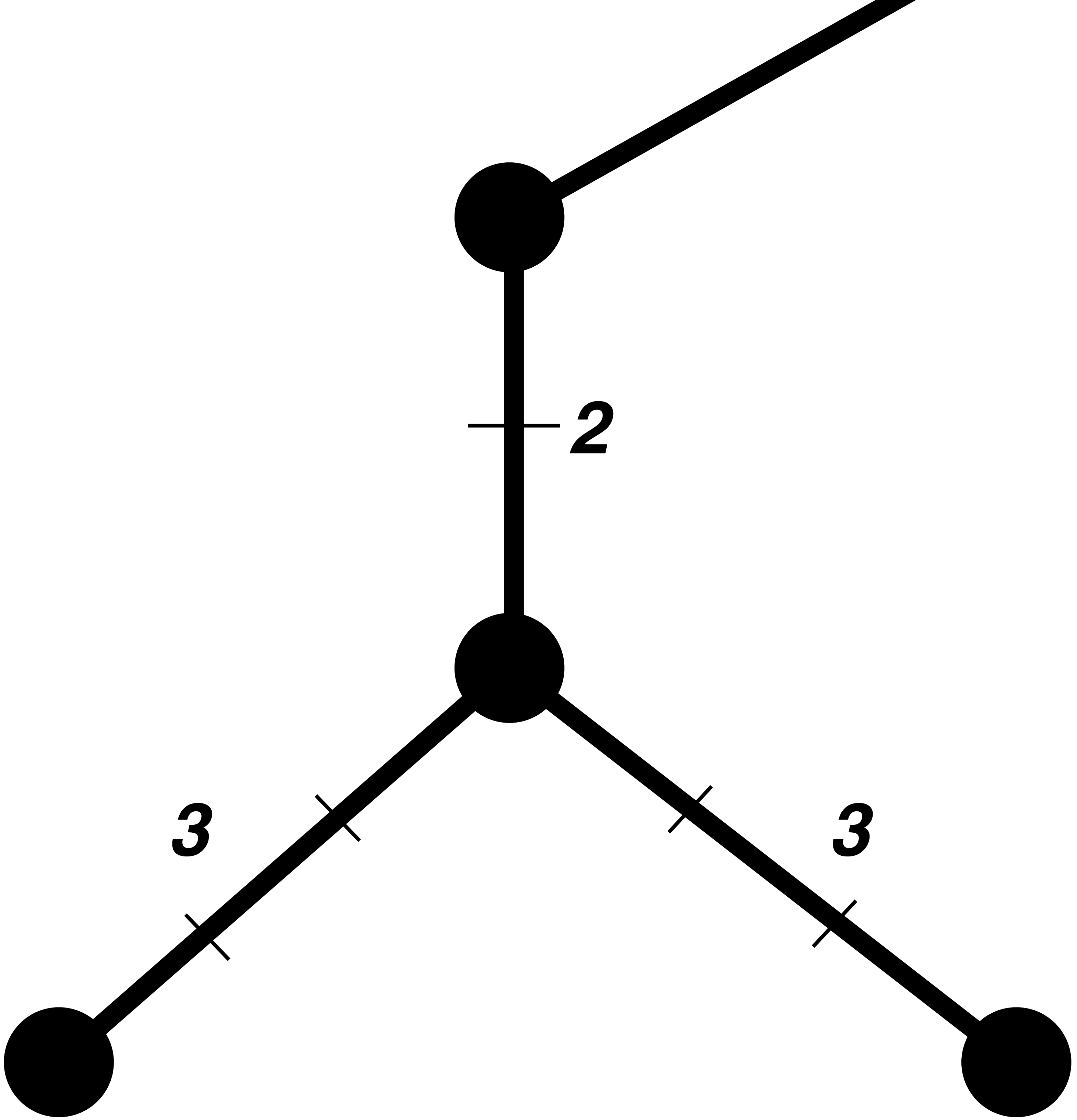
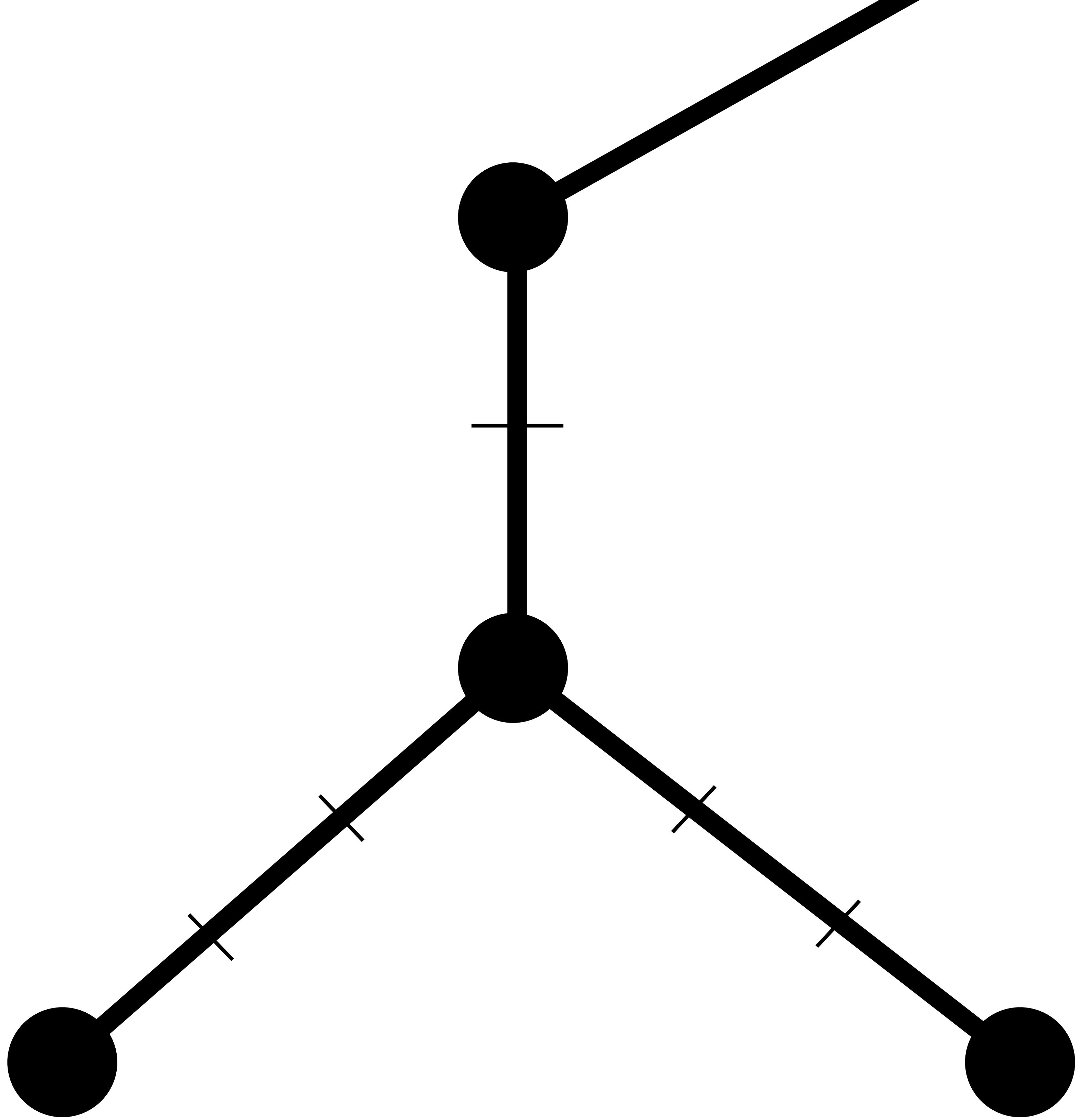
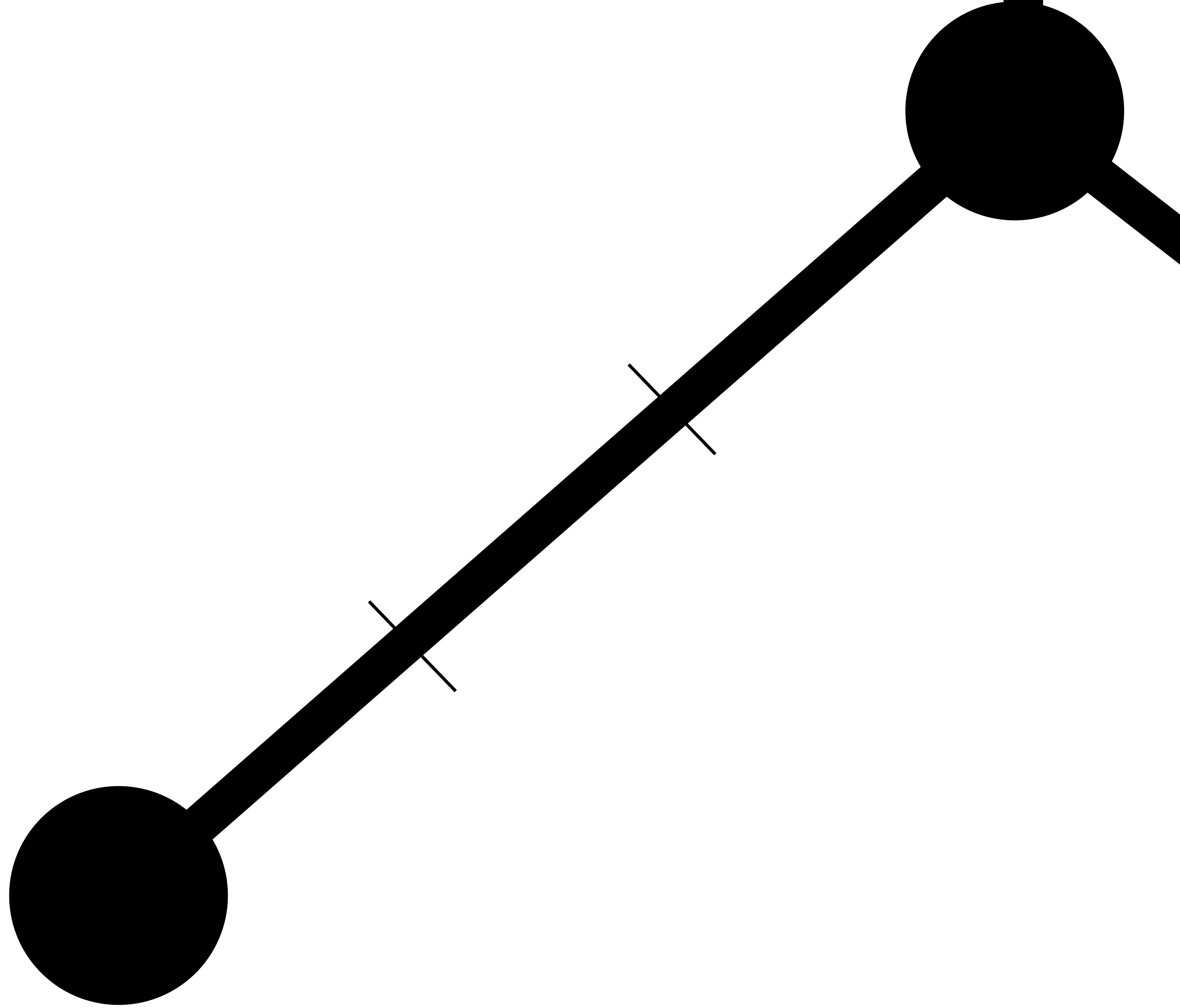
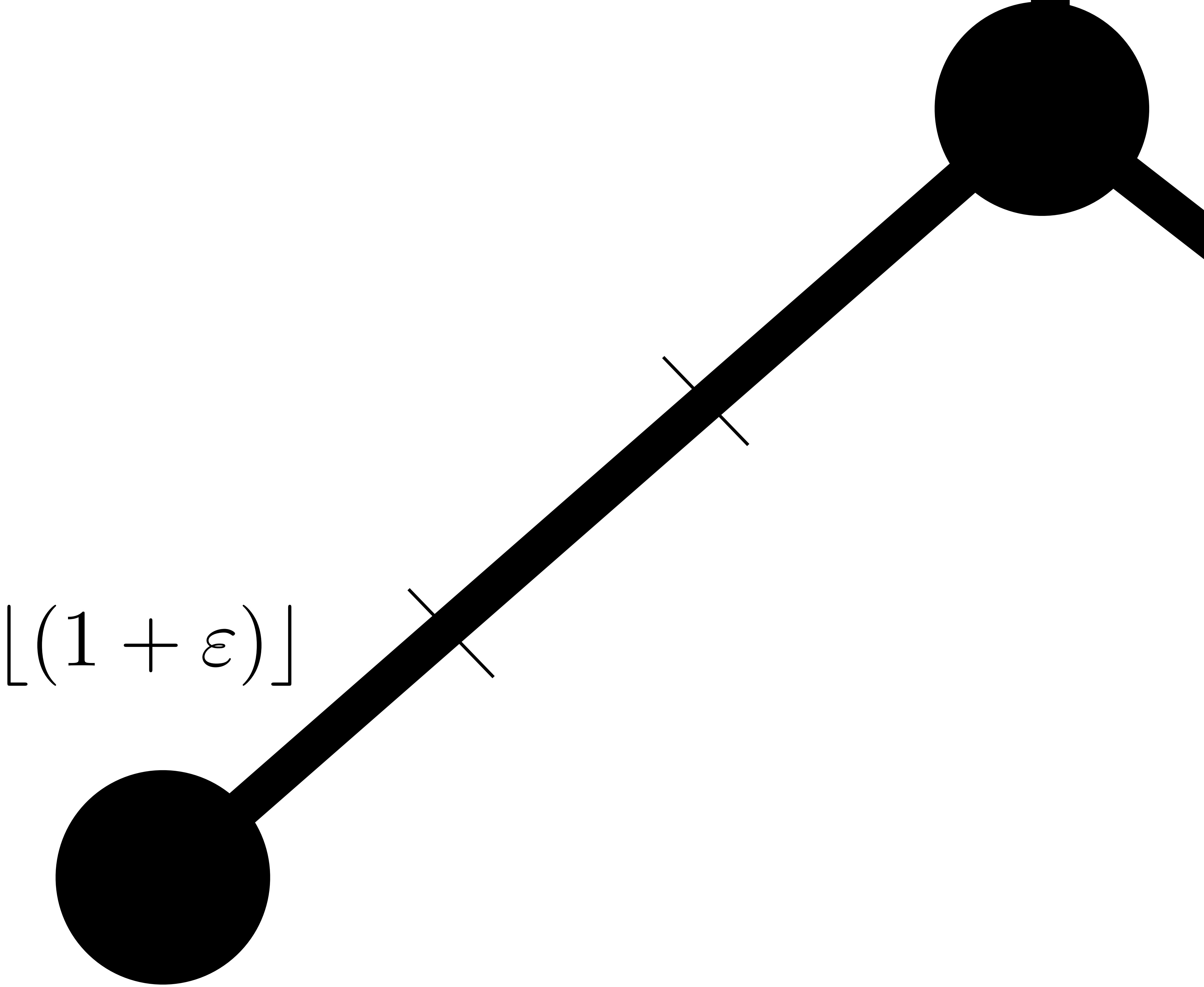

2
3
3

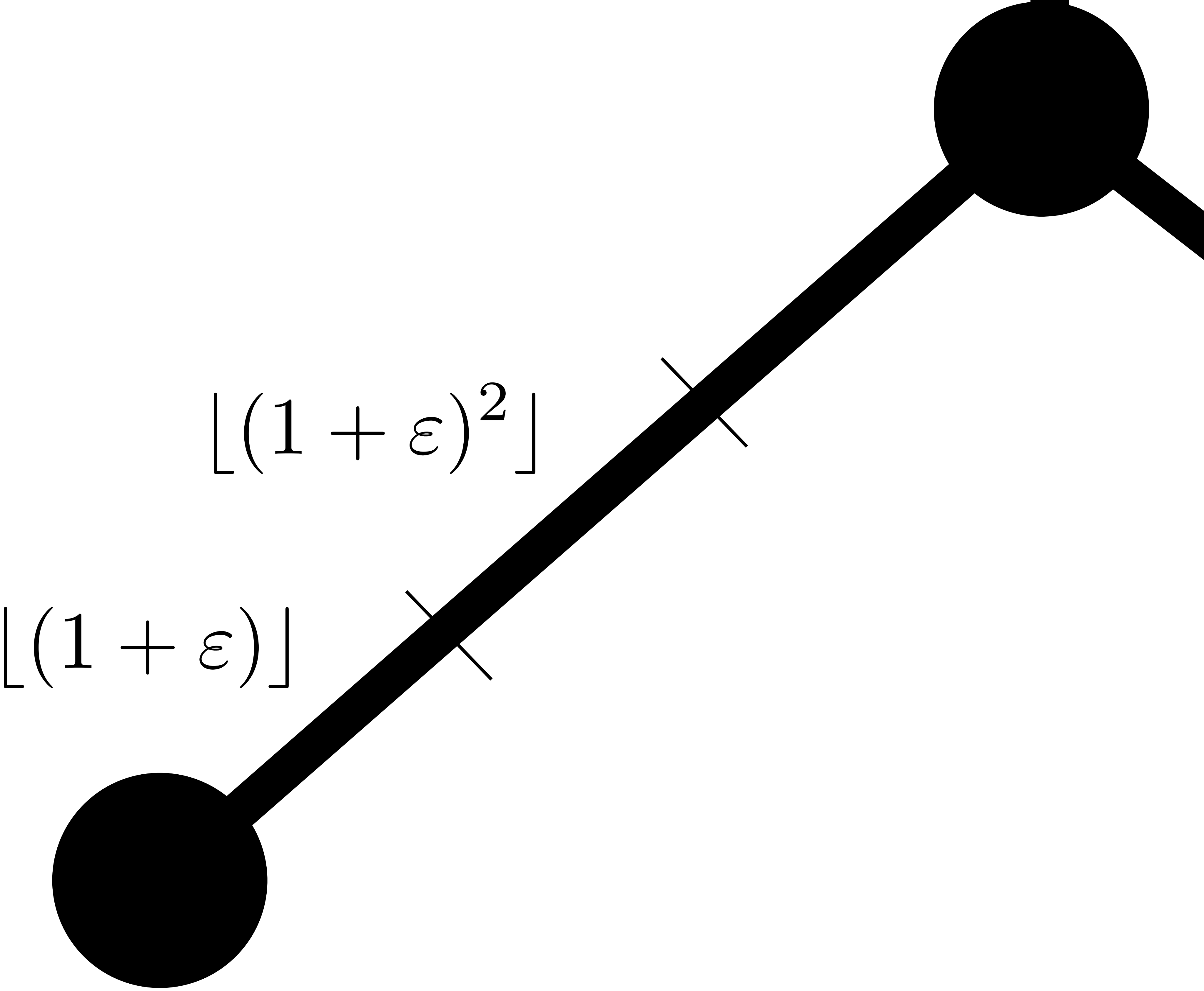


2
3
3

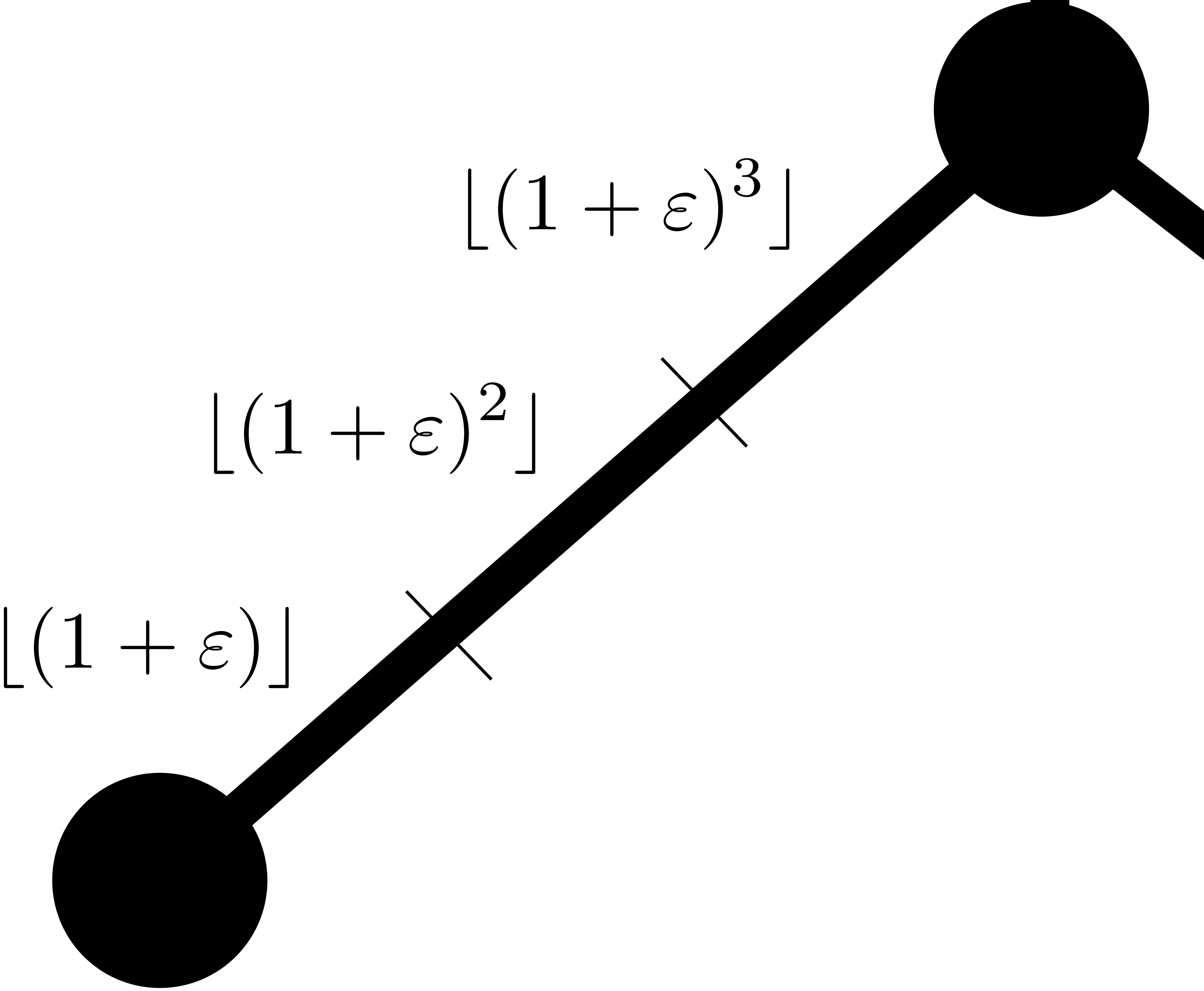


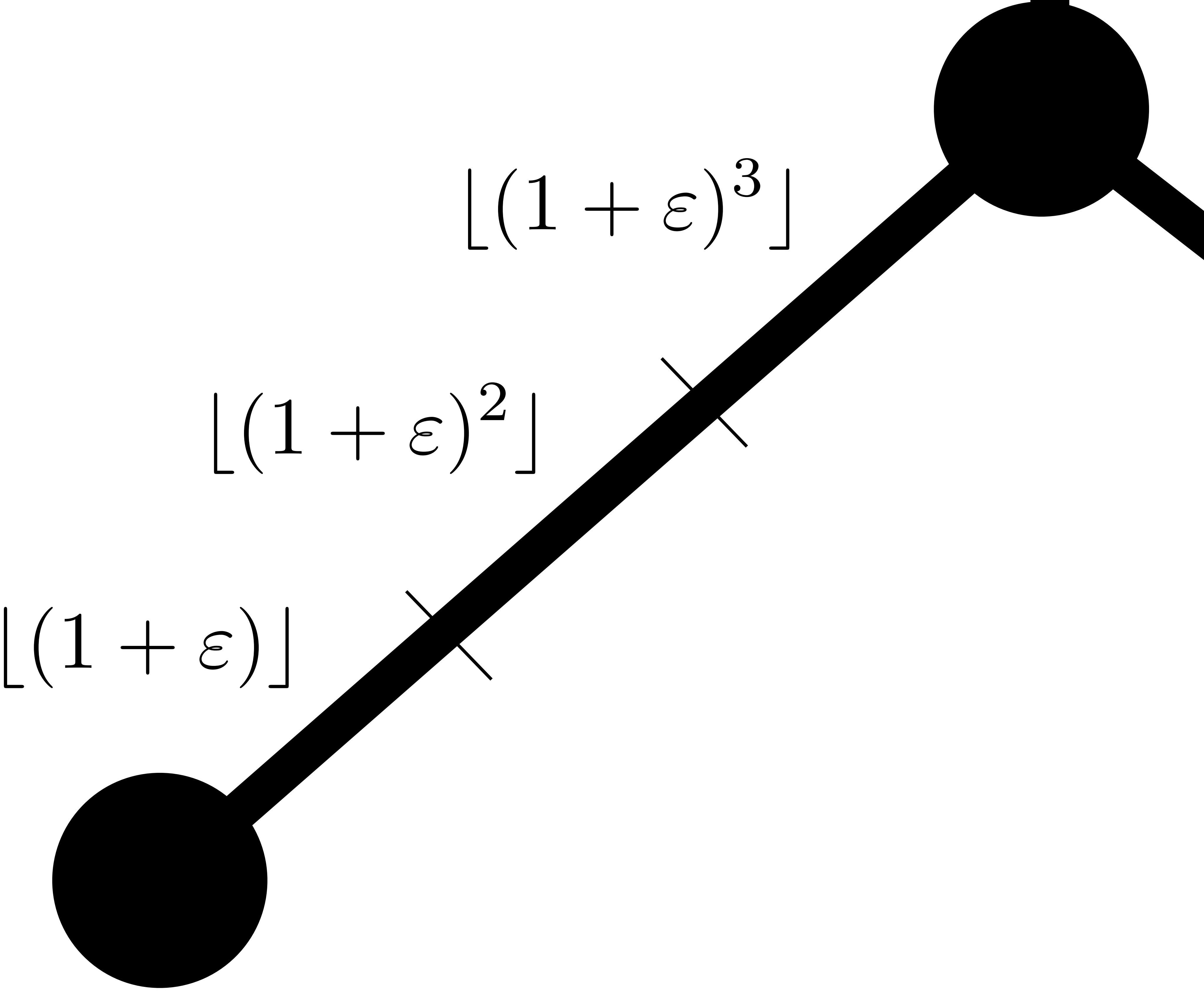



$\lfloor(1+\varepsilon)\rfloor$

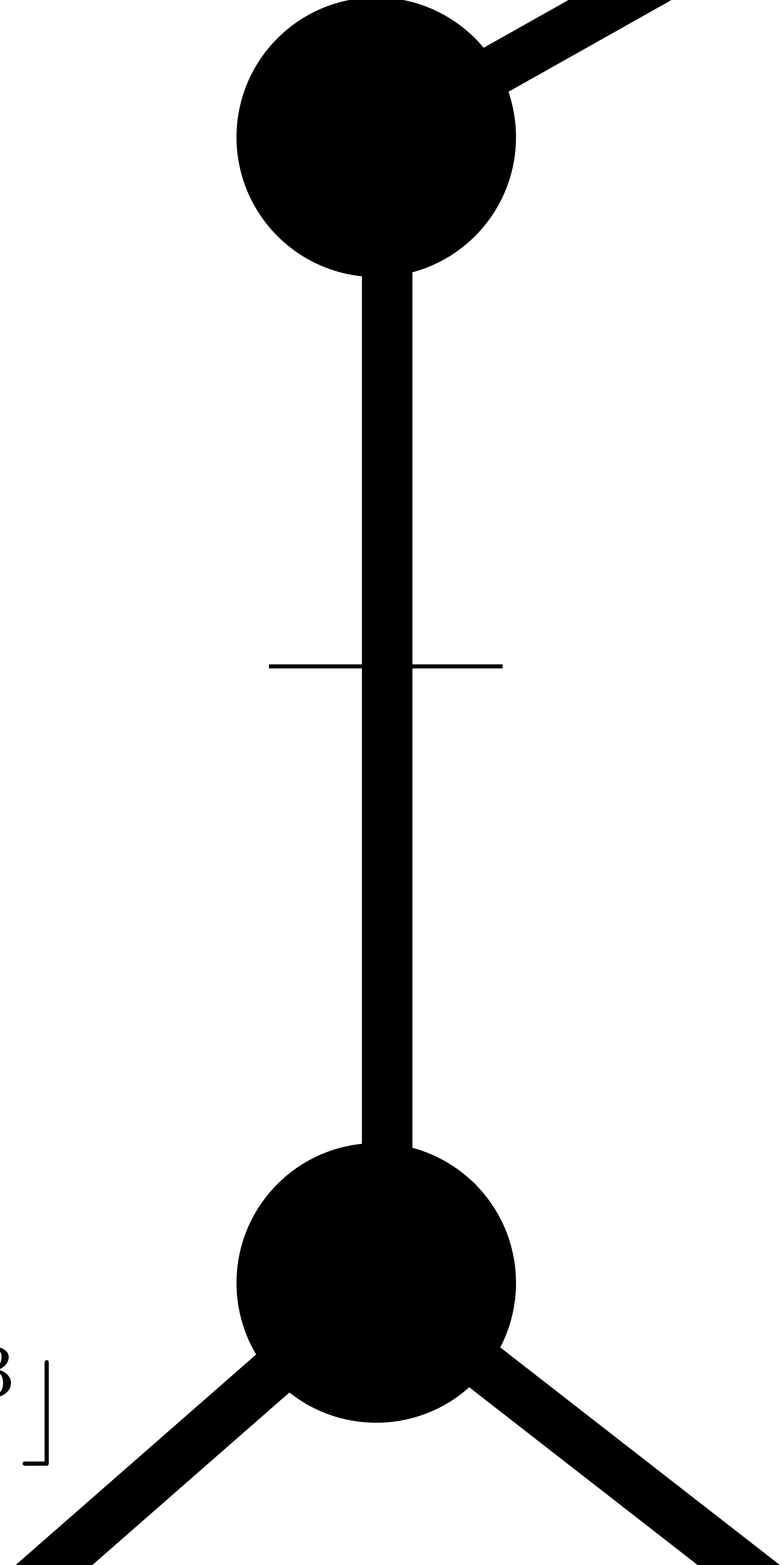


$\lfloor(1+\varepsilon)^2\rfloor$
$\lfloor(1+\varepsilon)\rfloor$

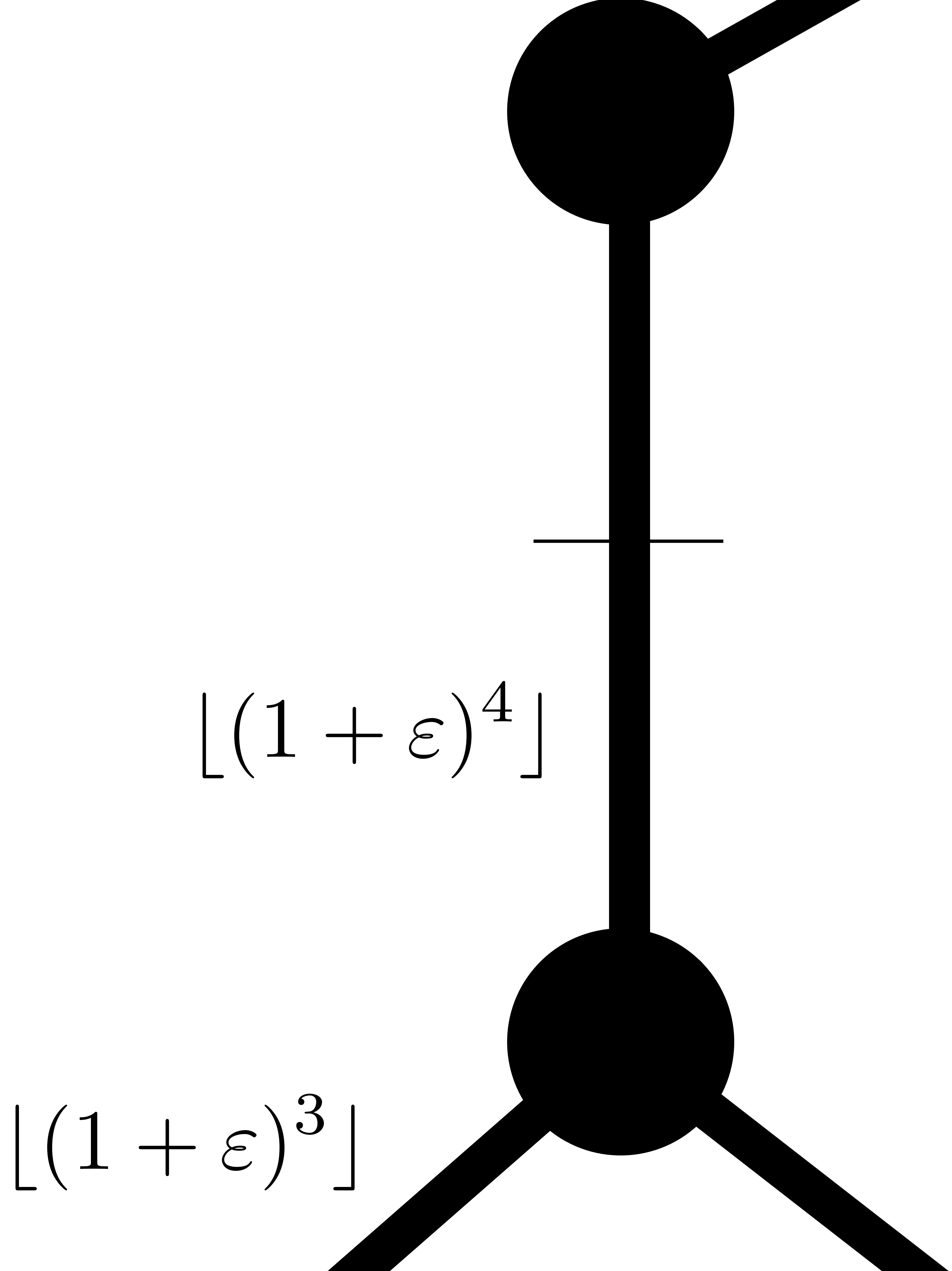

$\lfloor (1+\varepsilon)^3 \rfloor$
$\lfloor (1+\varepsilon)^2 \rfloor$
$\lfloor (1+\varepsilon) \rfloor$

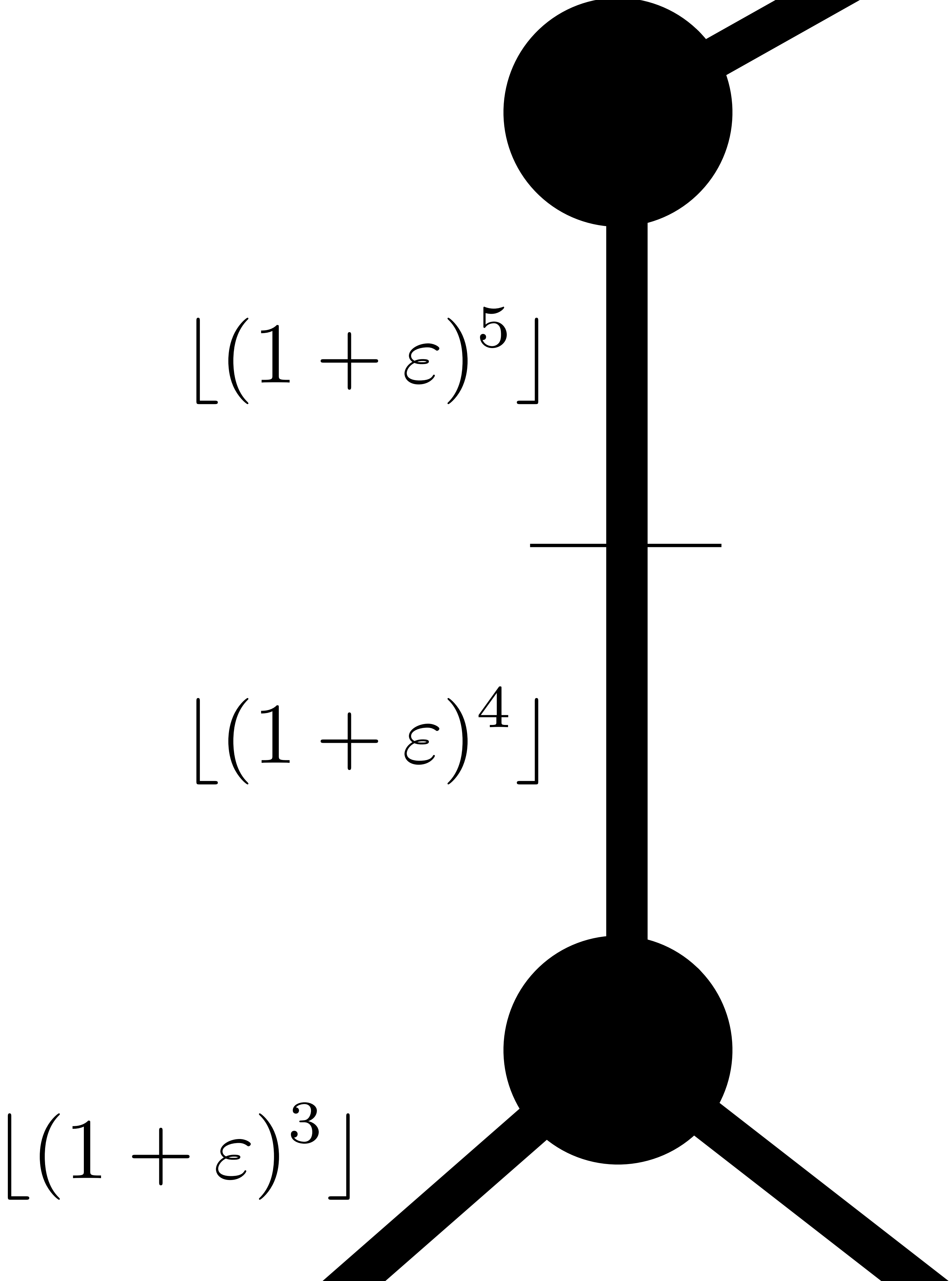


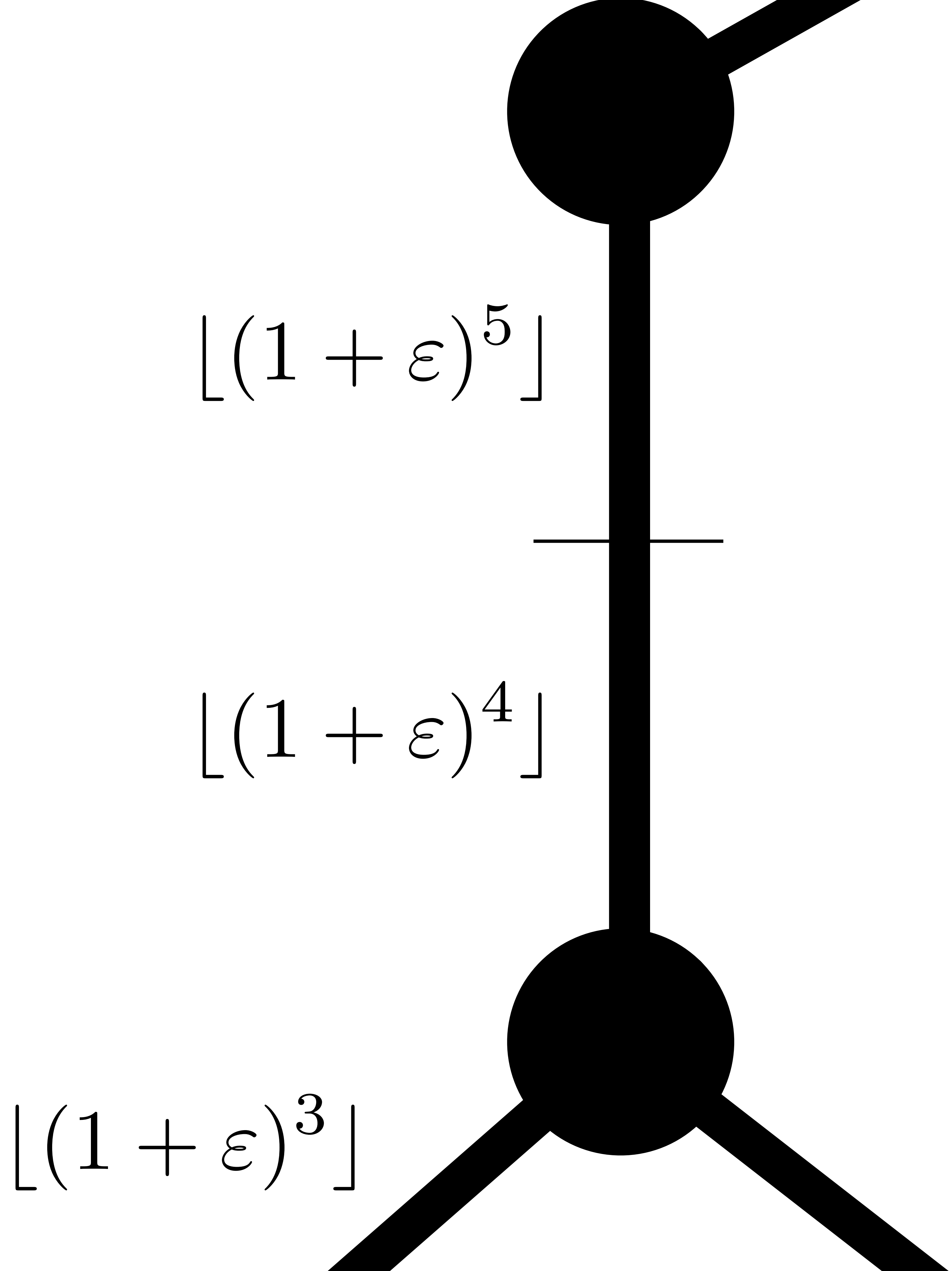
$\lfloor (1+\varepsilon)^3 \rfloor$
$\lfloor (1+\varepsilon)^2 \rfloor$
$\lfloor (1+\varepsilon) \rfloor$

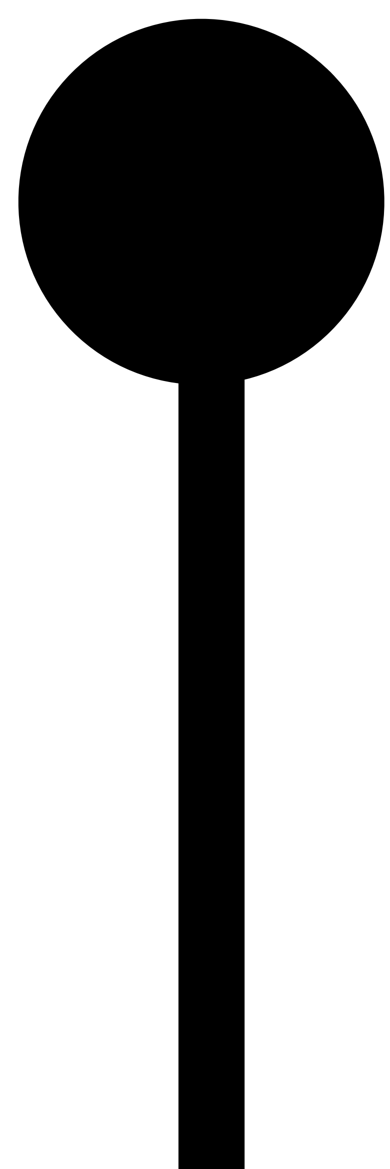


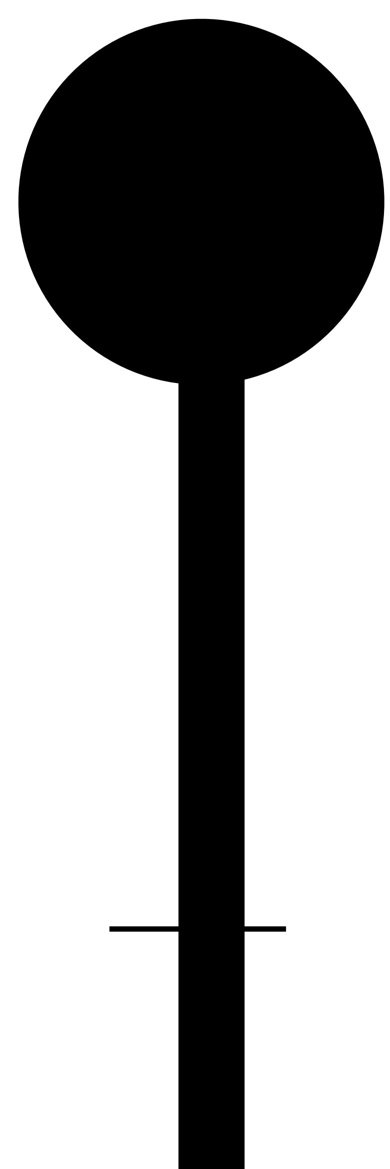
$\lfloor (1+\varepsilon)^3 \rfloor$

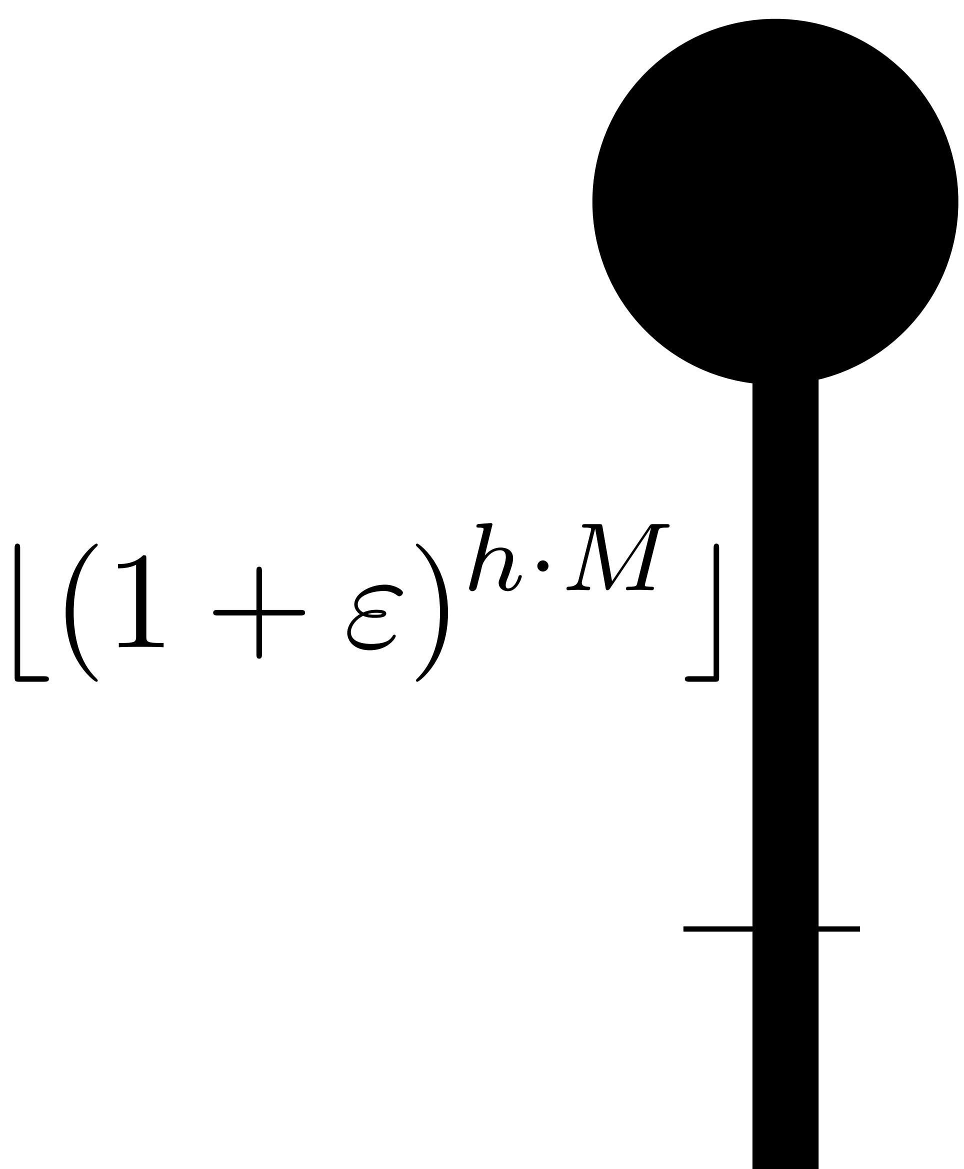

$\lfloor (1+\varepsilon)^4 \rfloor$
$\lfloor (1+\varepsilon)^3 \rfloor$

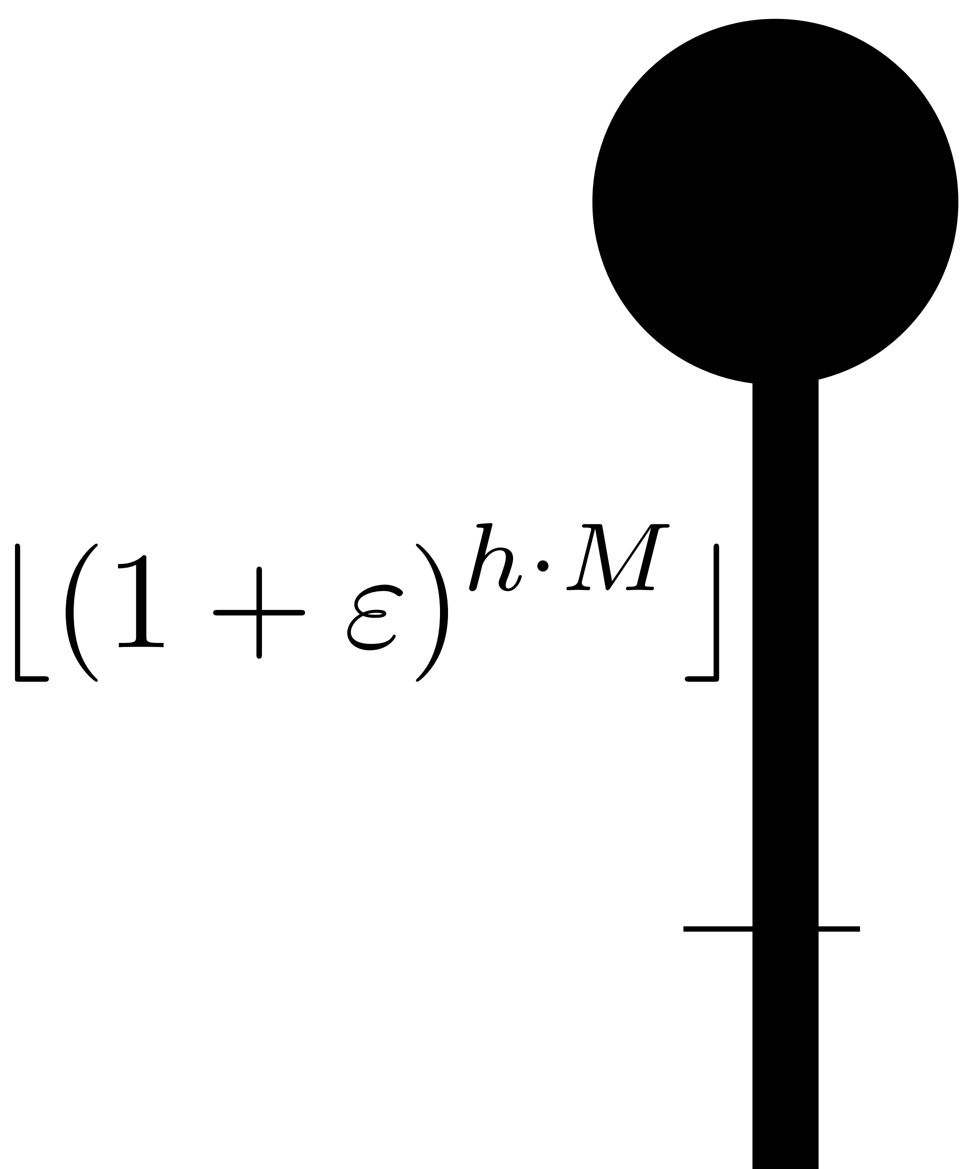

$\lfloor(1+\varepsilon)^5\rfloor$
$\lfloor(1+\varepsilon)^4\rfloor$
$\lfloor(1+\varepsilon)^3\rfloor$


$\lfloor (1+\varepsilon)^5 \rfloor$
$\lfloor (1+\varepsilon)^4 \rfloor$
$\lfloor (1+\varepsilon)^3 \rfloor$

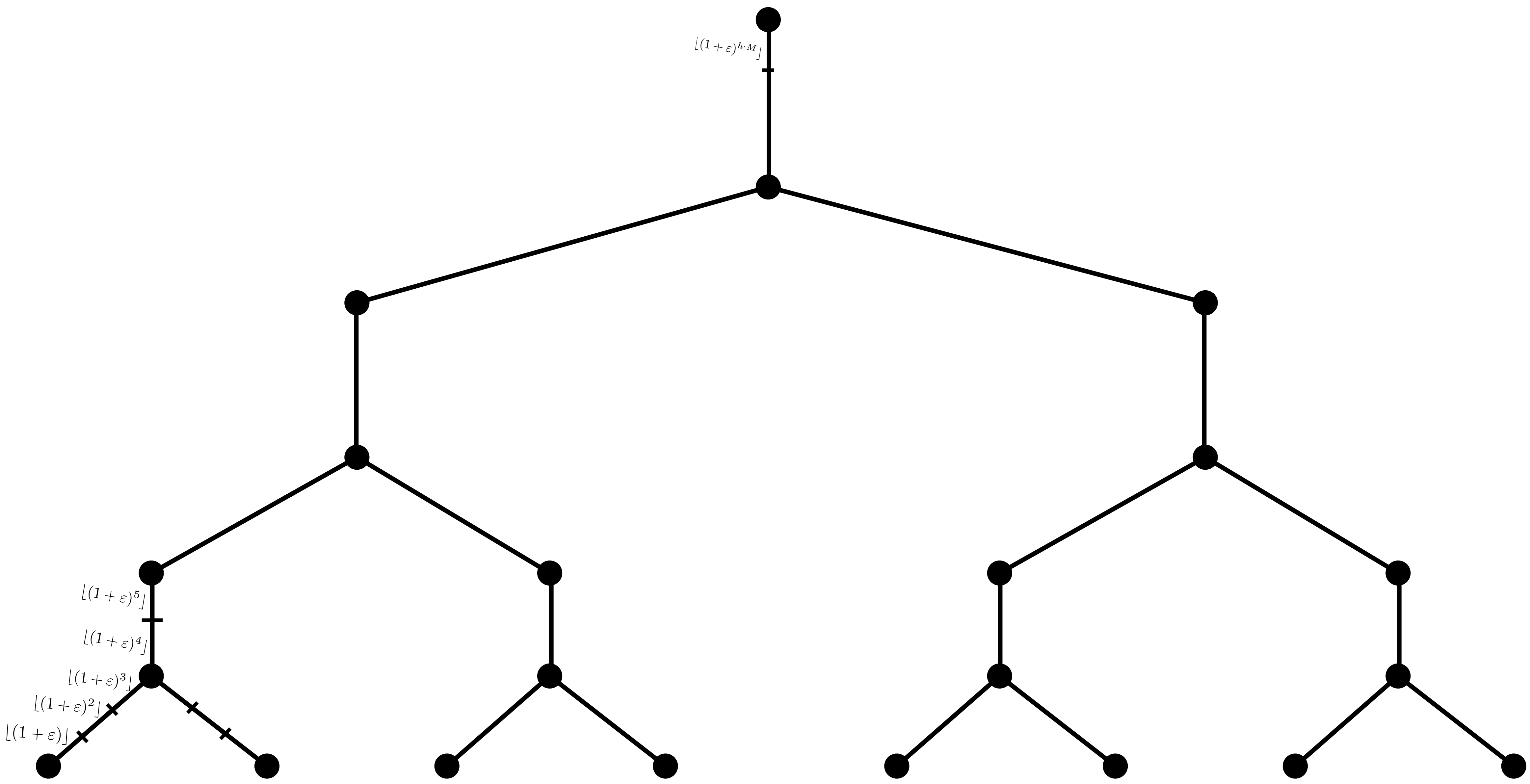




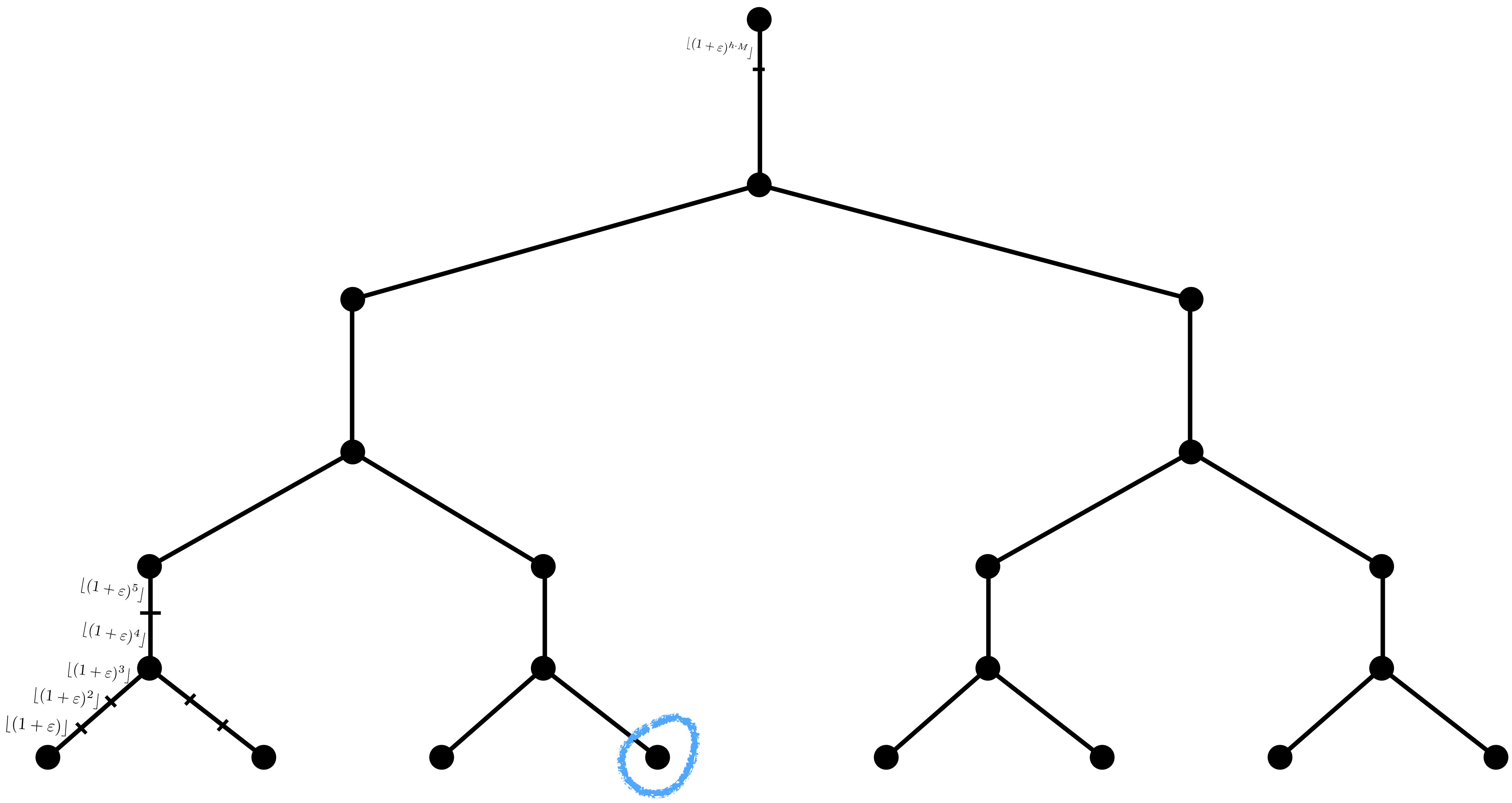

$\lfloor (1+\varepsilon)^{h \cdot M} \rfloor$


$\lfloor (1+\varepsilon)^{h \cdot M} \rfloor$

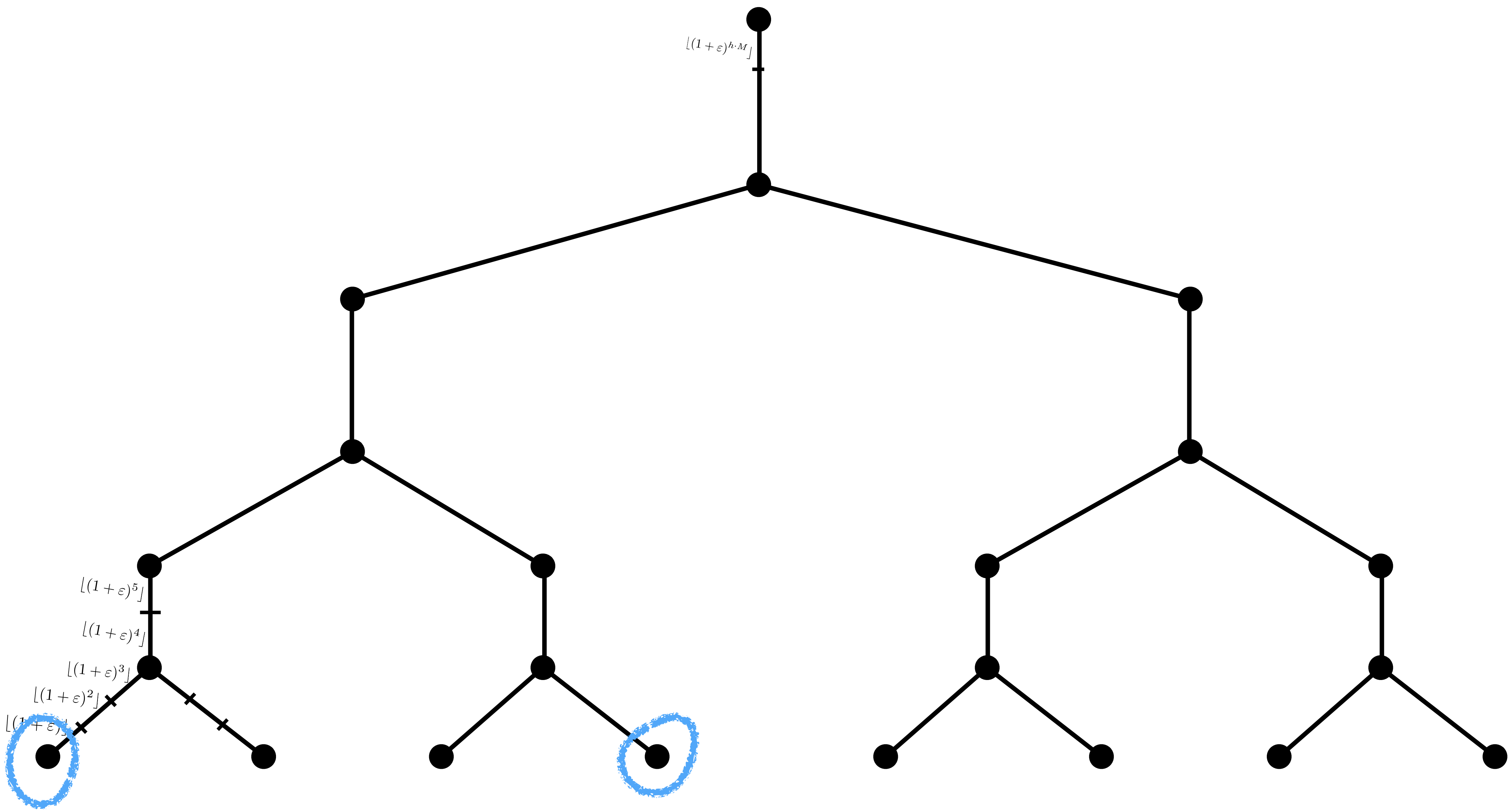
Approximate Lower Bound



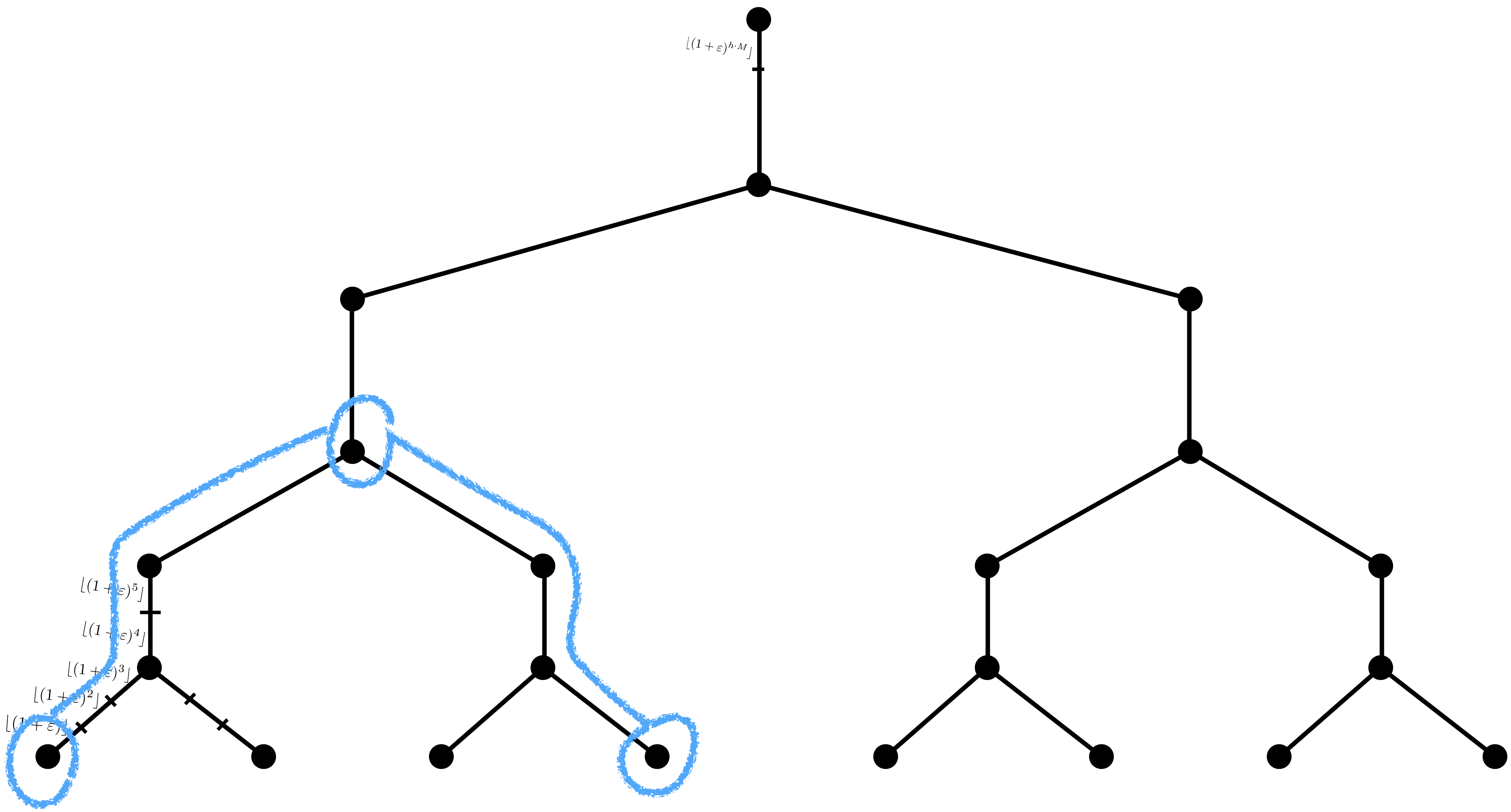
Approximate Lower Bound



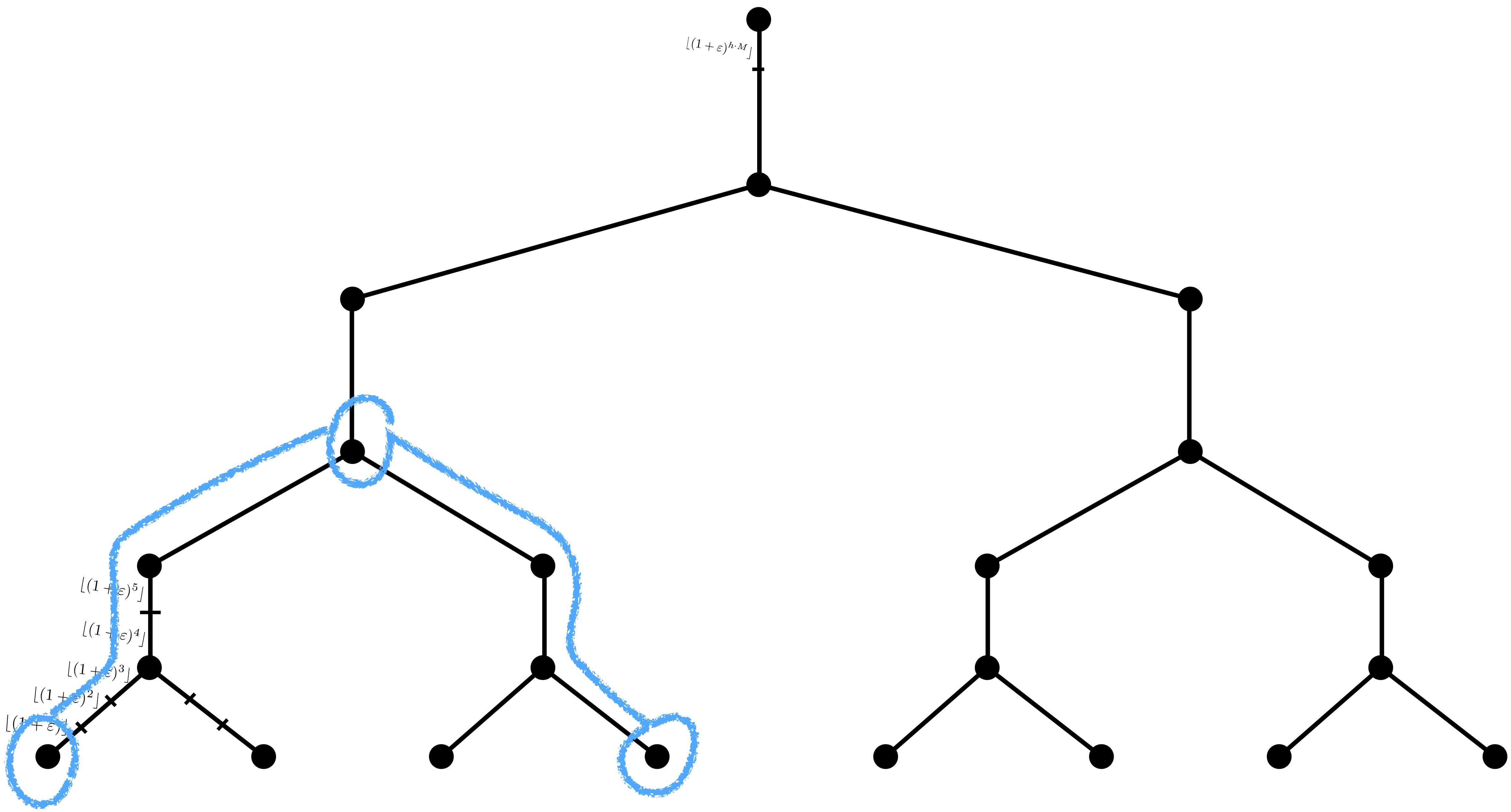
Approximate Lower Bound



Approximate Lower Bound

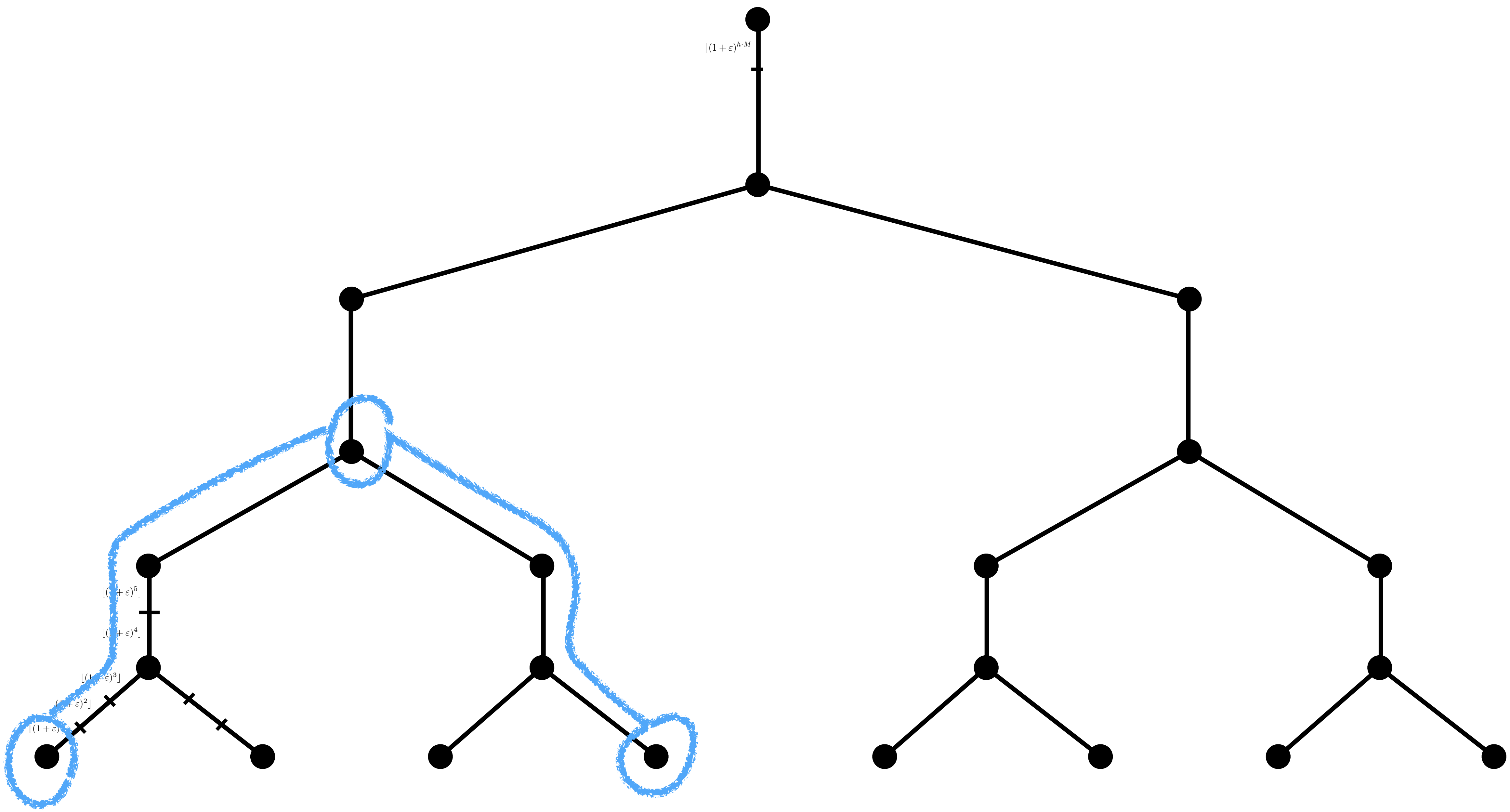


Approximate Lower Bound



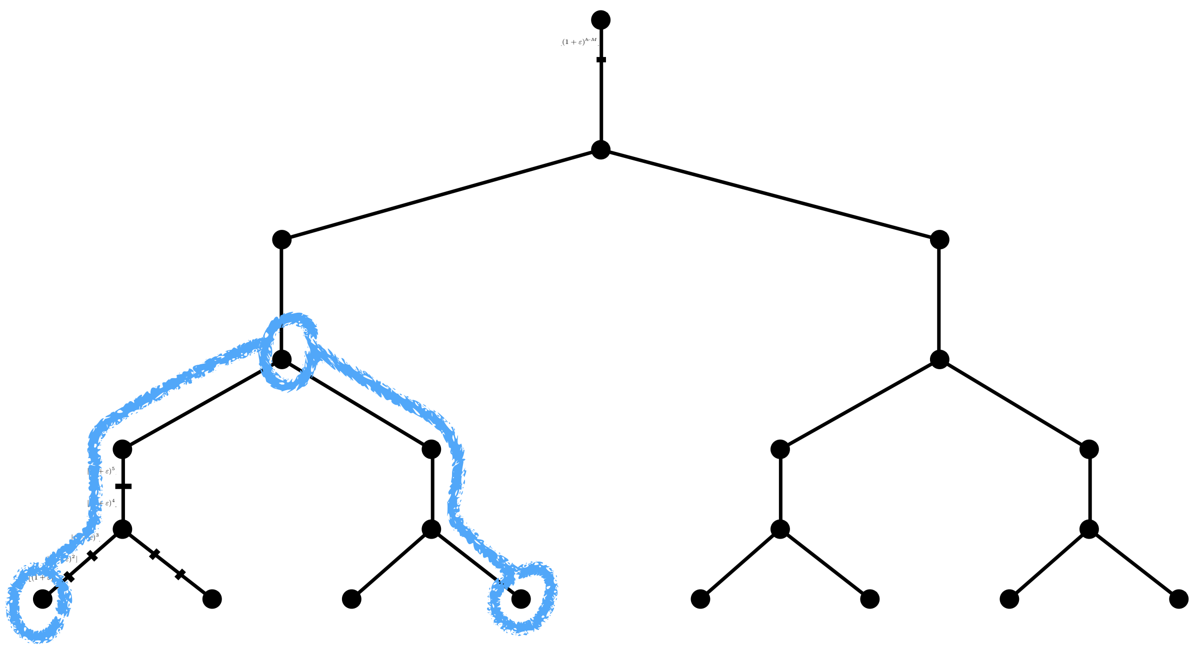
$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

Approximate Lower Bound



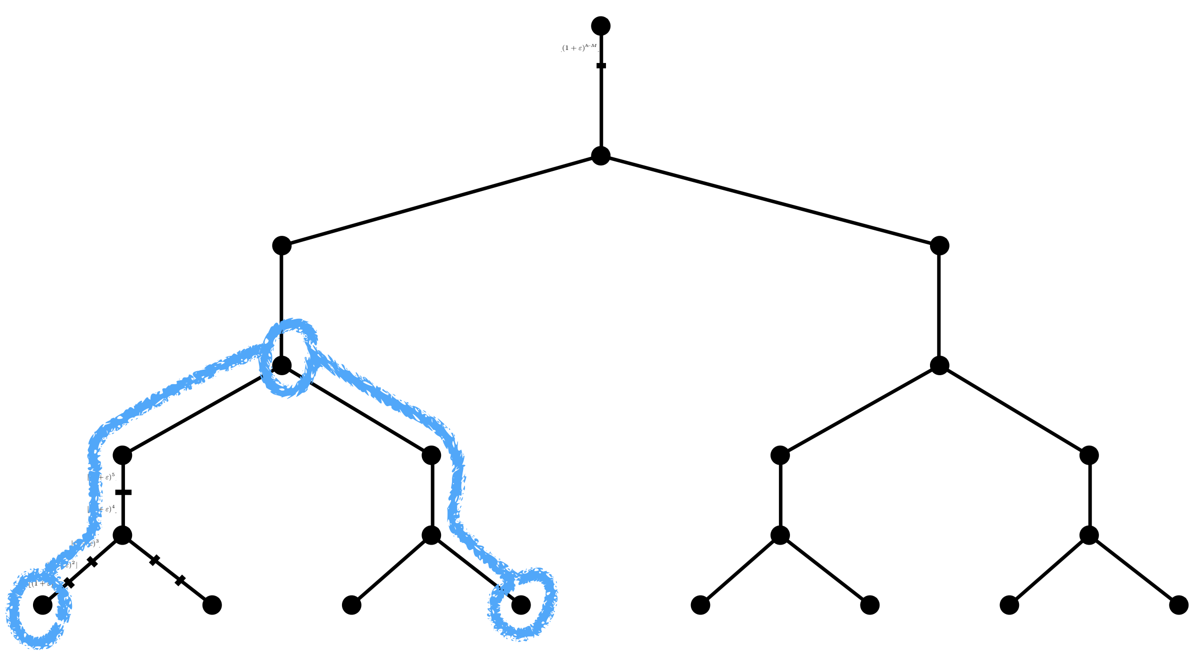
$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

Approximate Lower Bound



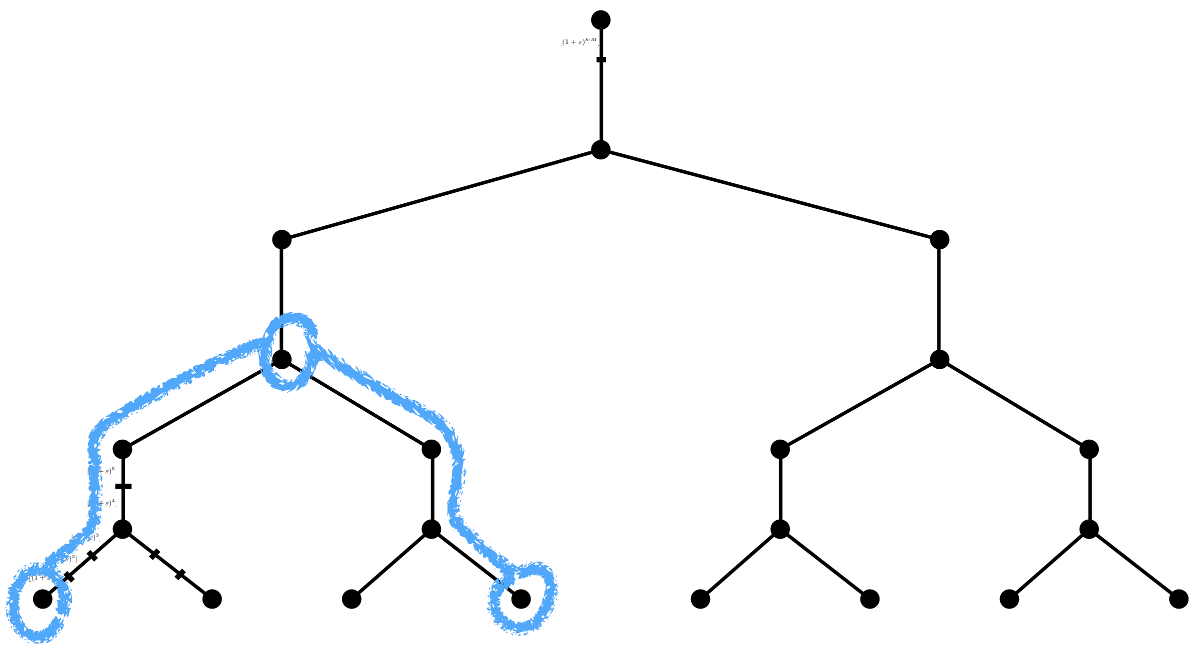
$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

Approximate Lower Bound



$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

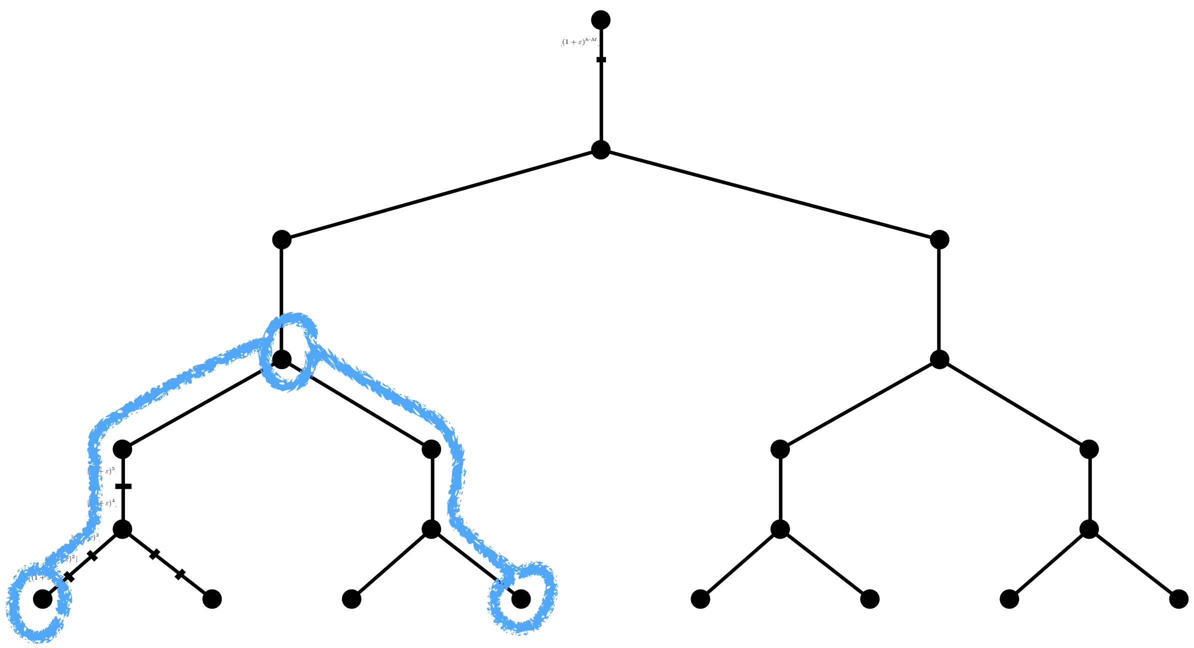
Approximate Lower Bound



$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

Approximate Lower Bound

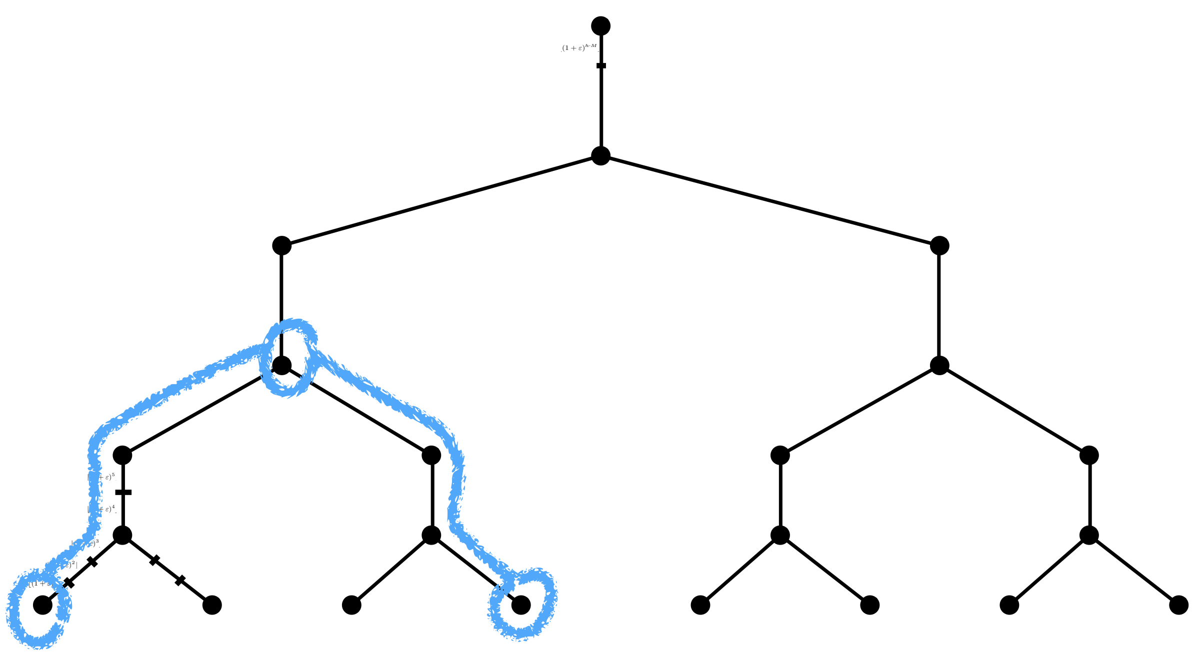


$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

Approximate Lower Bound



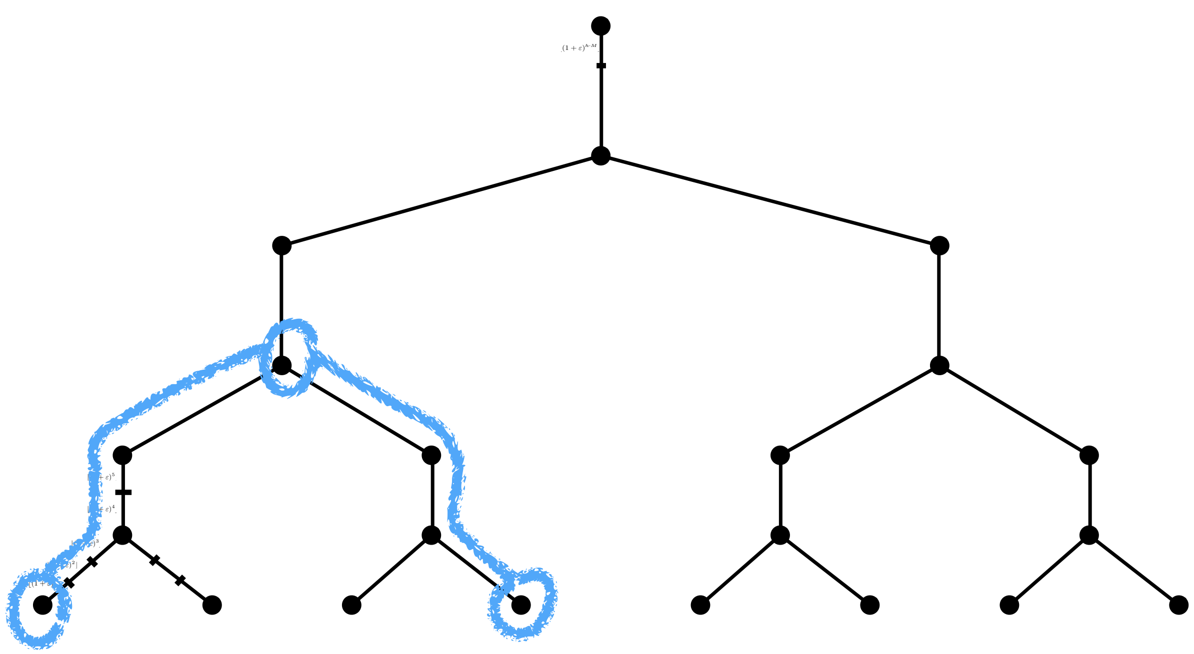
$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

Approximate Lower Bound



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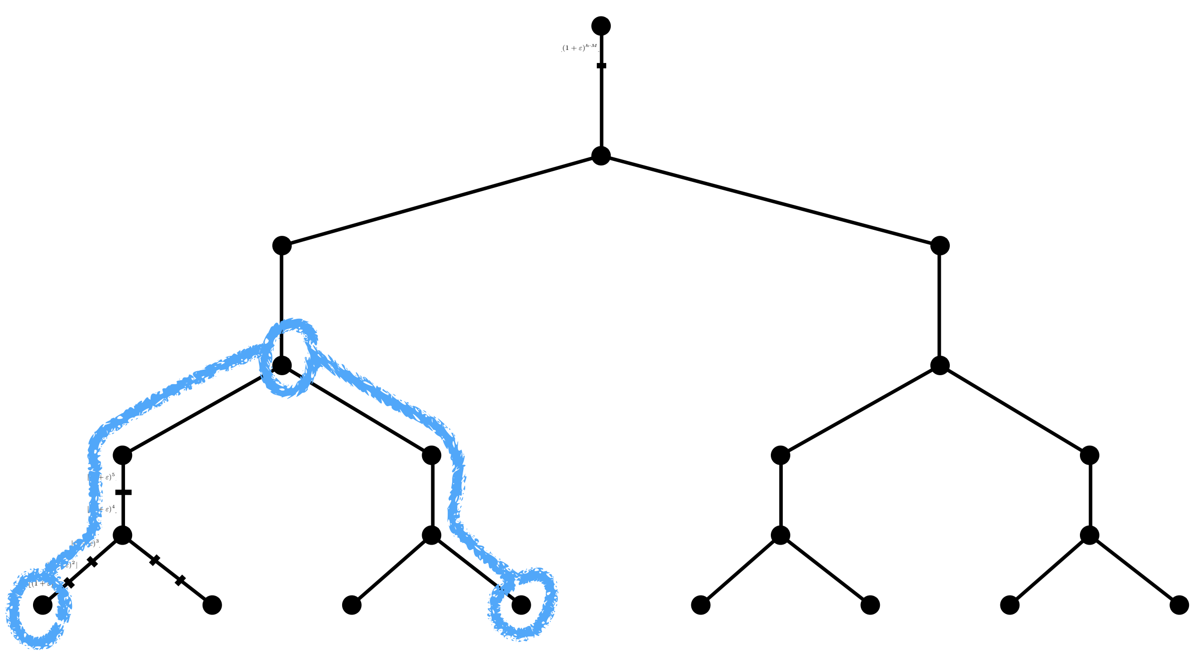
$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

(Intervals pairwise disjoint)

Approximate Lower Bound



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$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

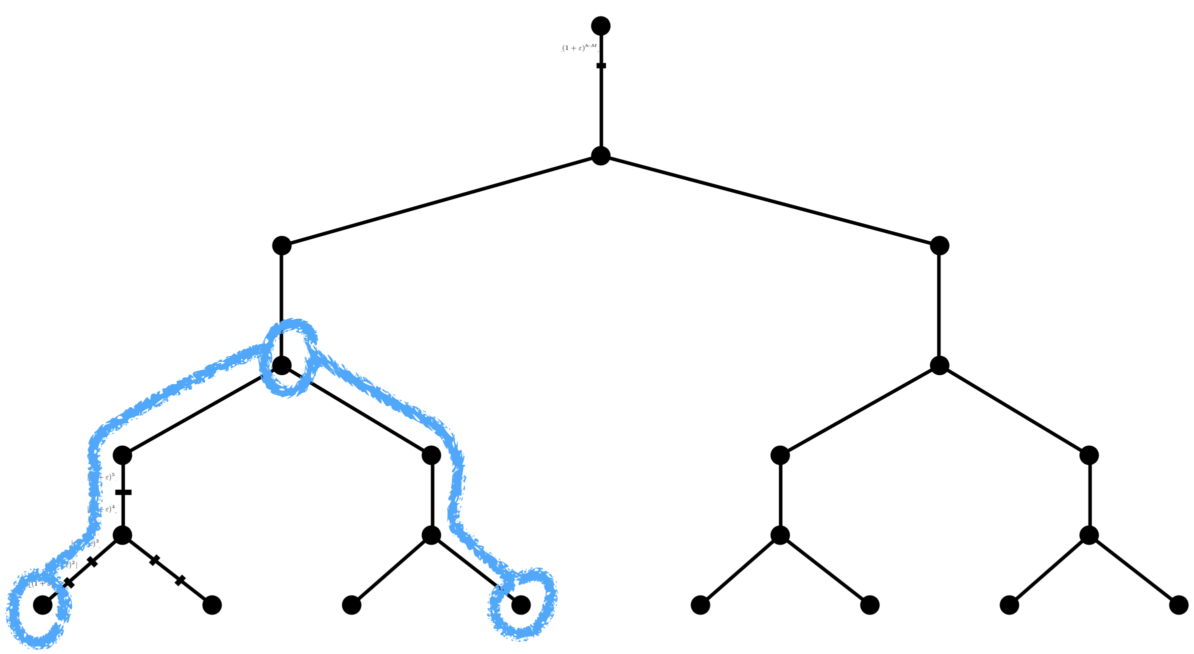
(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

(Intervals pairwise disjoint)

$$\Downarrow$$

Approximate Lower Bound



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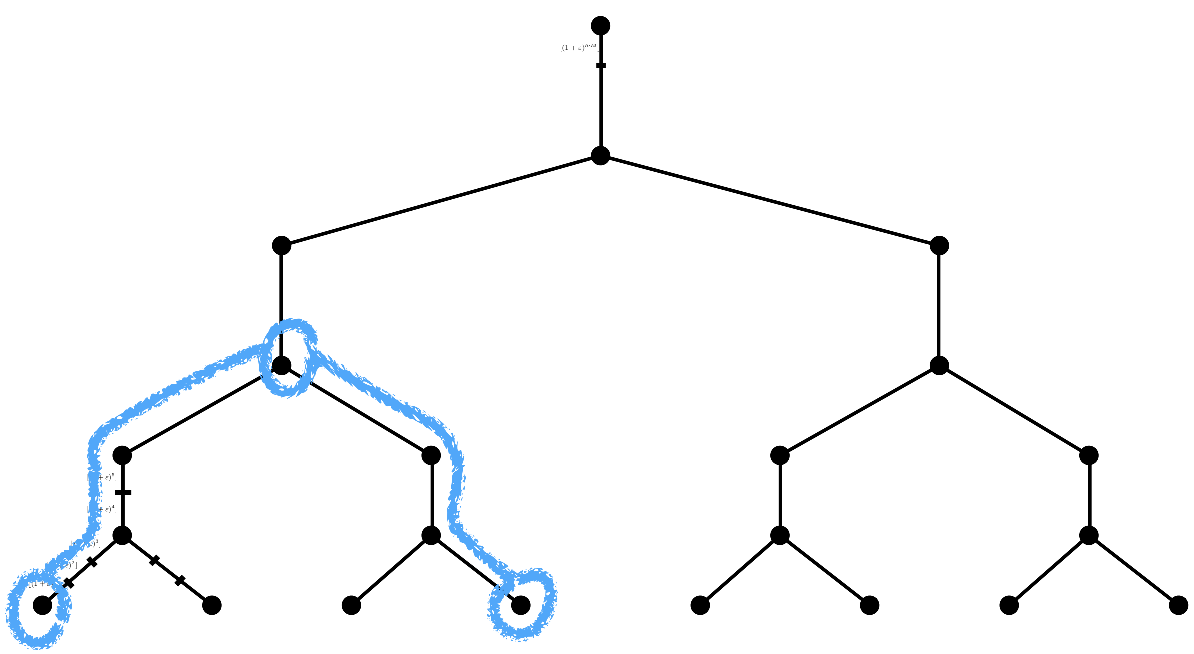
(Intervals pairwise disjoint)

$$\Downarrow$$

Approximate Lower Bound



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$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

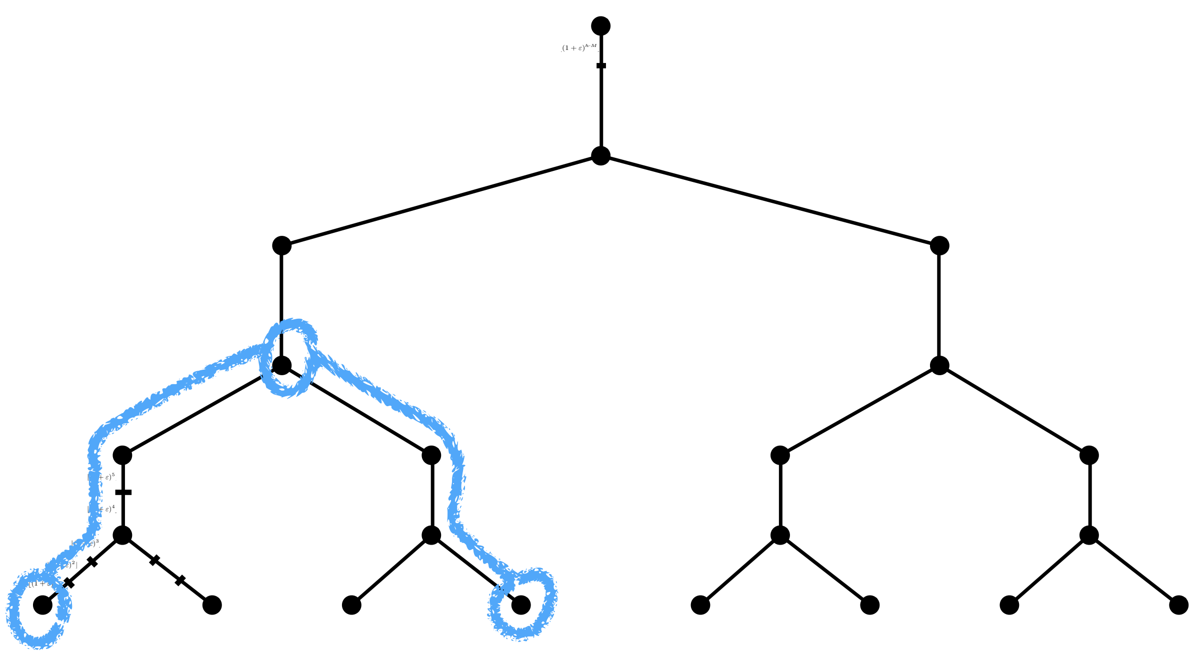
(Intervals pairwise disjoint)

$$\Downarrow$$

Approx.

(trans. tree)

Approximate Lower Bound



$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

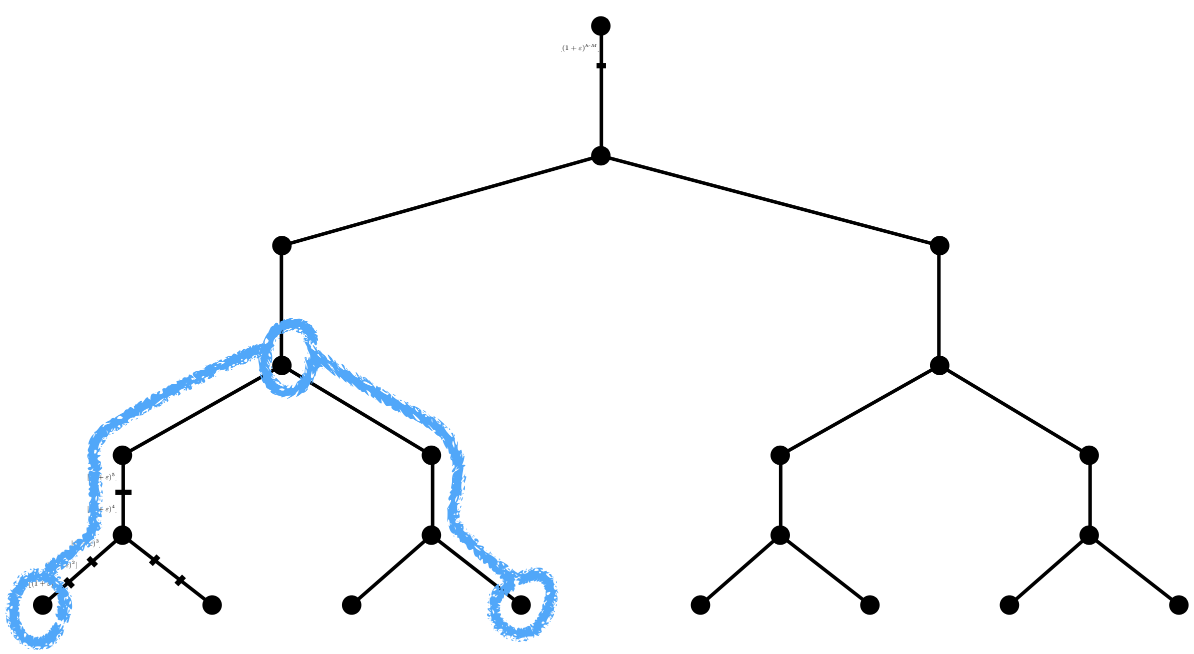
(Intervals pairwise disjoint)

$$\Downarrow$$

Approx. $\Longrightarrow$

(trans. tree)

Approximate Lower Bound



$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

(Intervals pairwise disjoint)

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree)

$f(k)$ monotone

Enough that

$$(1+\varepsilon)f(k) < f(k+1)$$

$f(k)$ monotone

Enough that

$$(1+\varepsilon)f(k) < f(k+1)$$

I_k right end

$f(k)$ monotone

Enough that

$$(1+\varepsilon)f(k) < f(k+1)$$

I_k right end ***I_{k+1} left end***

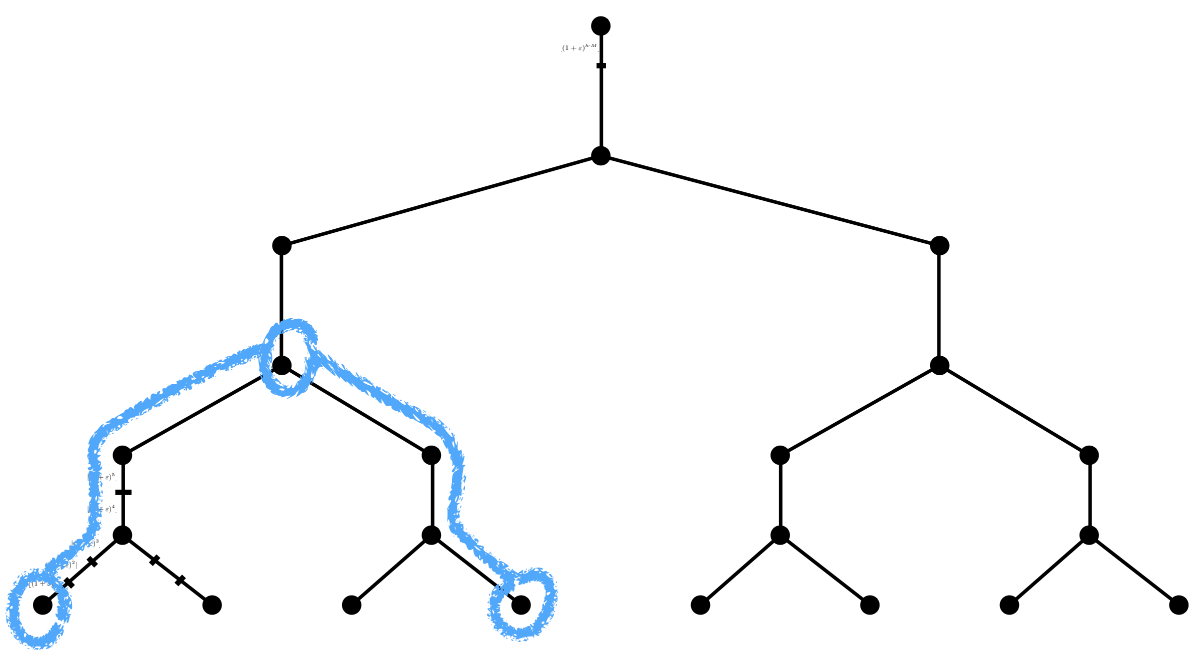
$f(k)$ monotone

Enough that

$$(1+\varepsilon)f(k) < f(k+1)$$

I_k right end ***I_{k+1} left end***

Approximate Lower Bound



$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

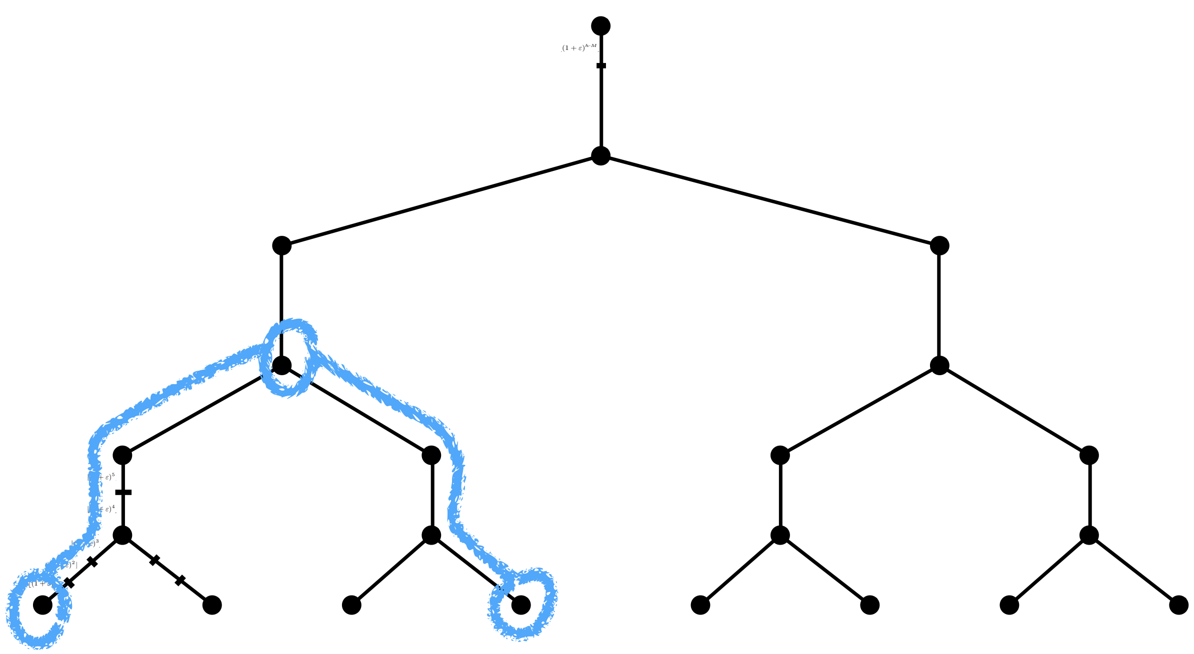
$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

Intervals pairwise disjoint

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree)

Approximate Lower Bound



$$f(k) = 2\sum_{i=1}^{k} \lfloor (1+\varepsilon)^i \rfloor$$

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

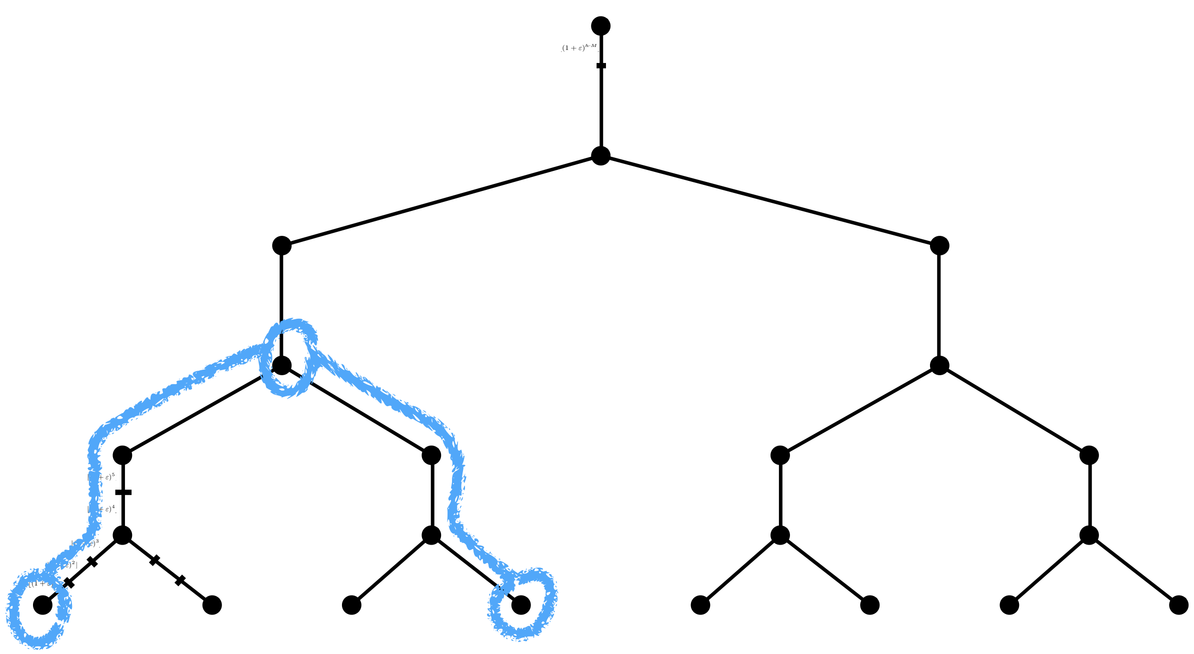
$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree)

Approximate Lower Bound



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$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

Intervals pairwise disjoint

$$\Downarrow$$

Approx. $\Longrightarrow$ Exact

(trans. tree) (orig. tree)

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

Intervals pairwise disjoint

$$\Downarrow$$

Approx. $\Longrightarrow$ Exact

(trans. tree) (orig. tree)

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$$\Downarrow$$

✓

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree)

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ ✓ Exact (orig. tree)

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$M := 1/\varepsilon$

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$

Intervals pairwise disjoint

$$\Downarrow$$

Approx. $\Longrightarrow$ Exact

(trans. tree) (orig. tree)

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$M := 1/\varepsilon$

$h := \log n$

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$\Downarrow$

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree) ✓

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$M := 1/\varepsilon$

$h := \log n$

$\Downarrow$

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon)\cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ ✓ Exact (orig. tree)

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$M := 1/\varepsilon$

$h := \log n$

$$\Downarrow$$

$$\Omega(\log \frac{1}{\varepsilon} \cdot \log n)$$

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$$\Downarrow$$

✓

Approx. $\Longrightarrow$ Exact

(trans. tree) (orig. tree)

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$M := 1/\varepsilon$

$h := \log n$

$$\Downarrow$$

$$\Omega(\log \frac{1}{\varepsilon} \cdot \log n)$$

Additionally:

Approximate Lower Bound

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k' \checkmark$$

Intervals pairwise disjoint

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree) $\checkmark$

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$M := 1/\varepsilon$

$h := \log n$

$$\Downarrow$$

$$\Omega(\log \frac{1}{\varepsilon} \cdot \log n)$$

Additionally:

$$O(\log \frac{1}{\varepsilon} \cdot \log n)$$

matching upper bound

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => [Alstrup et al. 2016]	
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Distance between leaves:

$$I_k := [f(k), (1+\varepsilon) \cdot f(k)]$$

(Approximate distance interval)

$$I_k \cap I_{k'} = \emptyset, \forall k \neq k'$$ ✓

Intervals pairwise disjoint

$$\Downarrow$$

Approx. (trans. tree) $\Longrightarrow$ Exact (orig. tree) ✓

Invoke lemma

$$g(h, M) \geq M^{\frac{h}{2}}$$

`maximize' when:

$$M := 1/\varepsilon$$

$$h := \log n$$

$$\Downarrow$$

$$\Omega(\log \frac{1}{\varepsilon} \cdot \log n)$$

Additionally:

$$O(\log \frac{1}{\varepsilon} \cdot \log n)$$

matching lower bound

k-Distance

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$		
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ + $O(k\log(k\log n/k))$	
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	
k-Distance $k > \log n$		

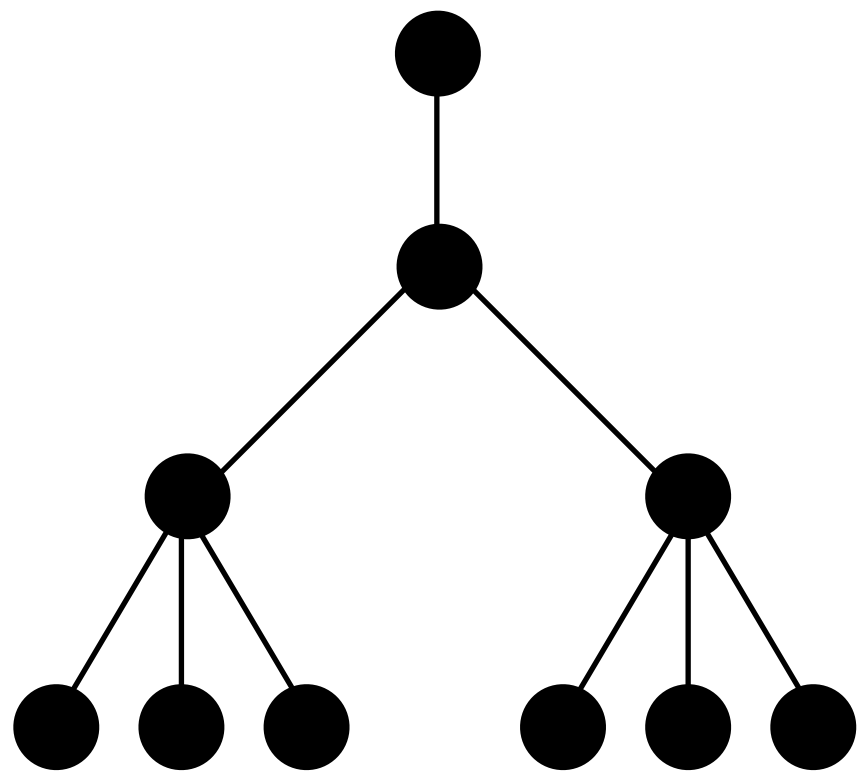
k-Distance

Lower Bound

$$k \leq \log n$$

Lower Bound

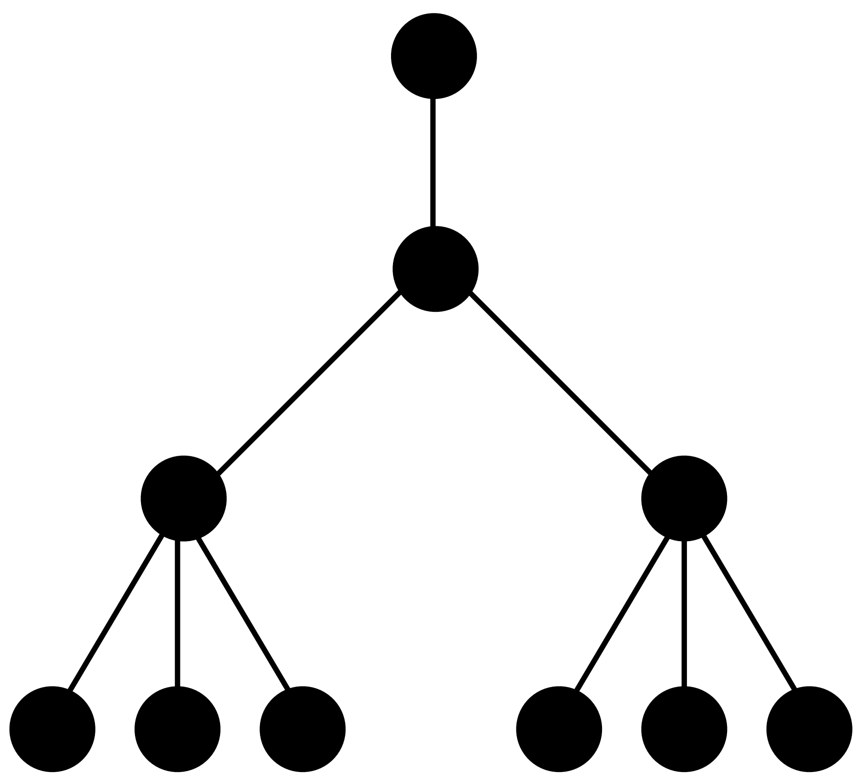
$k \leq \log n$



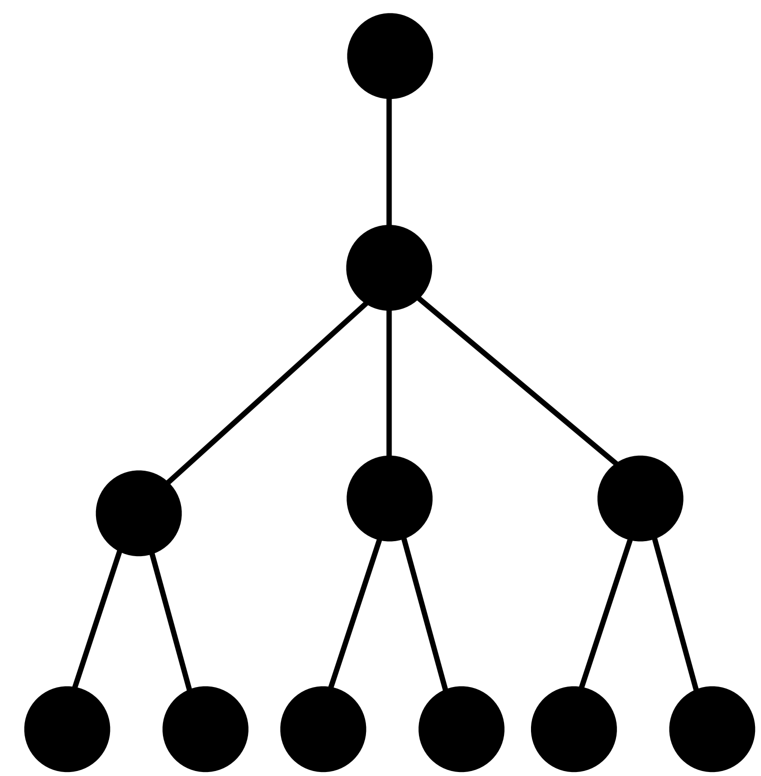
(1,2,3)-regular tree

Lower Bound

$$k \leq \log n$$



(1,2,3)-regular tree

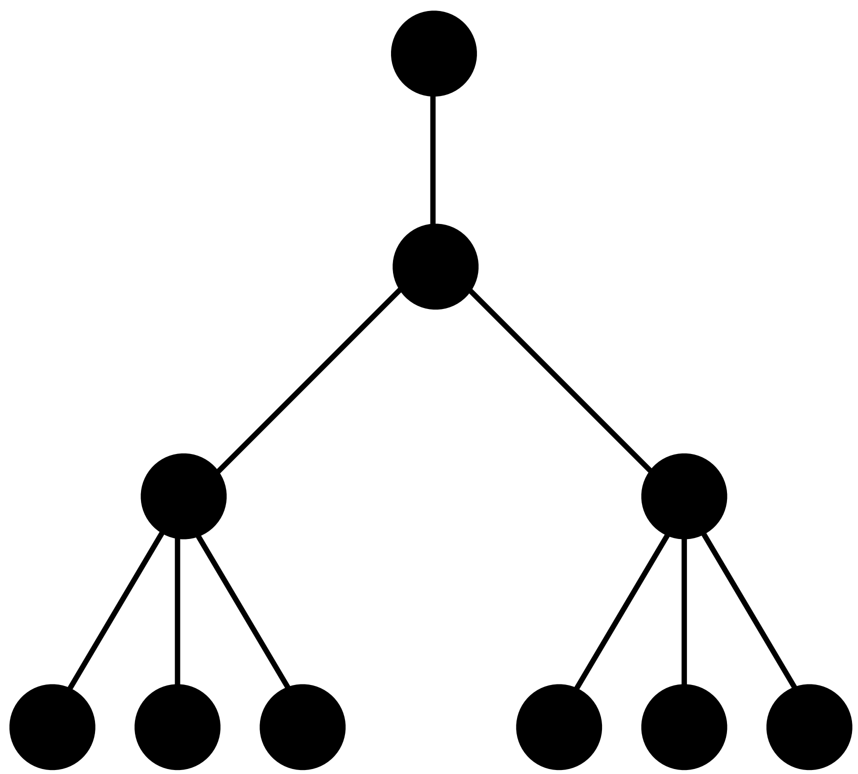


(1,3,2)-regular tree

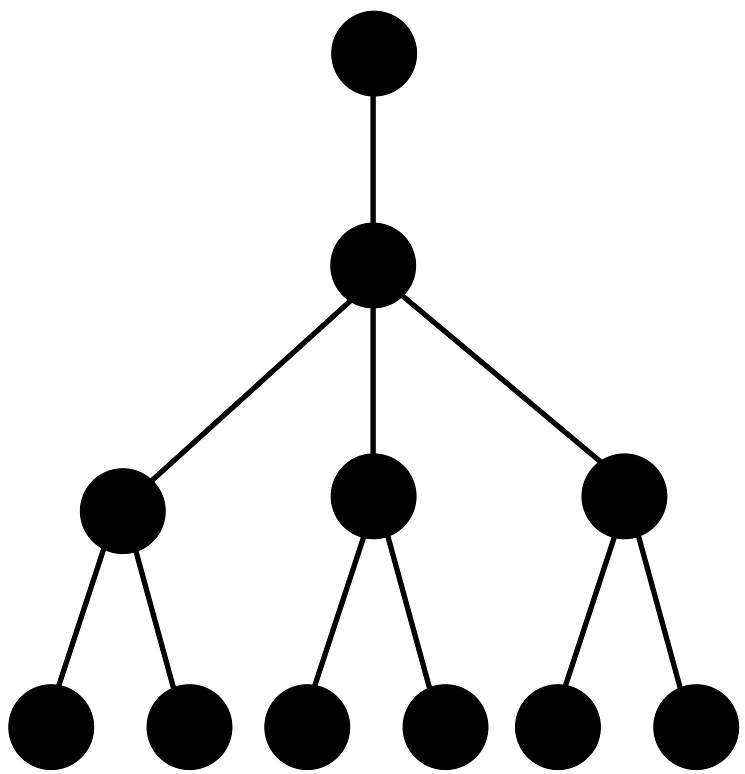
Lower Bound

$$k \leq \log n$$

Common labels



(1,2,3)-regular tree

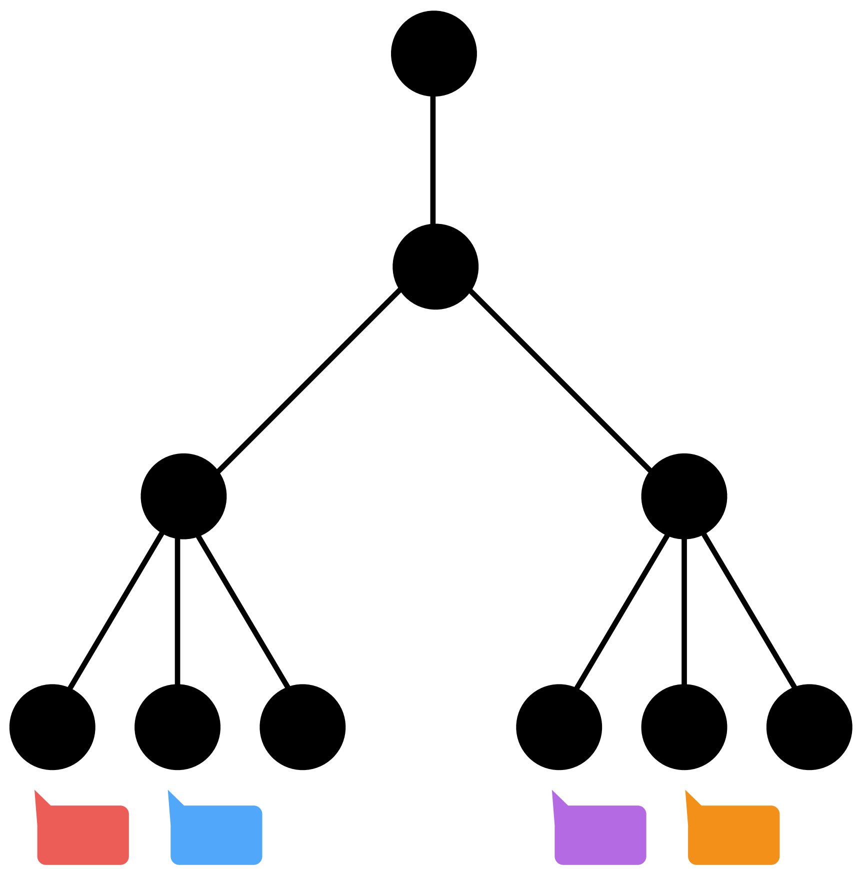


(1,3,2)-regular tree

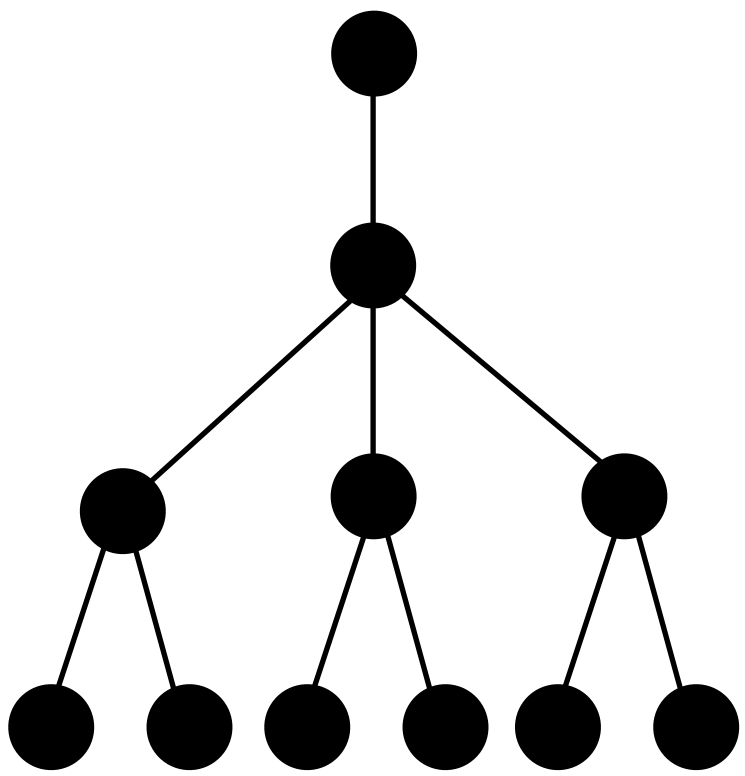
Lower Bound

$k \leq \log n$

Common labels



(1,2,3)-regular tree

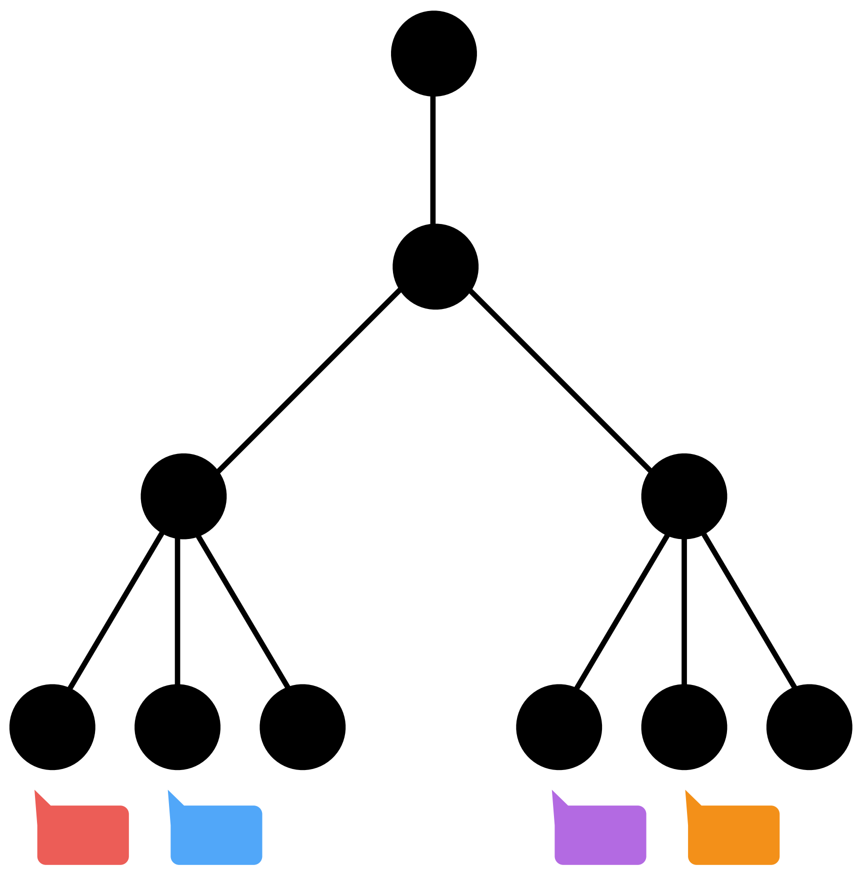


(1,3,2)-regular tree

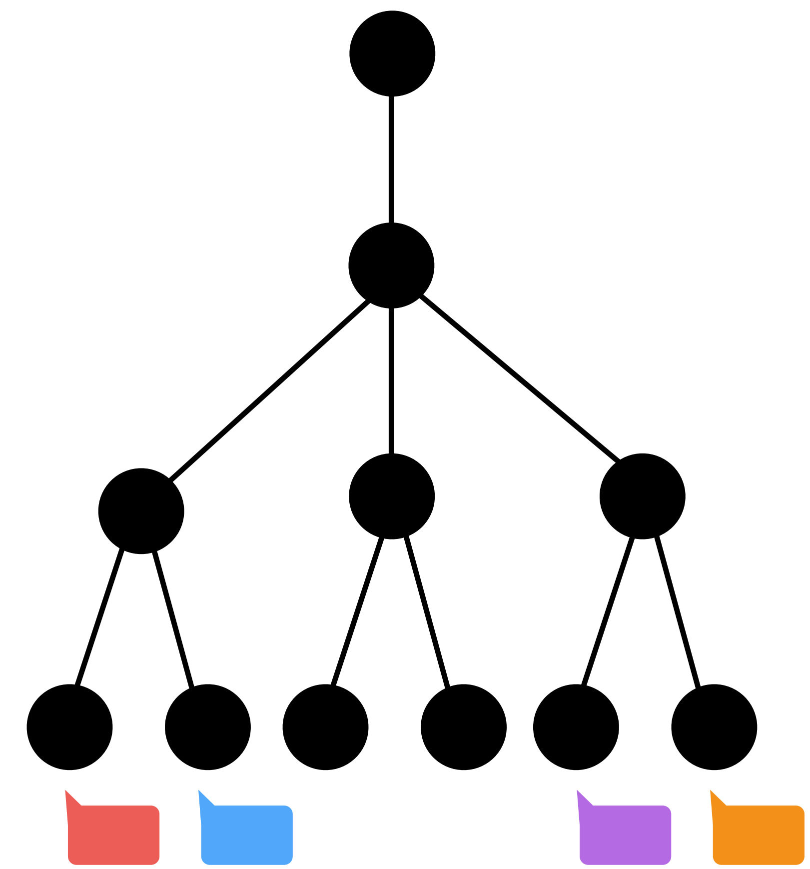
Lower Bound

$$k \leq \log n$$

Common labels



(1,2,3)-regular tree

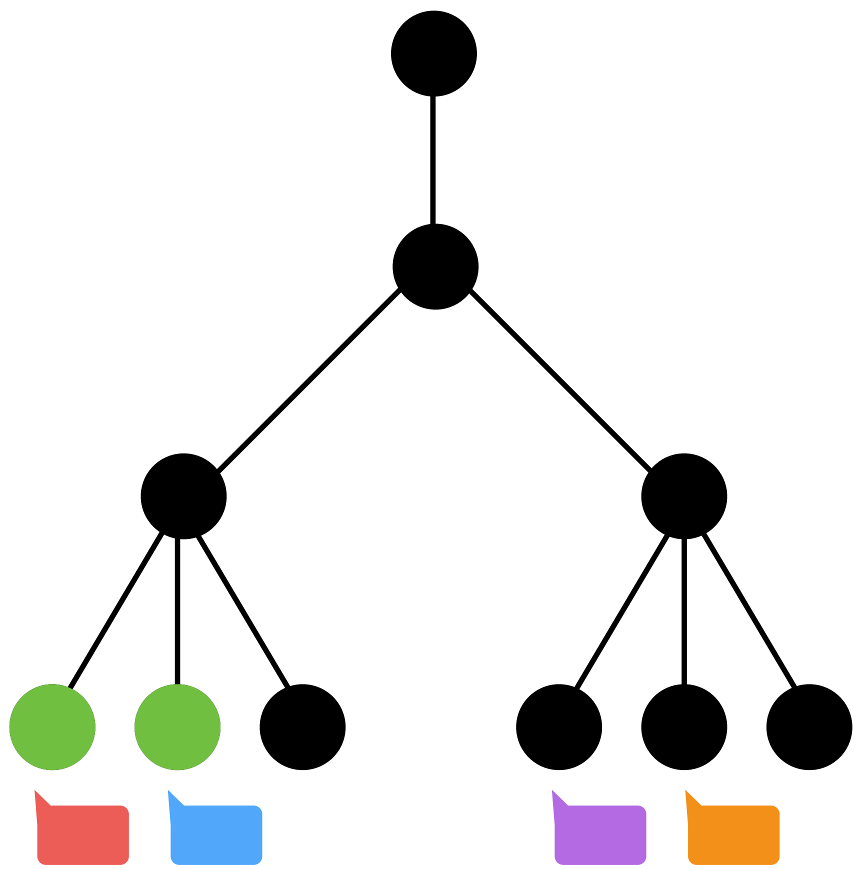


(1,3,2)-regular tree

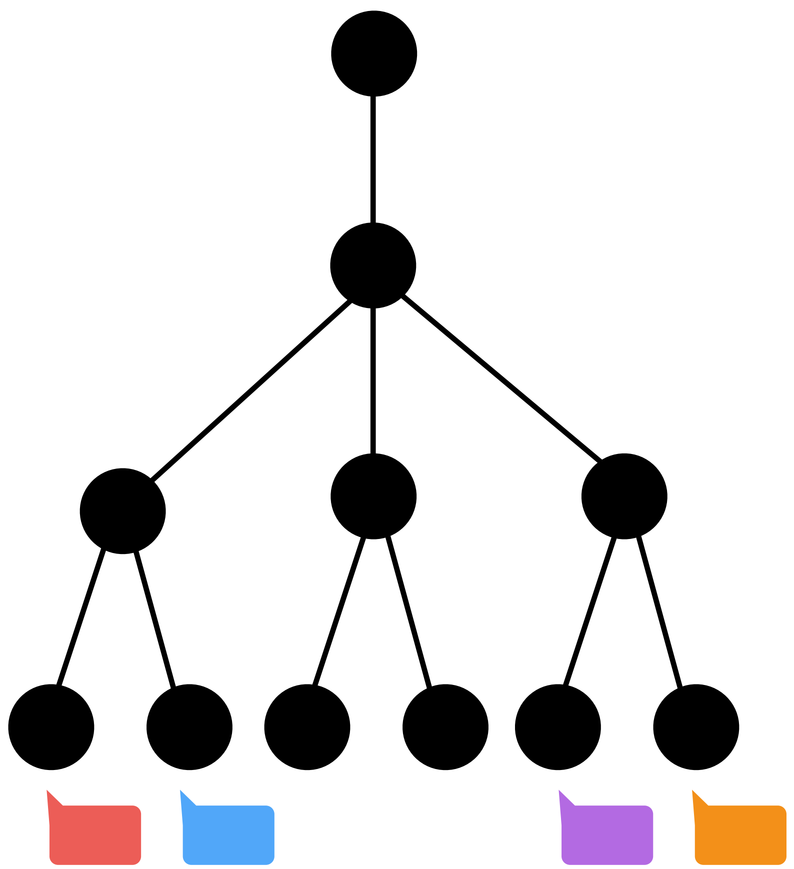
Lower Bound

$$k \leq \log n$$

Common labels



(1,2,3)-regular tree

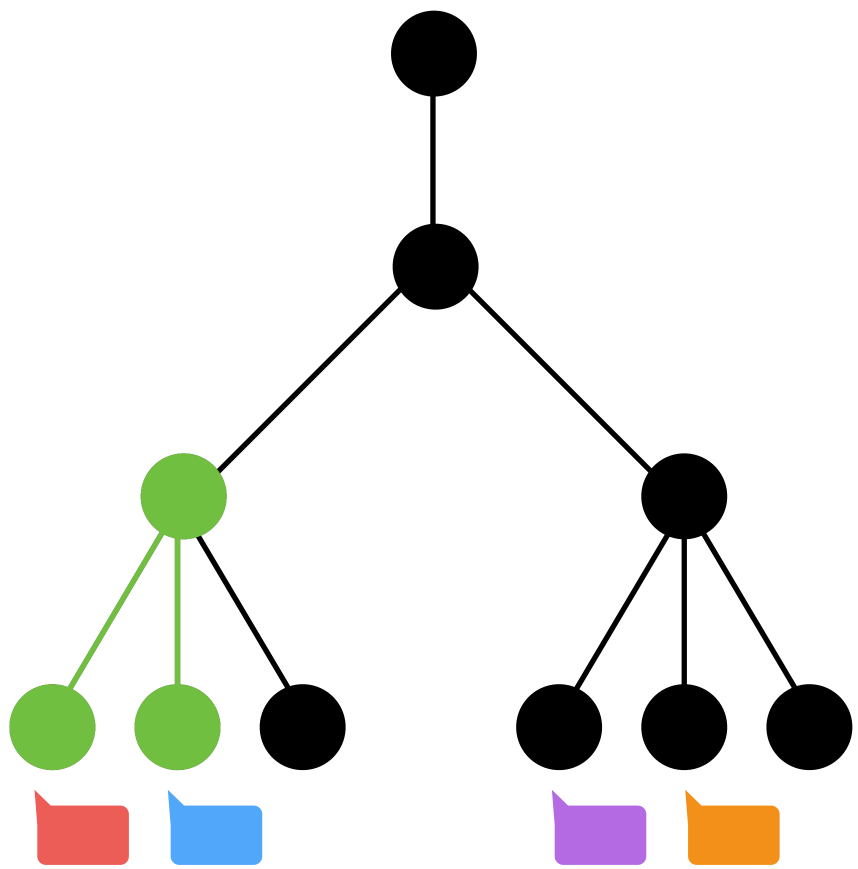


(1,3,2)-regular tree

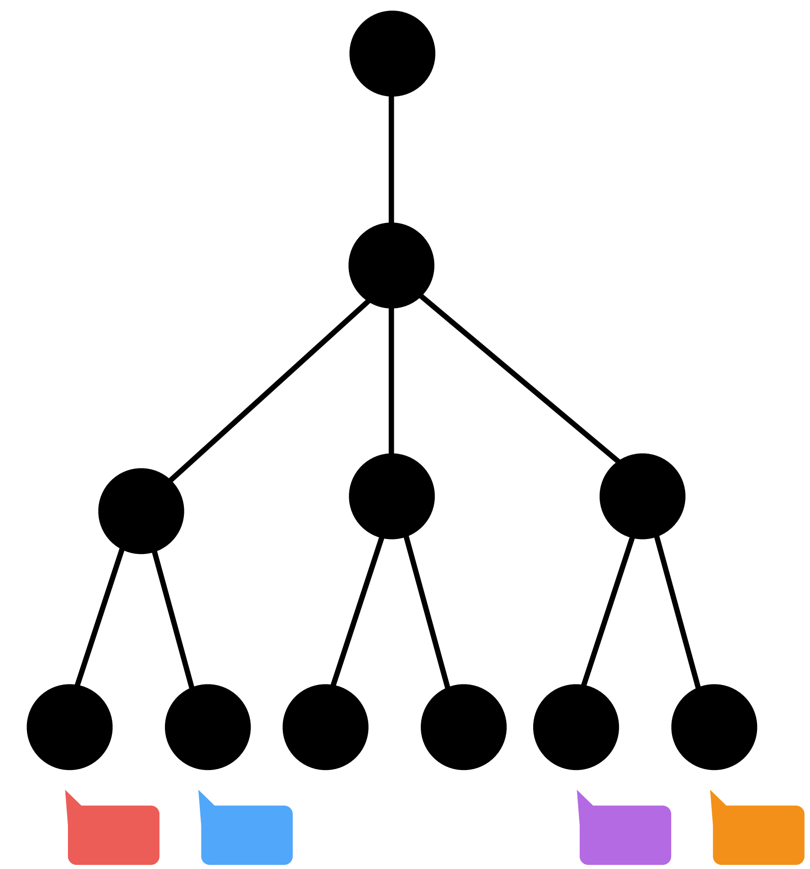
Lower Bound

$$k \leq \log n$$

Common labels



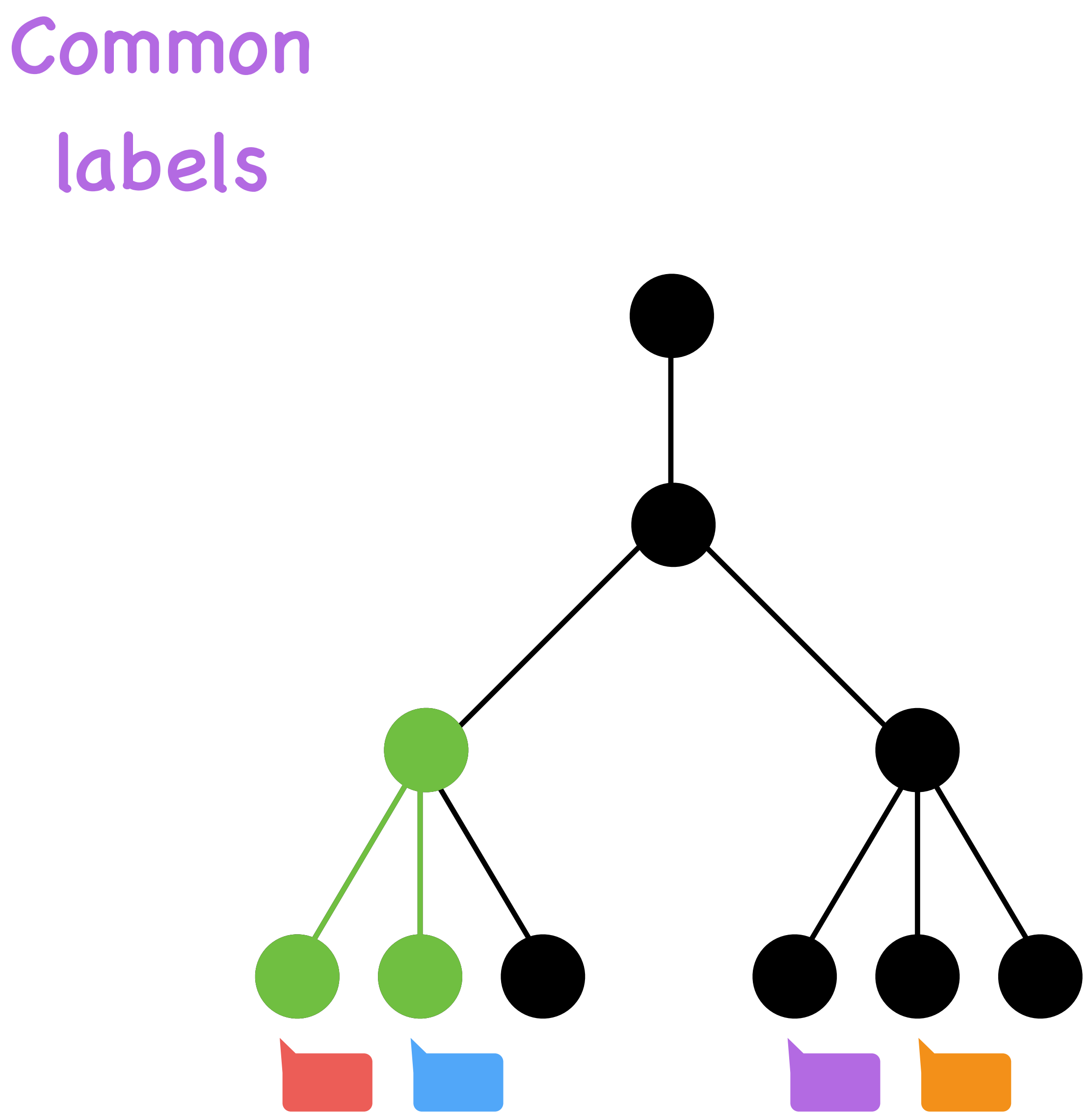
(1,2,3)-regular tree



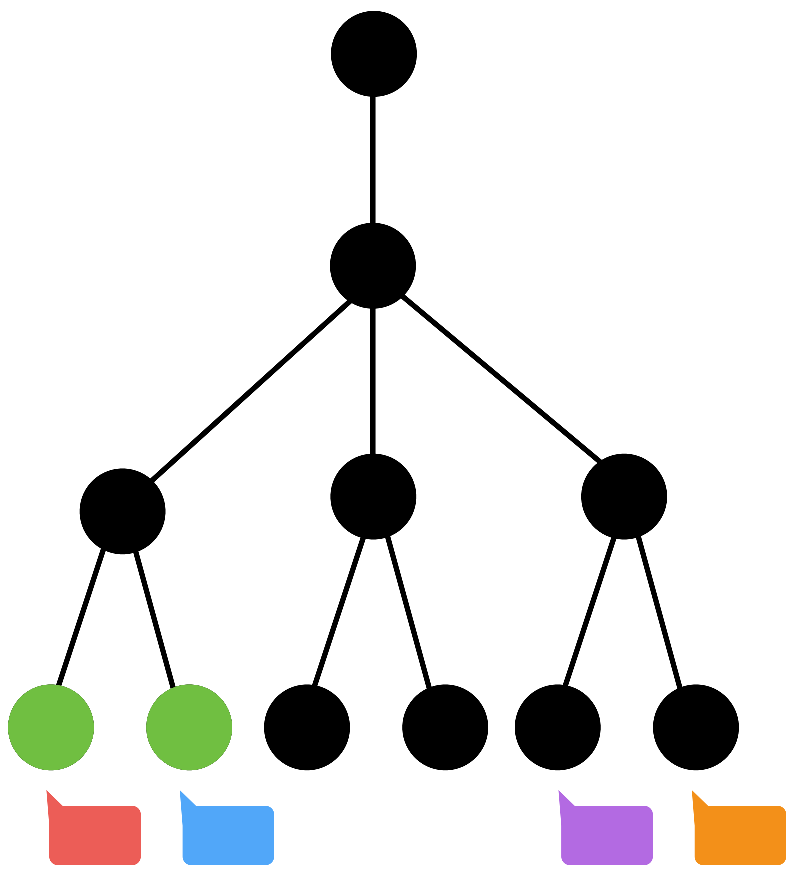
(1,3,2)-regular tree

Lower Bound

$$k \leq \log n$$



(1,2,3)-regular tree

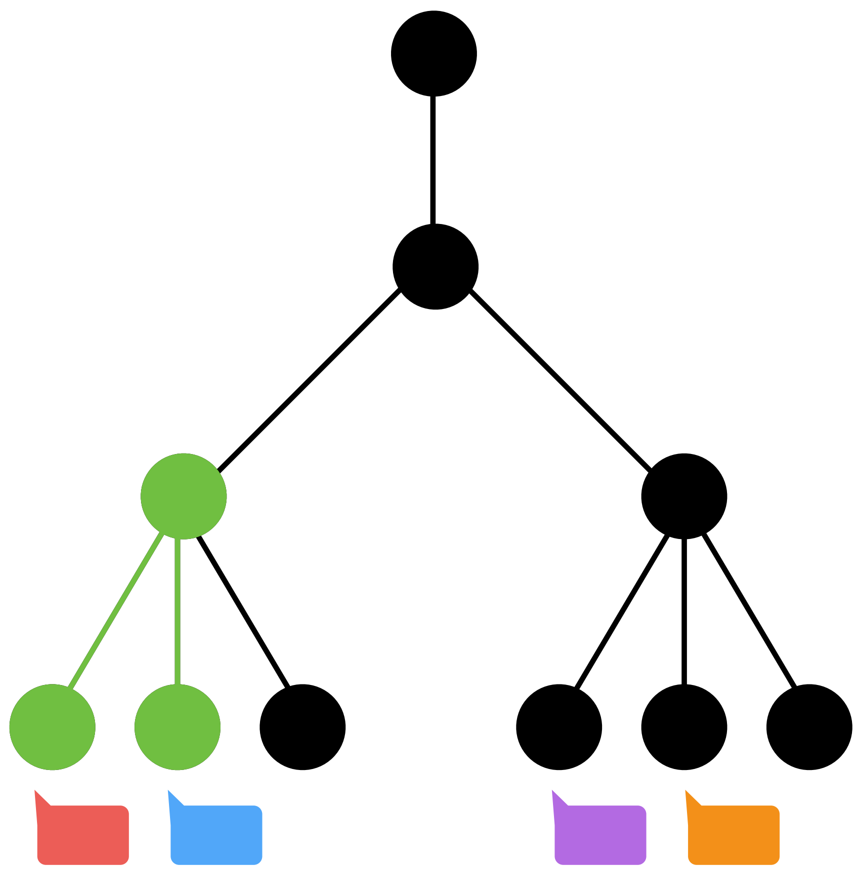


(1,3,2)-regular tree

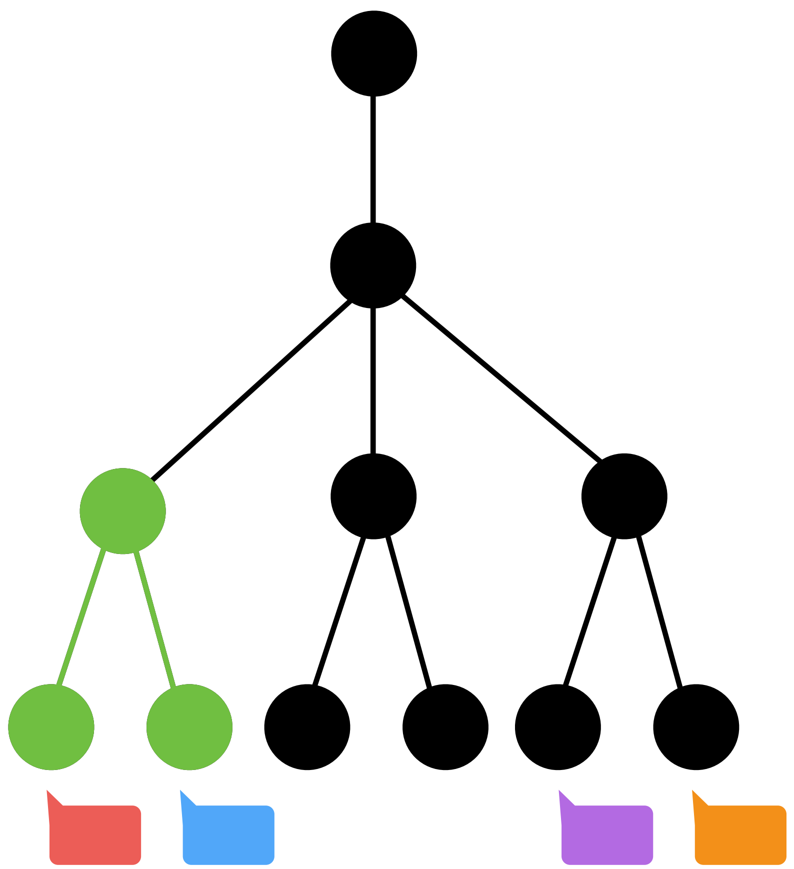
Lower Bound

$$k \leq \log n$$

Common
labels



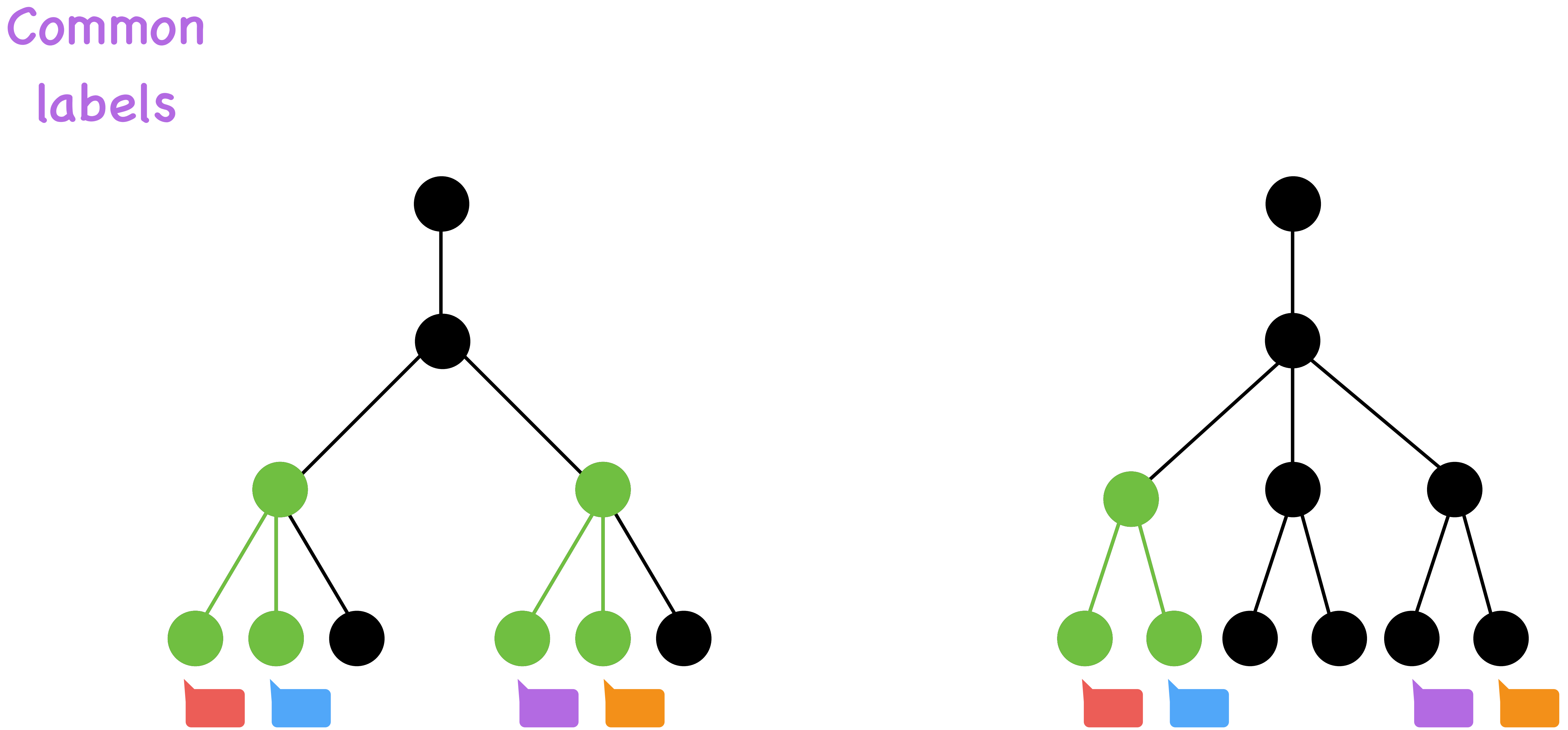
(1,2,3)-regular tree



(1,3,2)-regular tree

Lower Bound

$k \leq \log n$



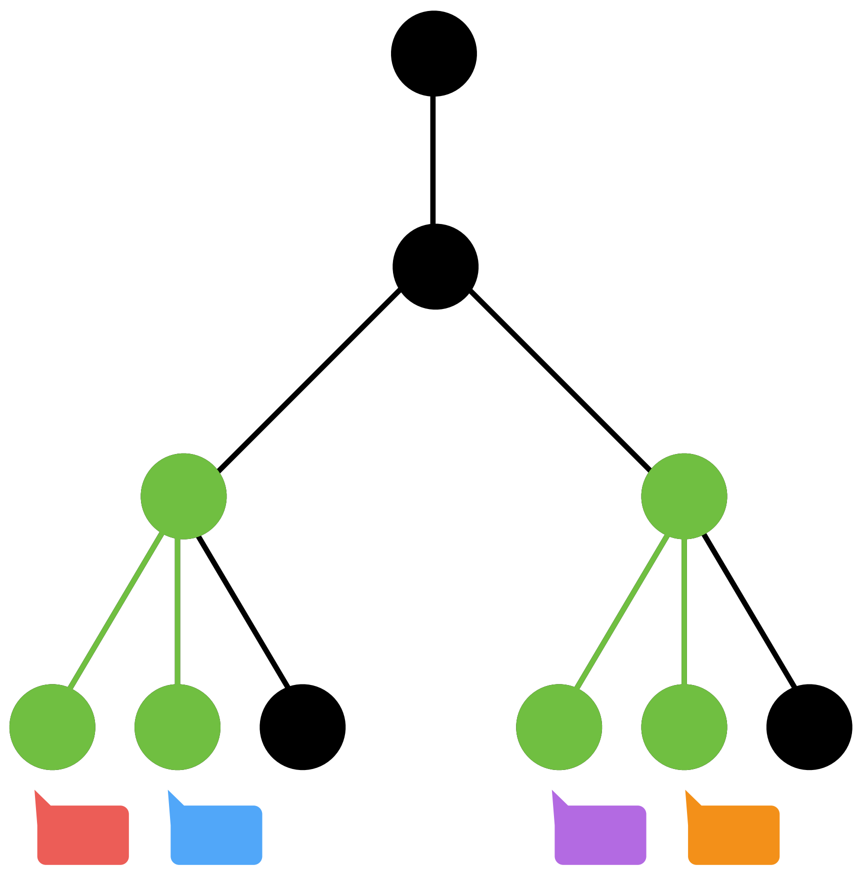
(1,2,3)-regular tree

(1,3,2)-regular tree

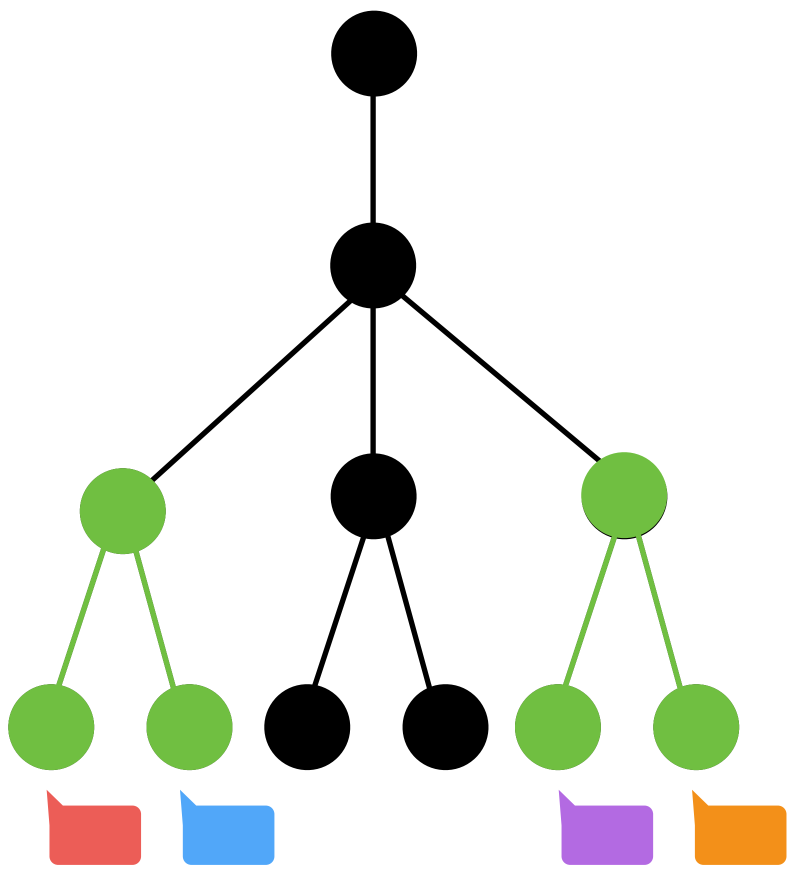
Lower Bound

$$k \leq \log n$$

Common labels



(1,2,3)-regular tree

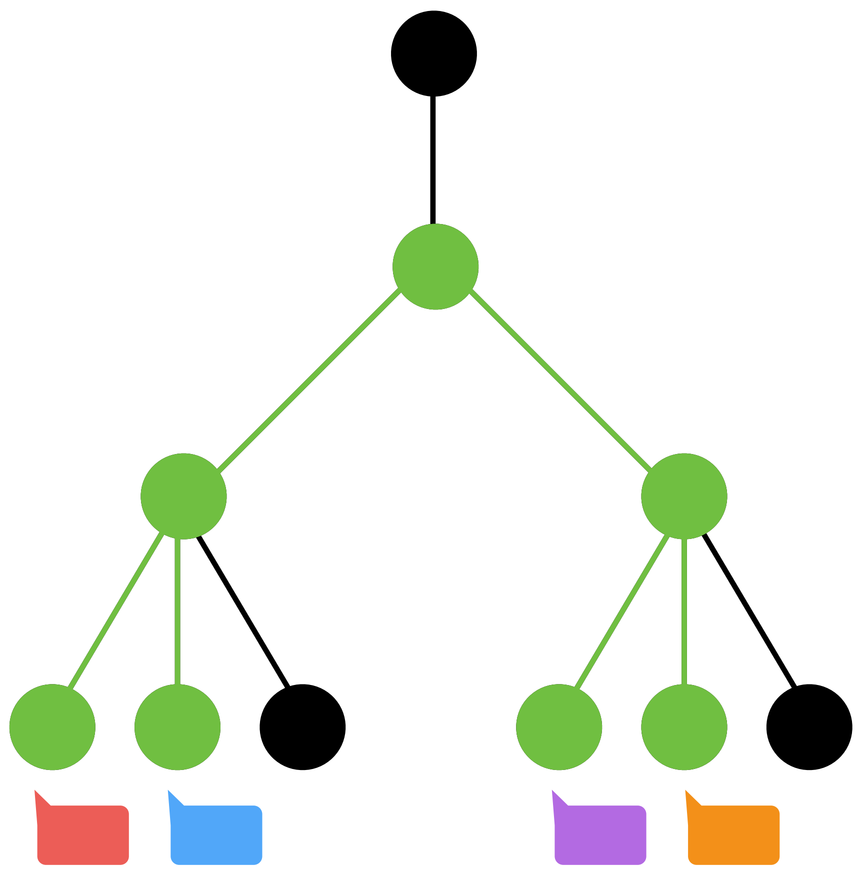


(1,3,2)-regular tree

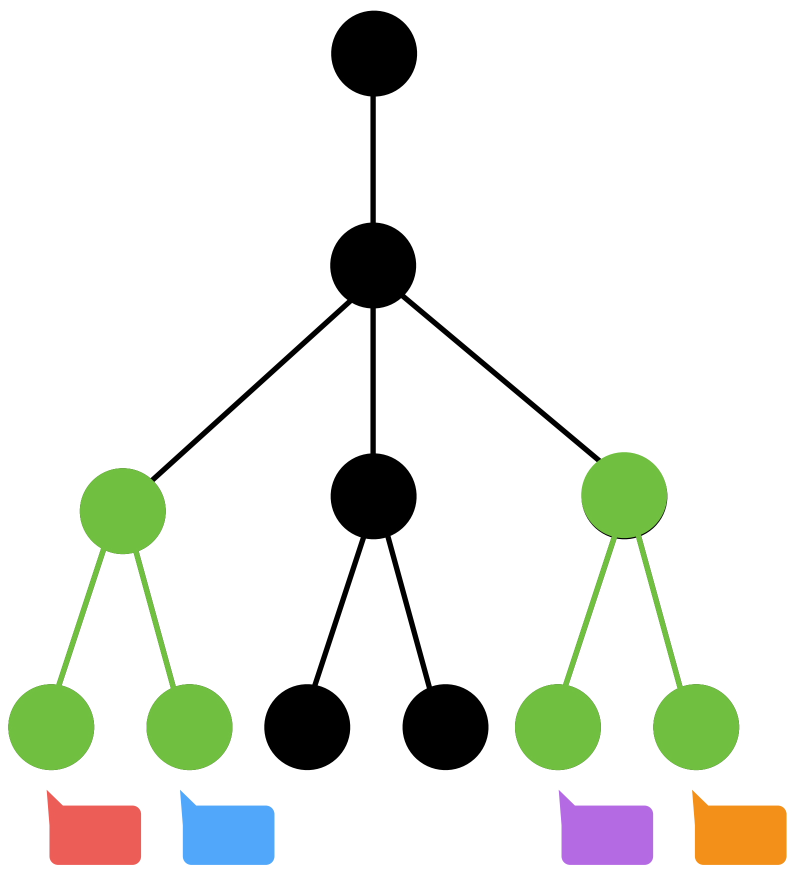
Lower Bound

$$k \leq \log n$$

Common labels



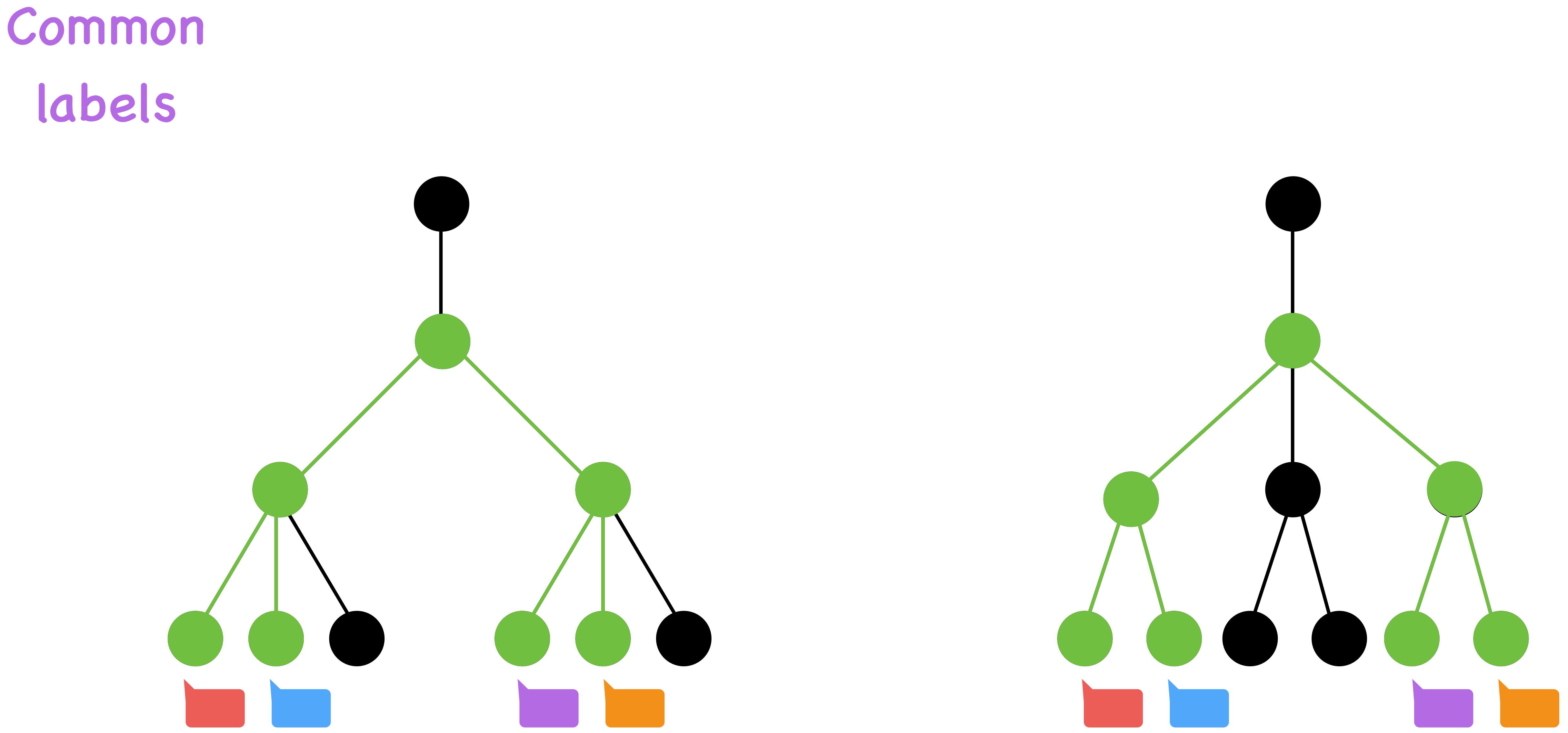
(1,2,3)-regular tree



(1,3,2)-regular tree

Lower Bound

$k \leq \log n$



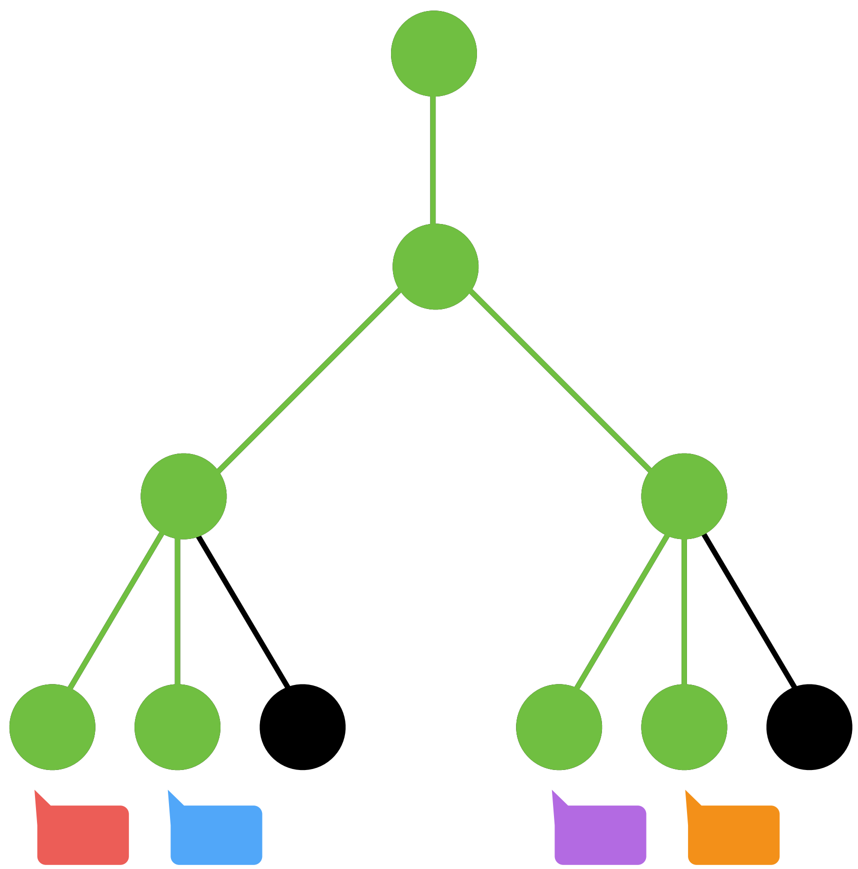
(1,2,3)-regular tree

(1,3,2)-regular tree

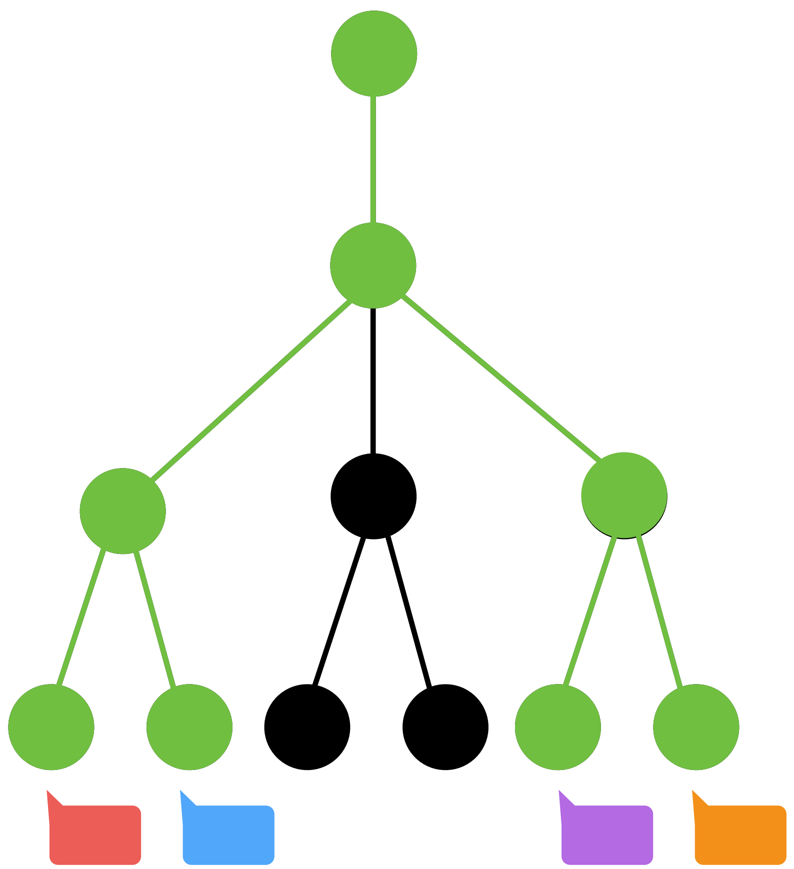
Lower Bound

$k \leq \log n$

Common labels



(1,2,3)-regular tree

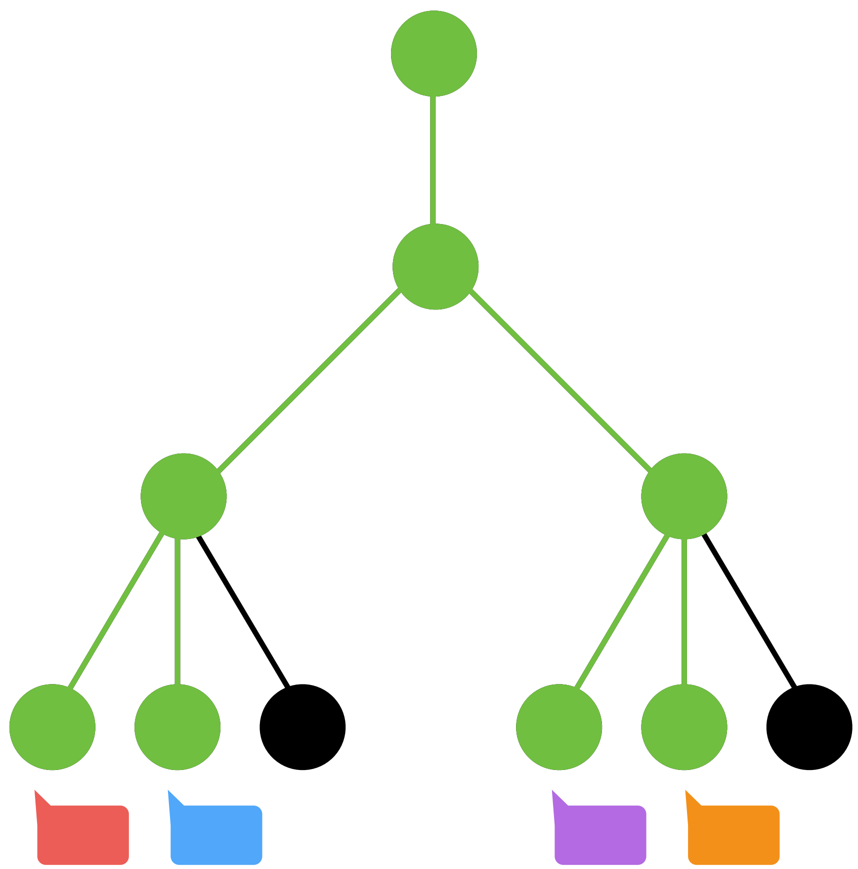


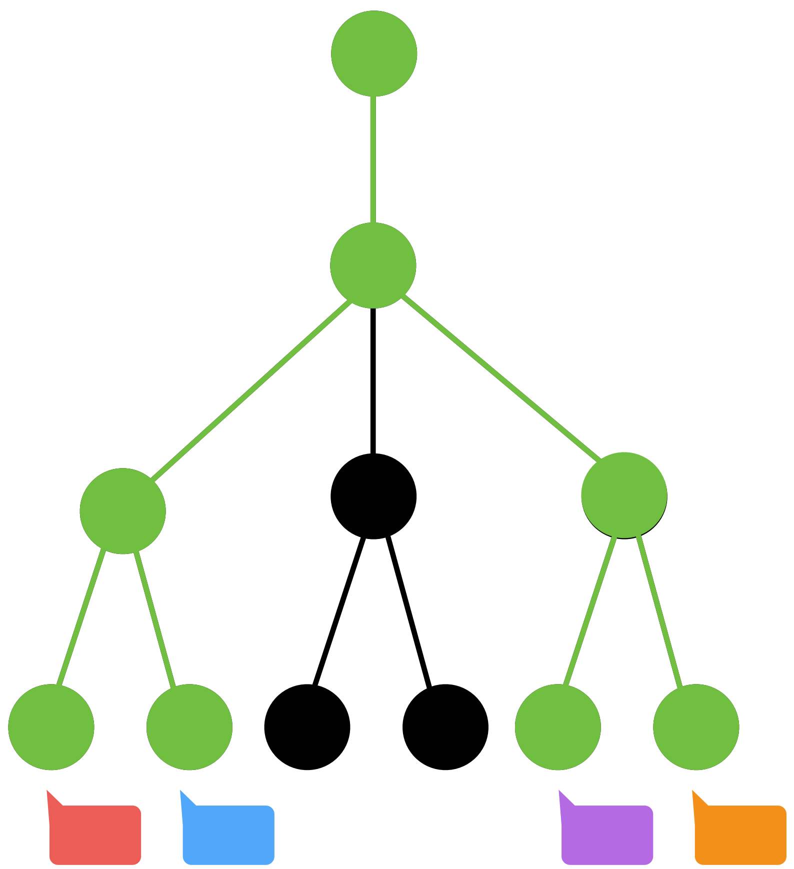
(1,3,2)-regular tree

Lower Bound

$$k \leq \log n$$

Common labels =>





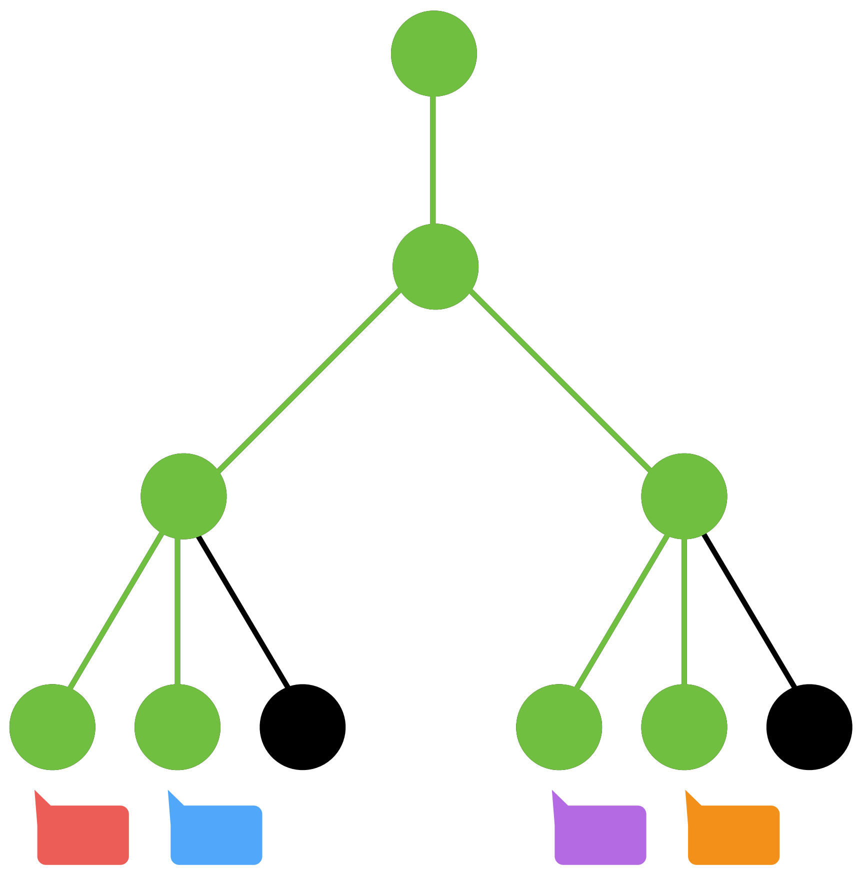
(1,2,3)-regular tree

(1,3,2)-regular tree

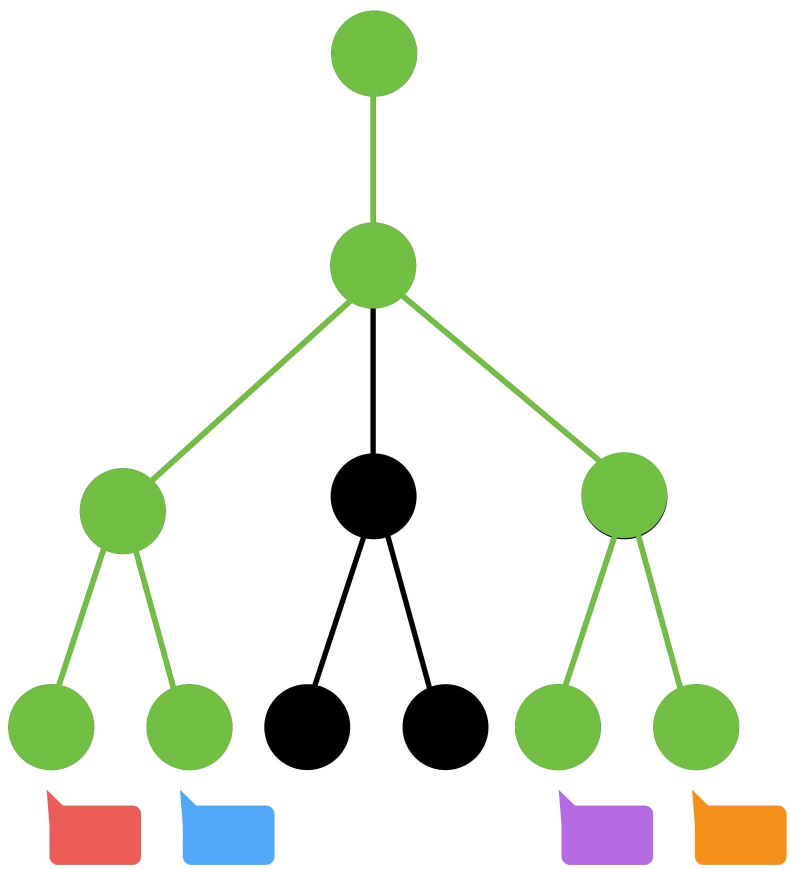
Lower Bound

$k \leq \log n$

Common labels => Common Subtree



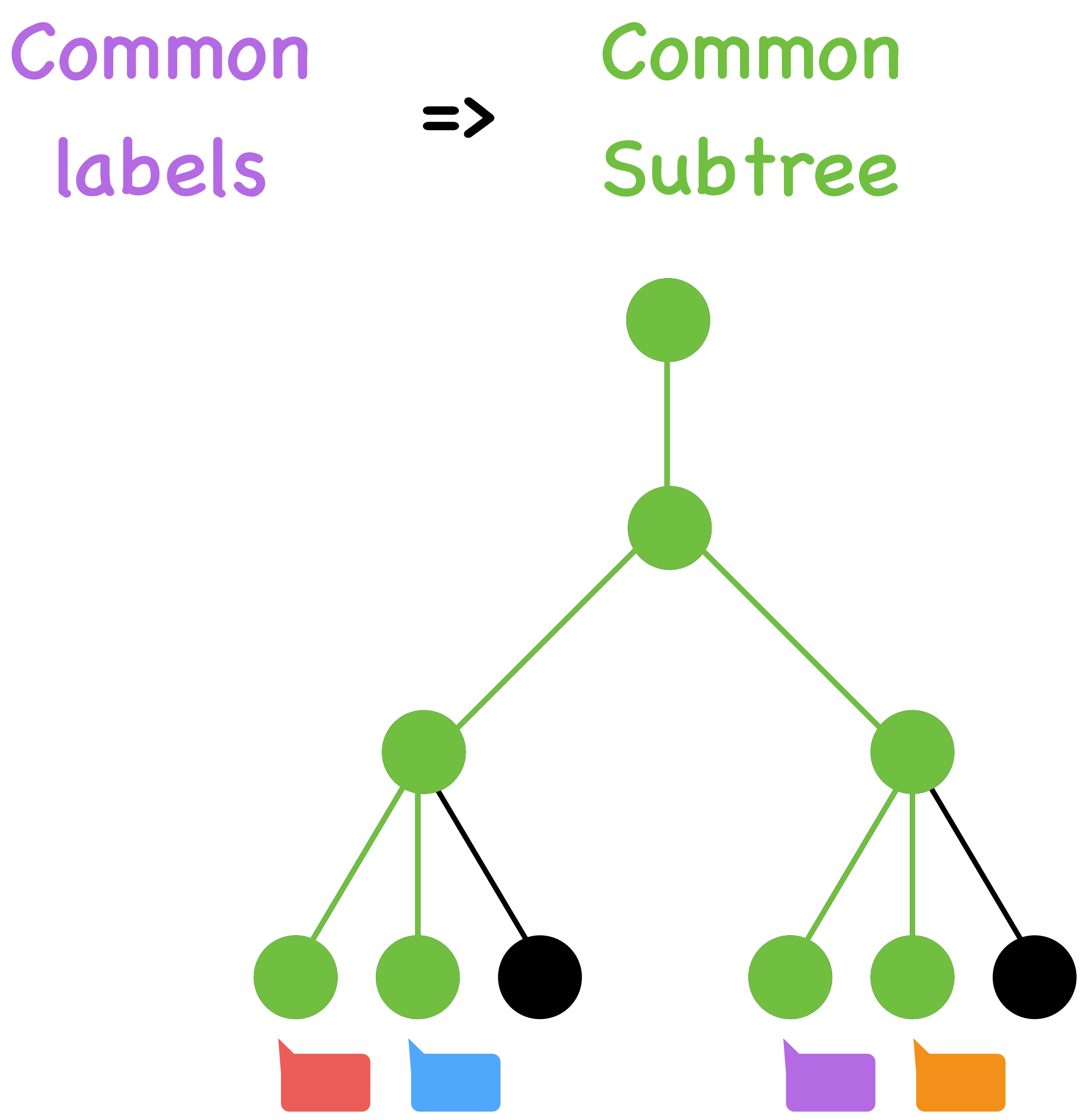
(1,2,3)-regular tree



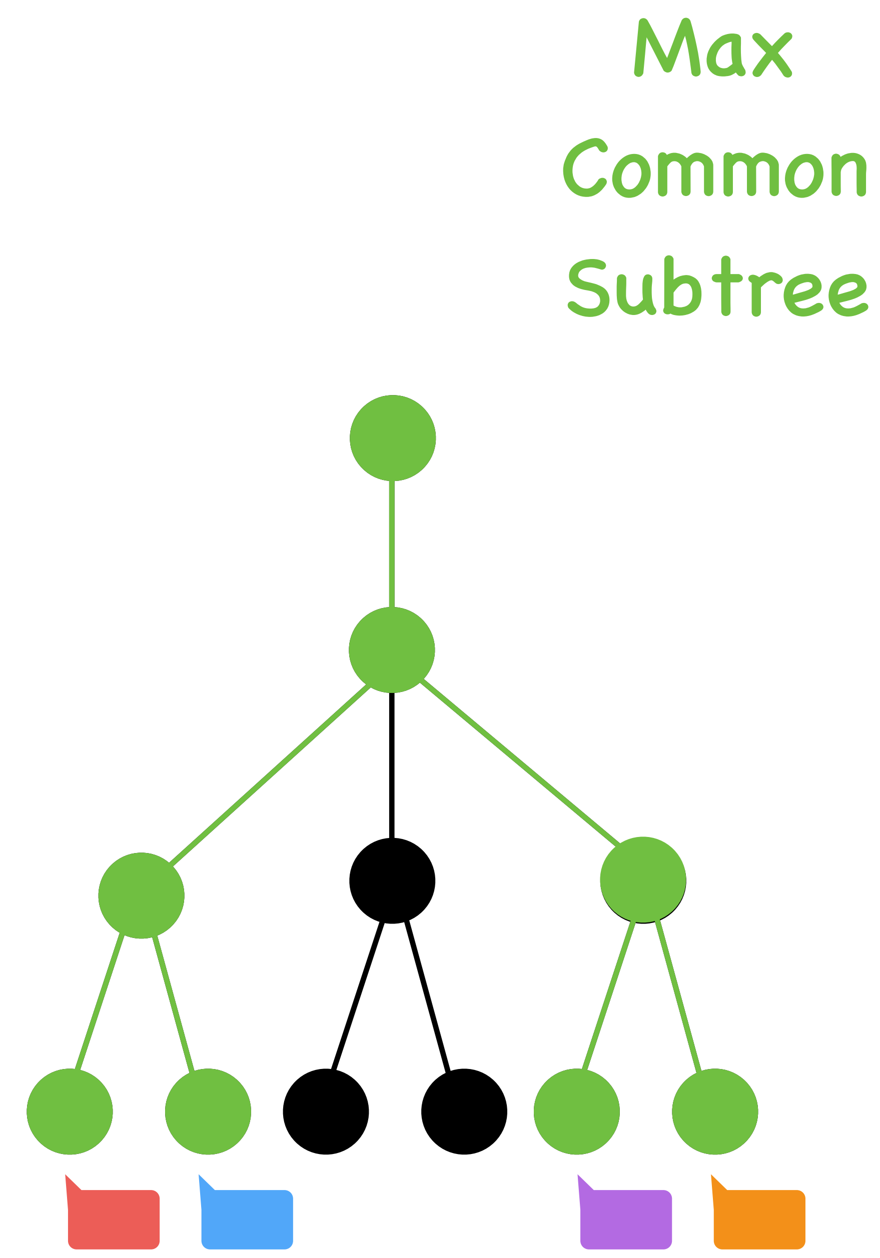
(1,3,2)-regular tree

Lower Bound

$k \leq \log n$



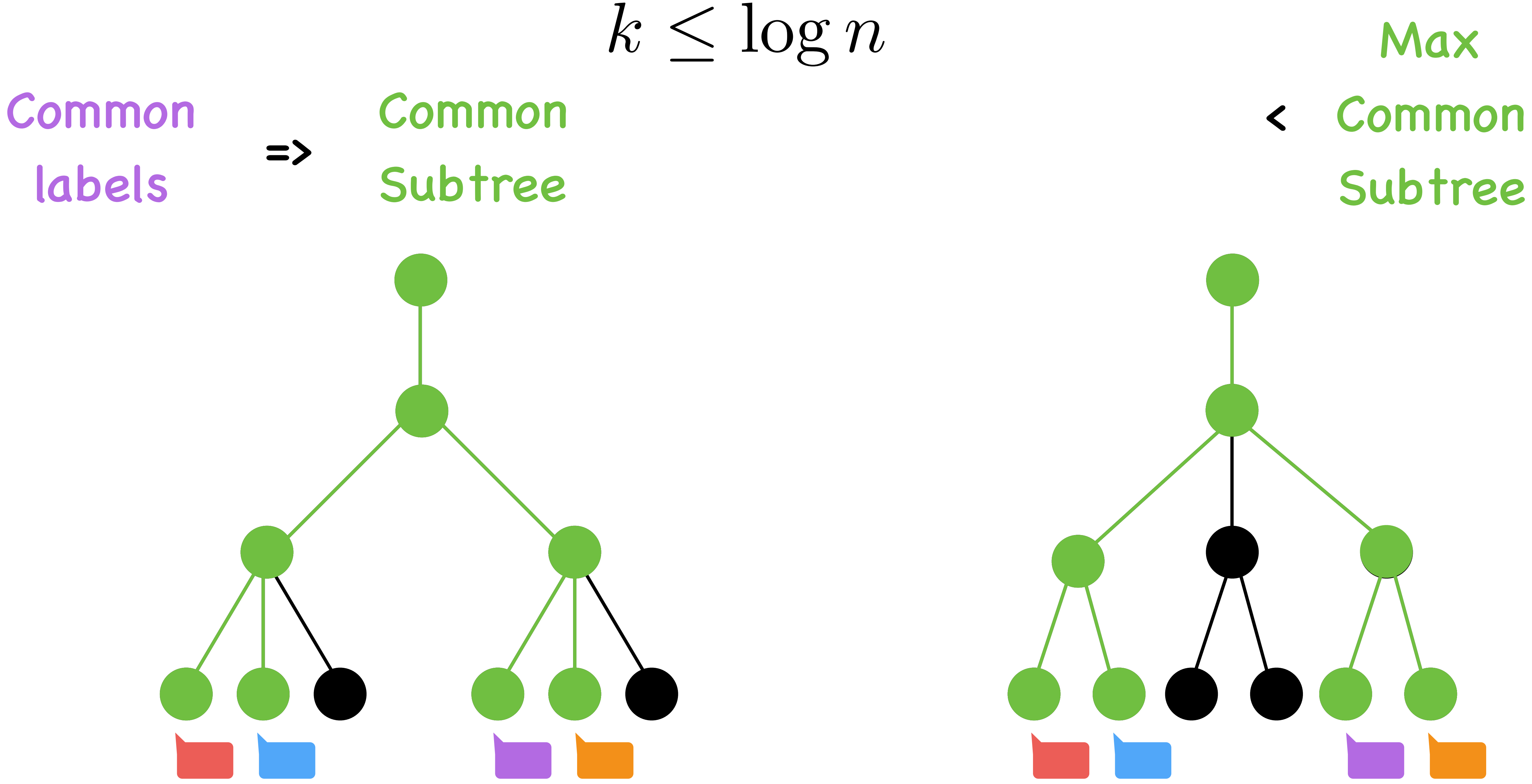
(1,2,3)-regular tree



(1,3,2)-regular tree

Lower Bound

$$k \leq \log n$$



(1,2,3)-regular tree

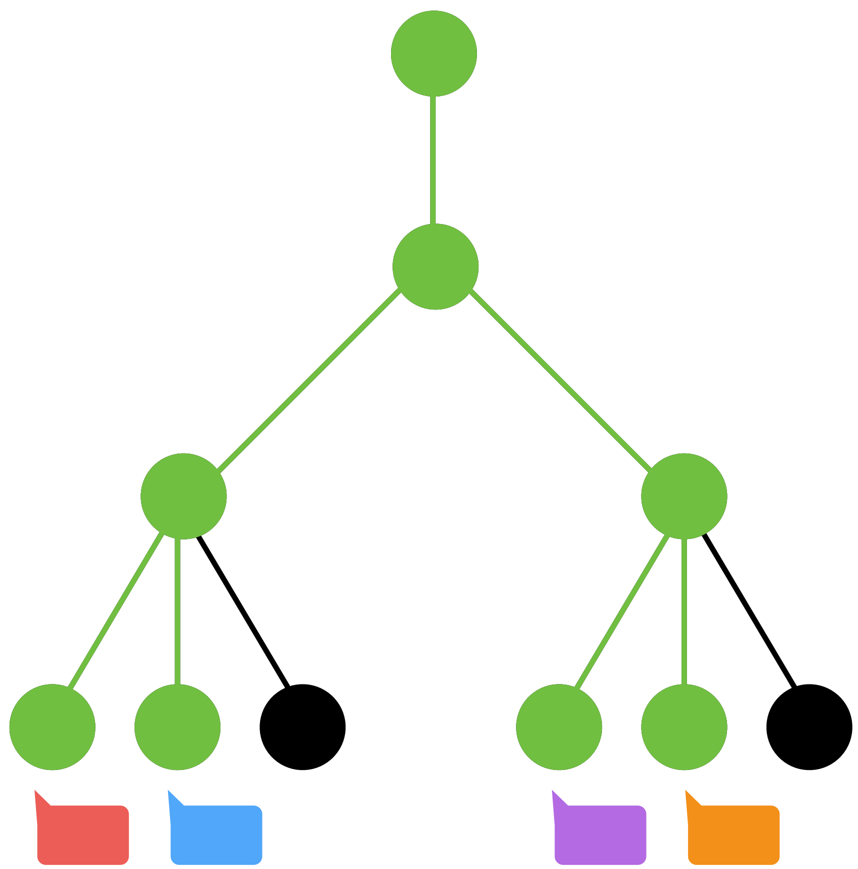
(1,3,2)-regular tree

Lower Bound

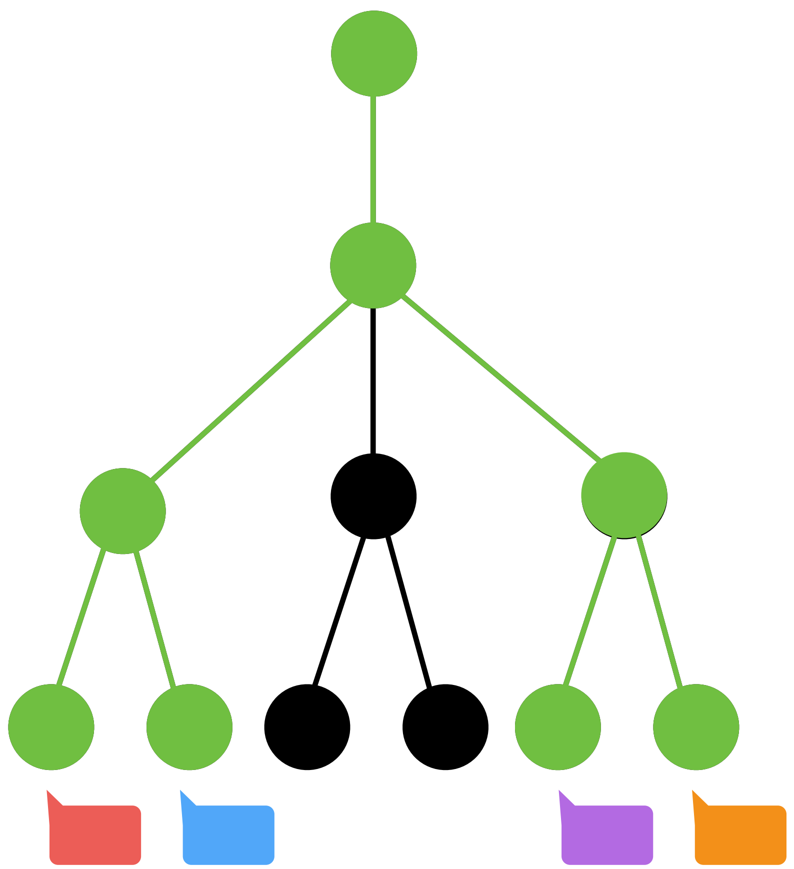
$k \leq \log n$

Common labels => Common Subtree

Max Common labels < Max Common Subtree



(1,2,3)-regular tree



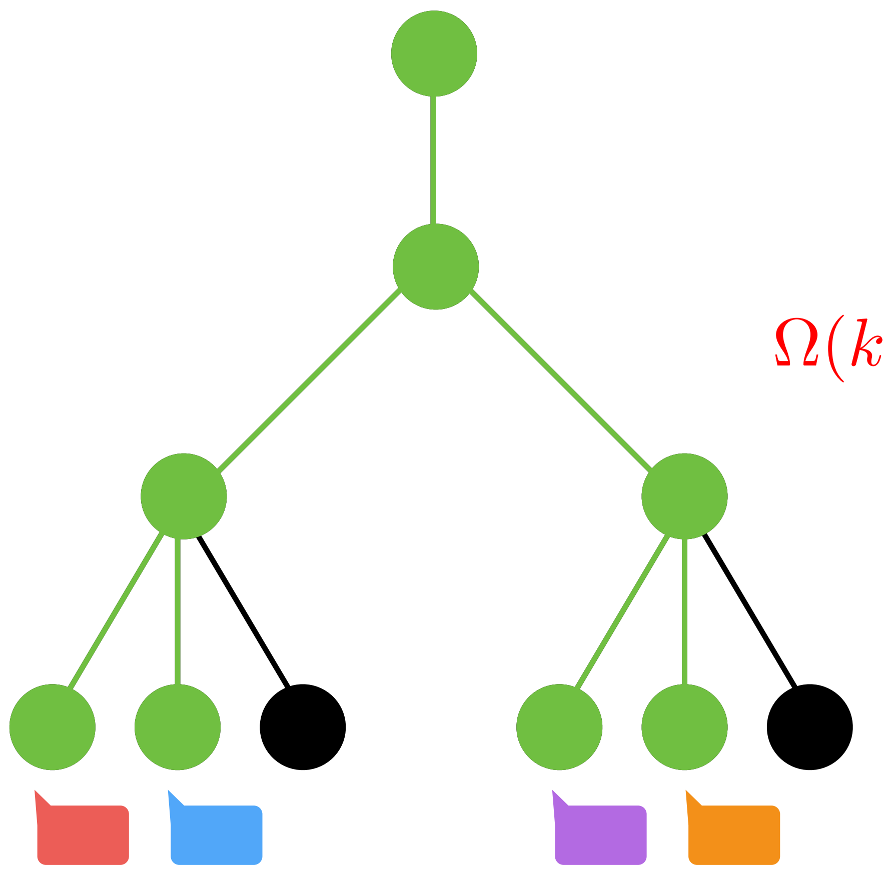
(1,3,2)-regular tree

Lower Bound

$k \leq \log n$

Common labels => Common Subtree

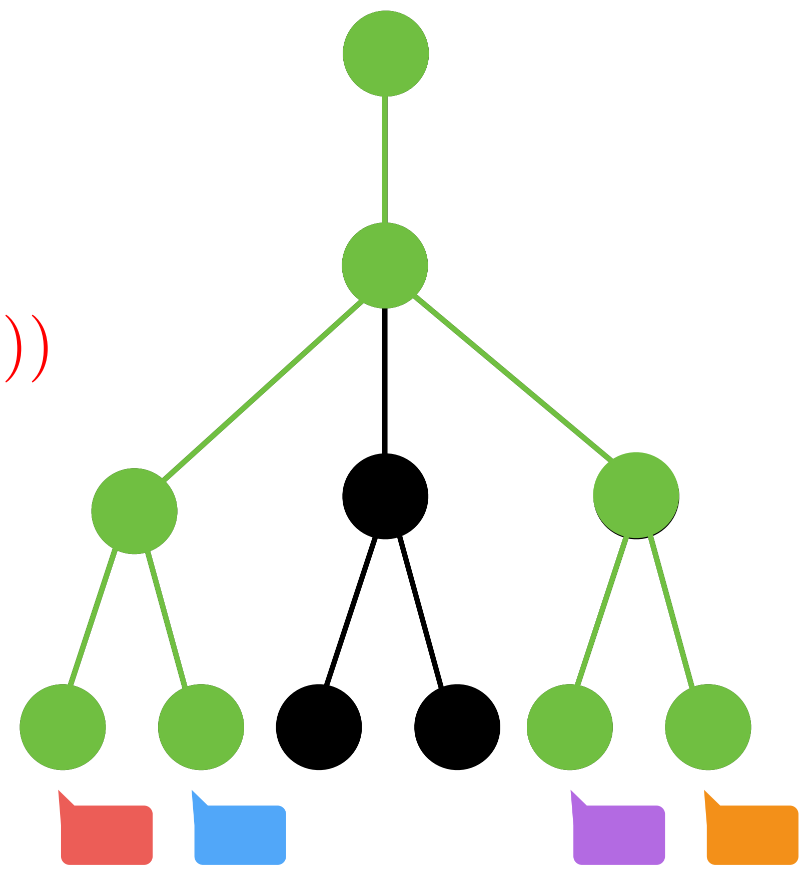
Max Common labels < Max Common Subtree



$\log n$

$+$

$\Omega(k \log(\log n / k \log k))$



(1,2,3)-regular tree

(1,3,2)-regular tree

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	
k-Distance $k > \log n$		

Results

	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	$\log n$ $+$ $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$		

Results

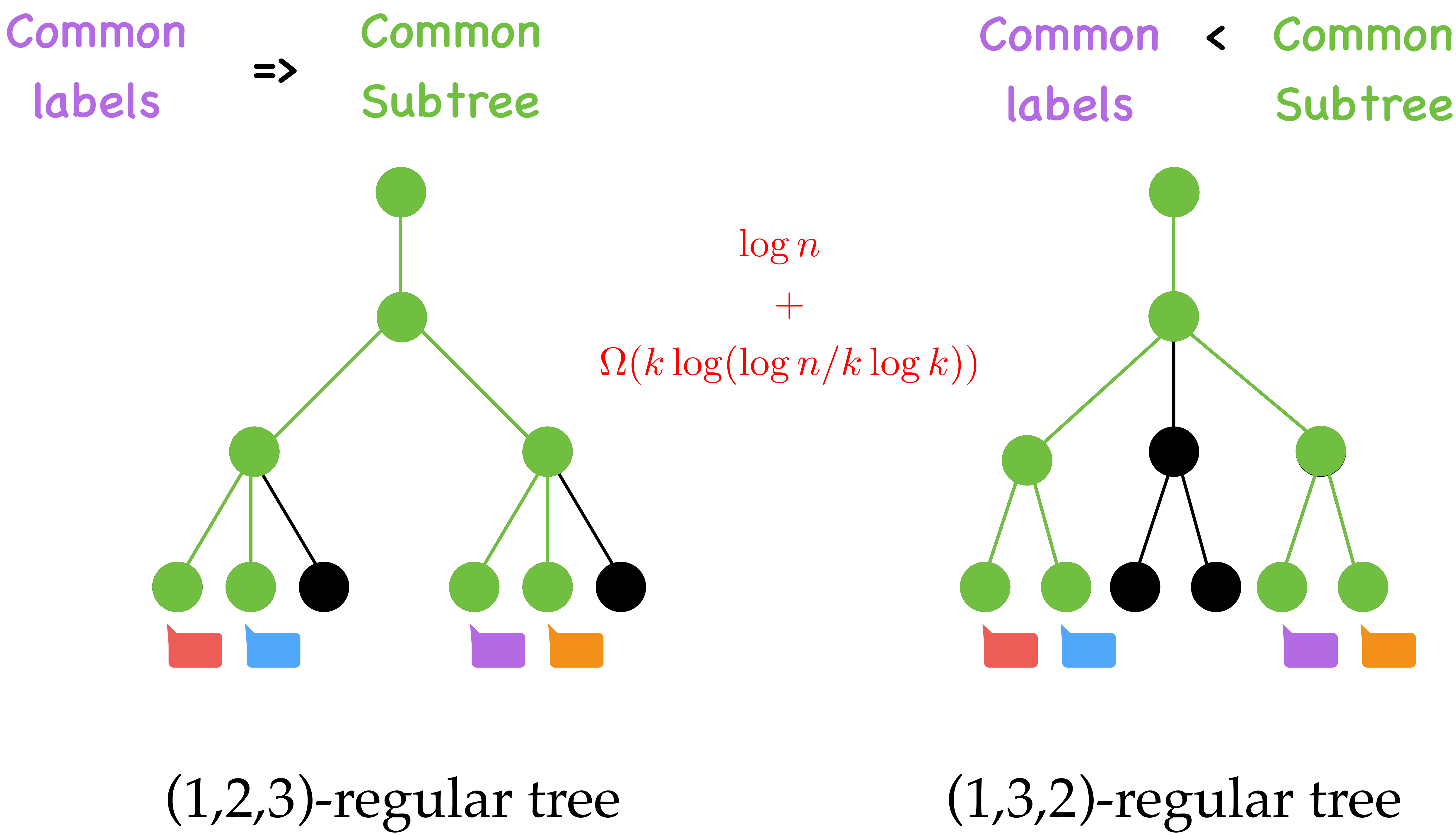
	Upper	Lower
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]	$\frac{1}{8}\log^2 n$ => $\frac{1}{4}\log^2 n$ [Gavoille et al. 2001] [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ => $\log\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ + $O(k\log(k\log n/k))$ => [Gavoille et al. 2007]	$\log n$ + $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$		

Results

	Upper			Lower		
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	=>	$\frac{1}{4}\log^2 n$	$\frac{1}{8}\log^2 n$ [Gavoille et al. 2001]	=>	$\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	=>	$\log\frac{1}{\varepsilon}\log n$			$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	=>	$\log n$ $+$ $O(k\log(\log n/k))$			$\log n$ $+$ $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$						

Lower Bound

$k \leq \log n$



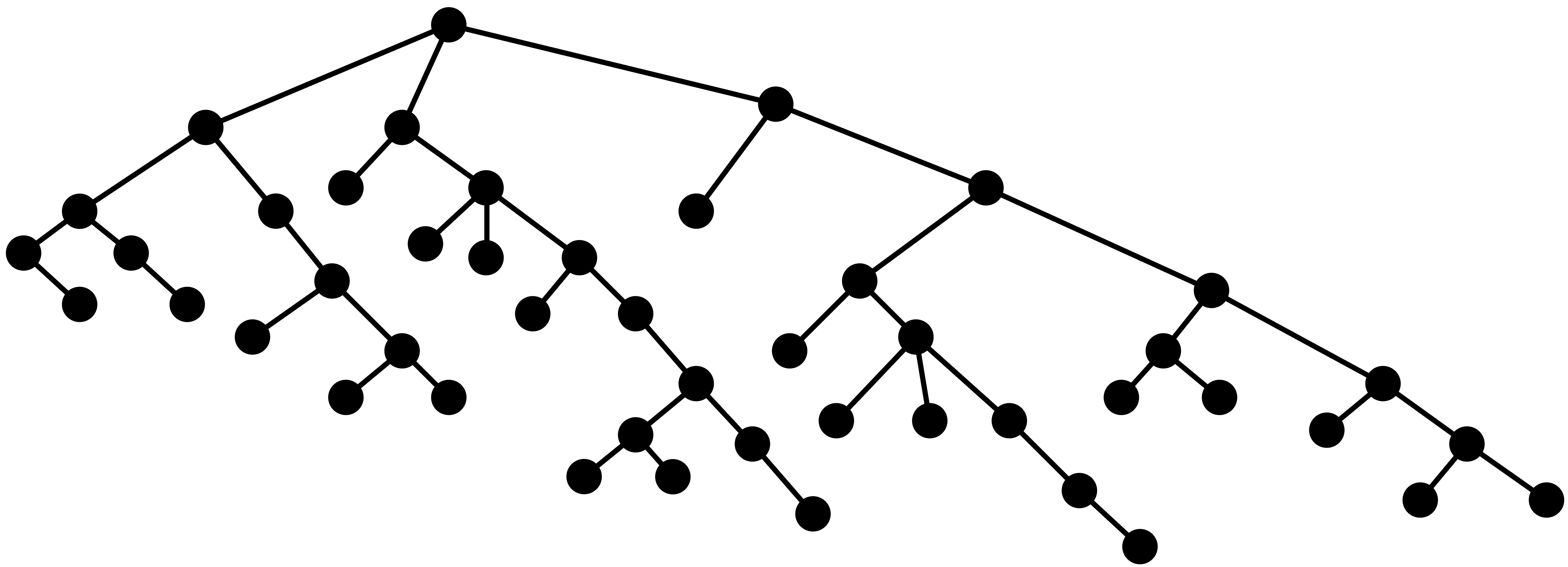
Upper Bound

Upper Bound

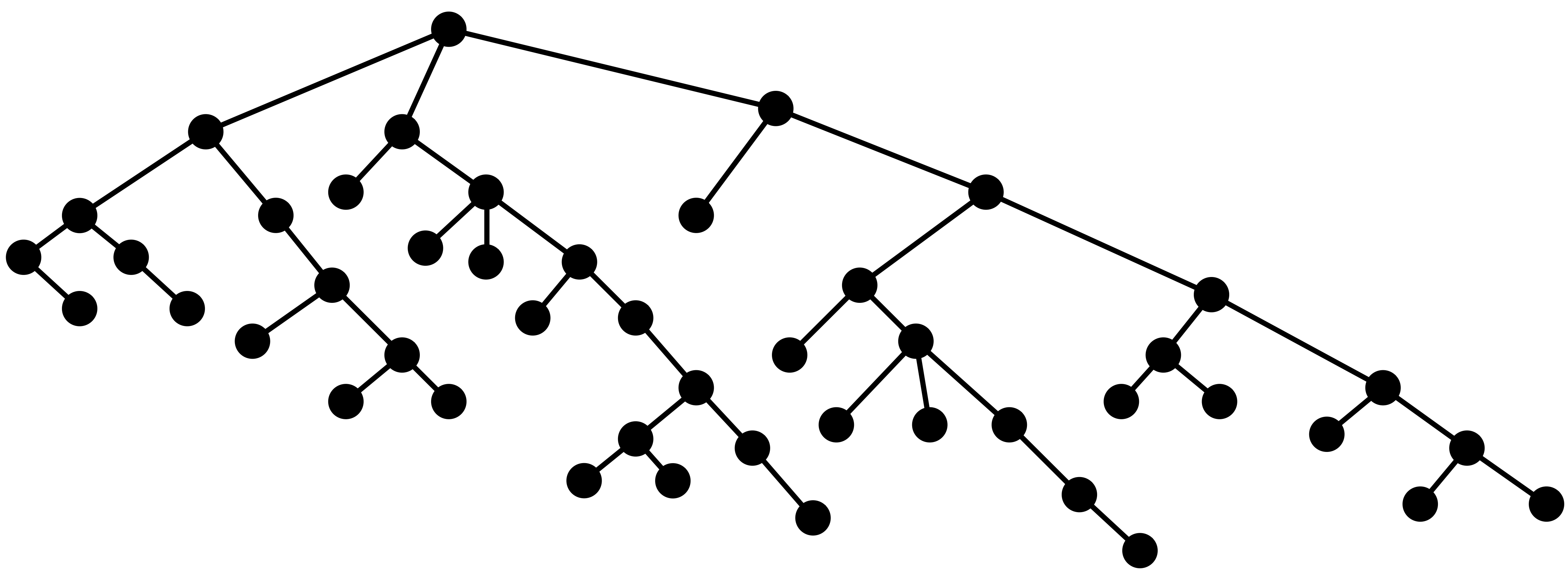
$k \geq \log n$

Heavy Light Decomposition

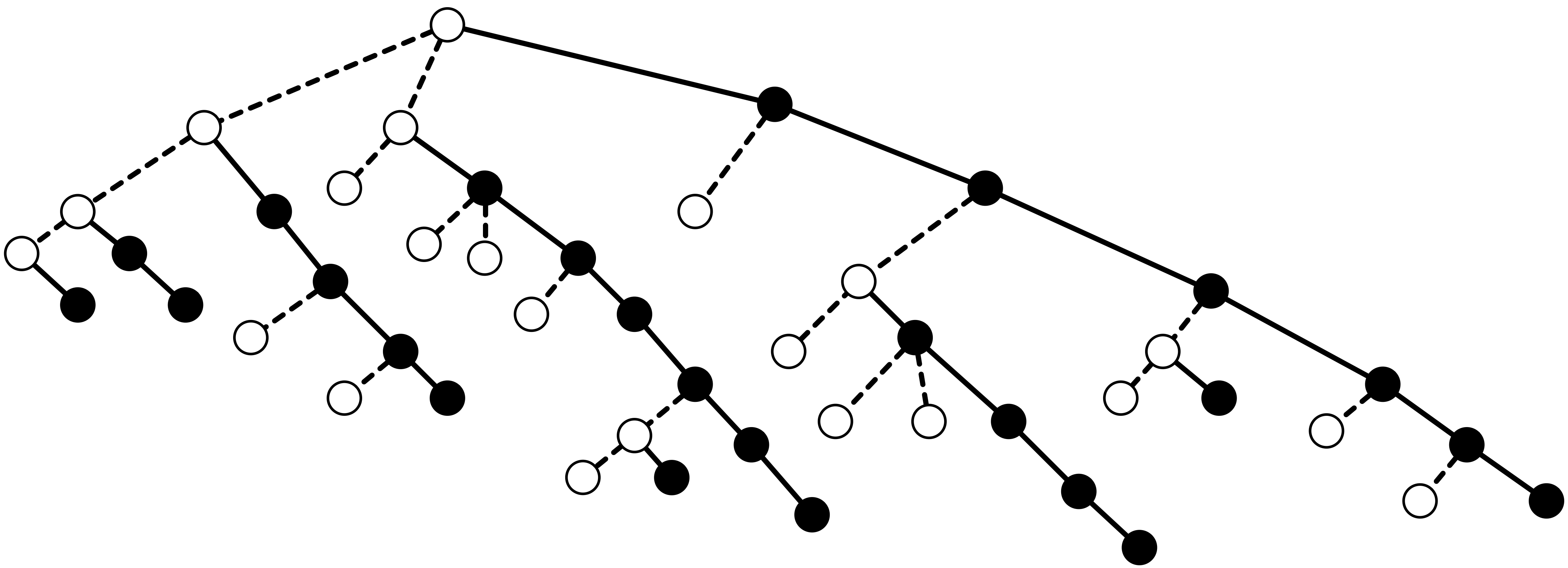
Heavy Light Decomposition



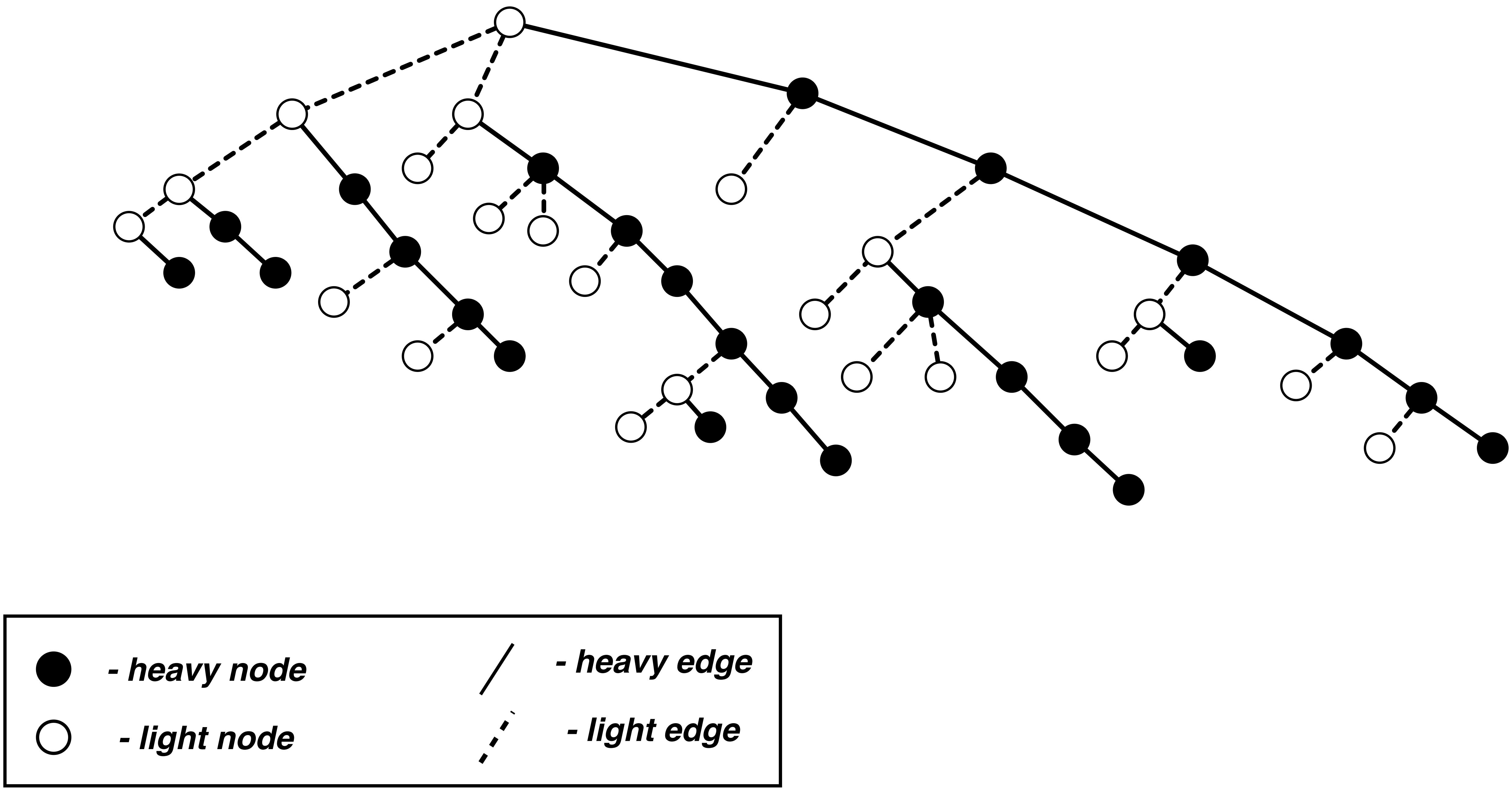
Heavy Light Decomposition

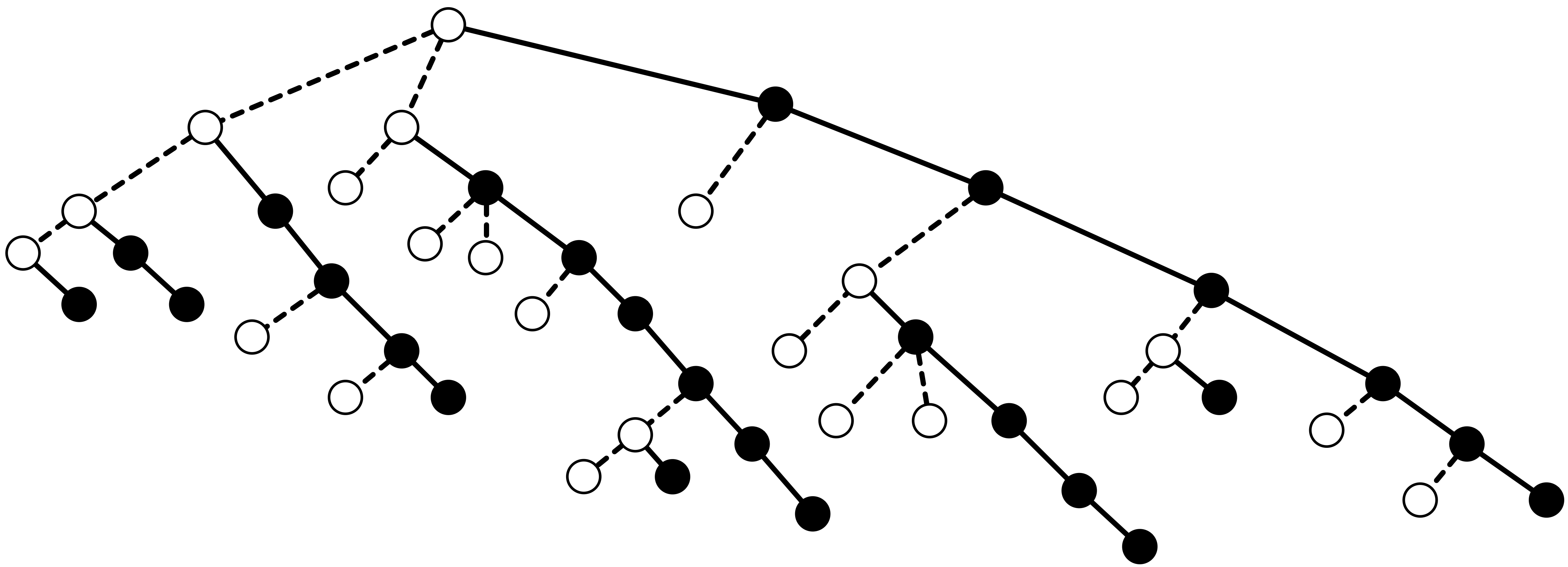


Heavy Light Decomposition

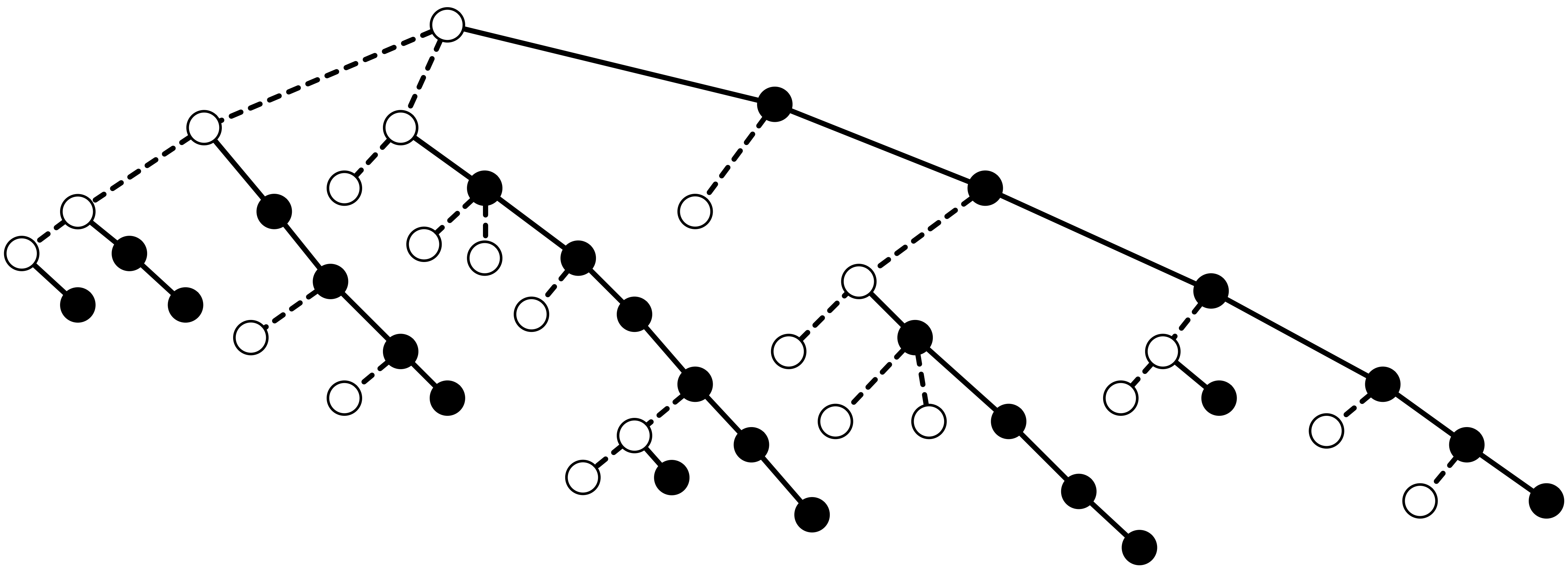


Heavy Light Decomposition

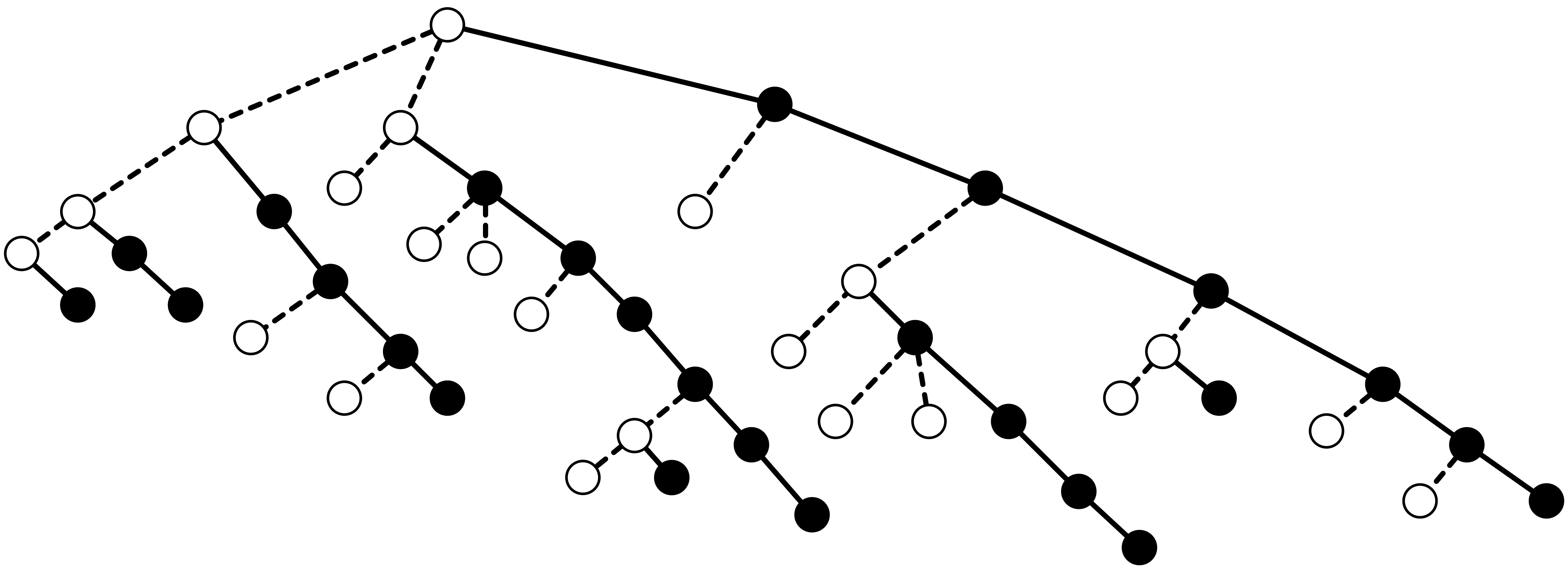




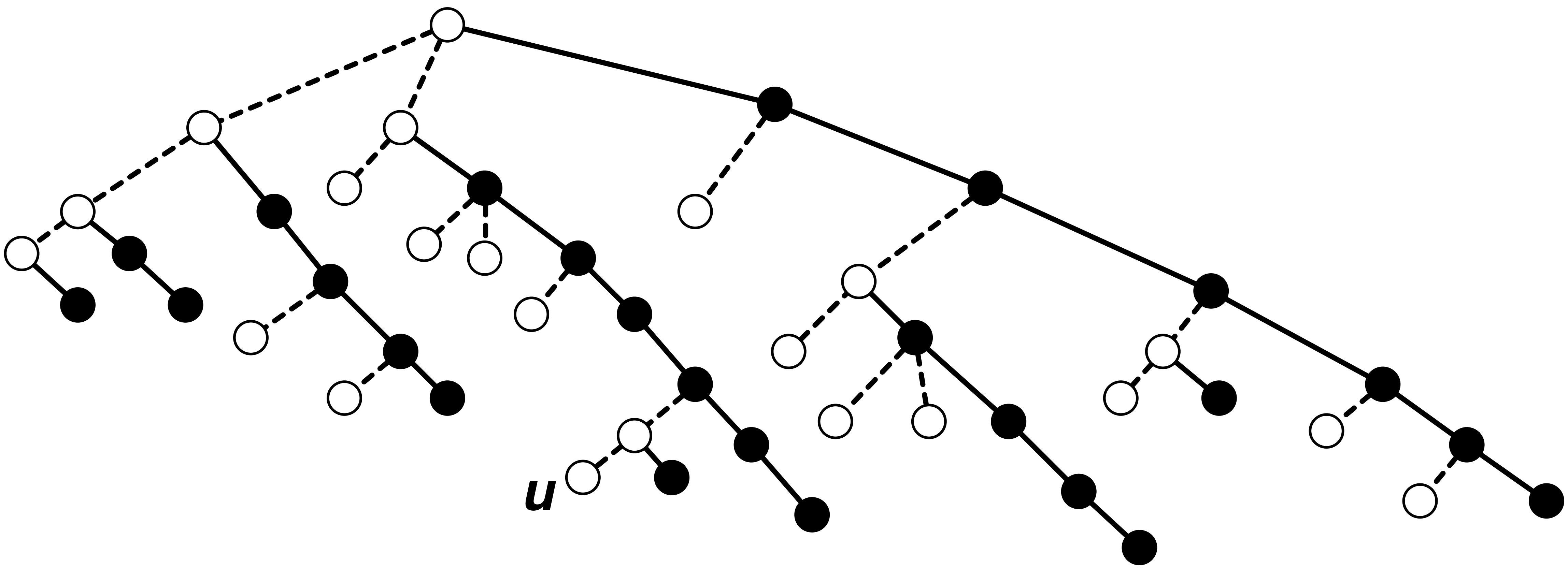
Light Depth



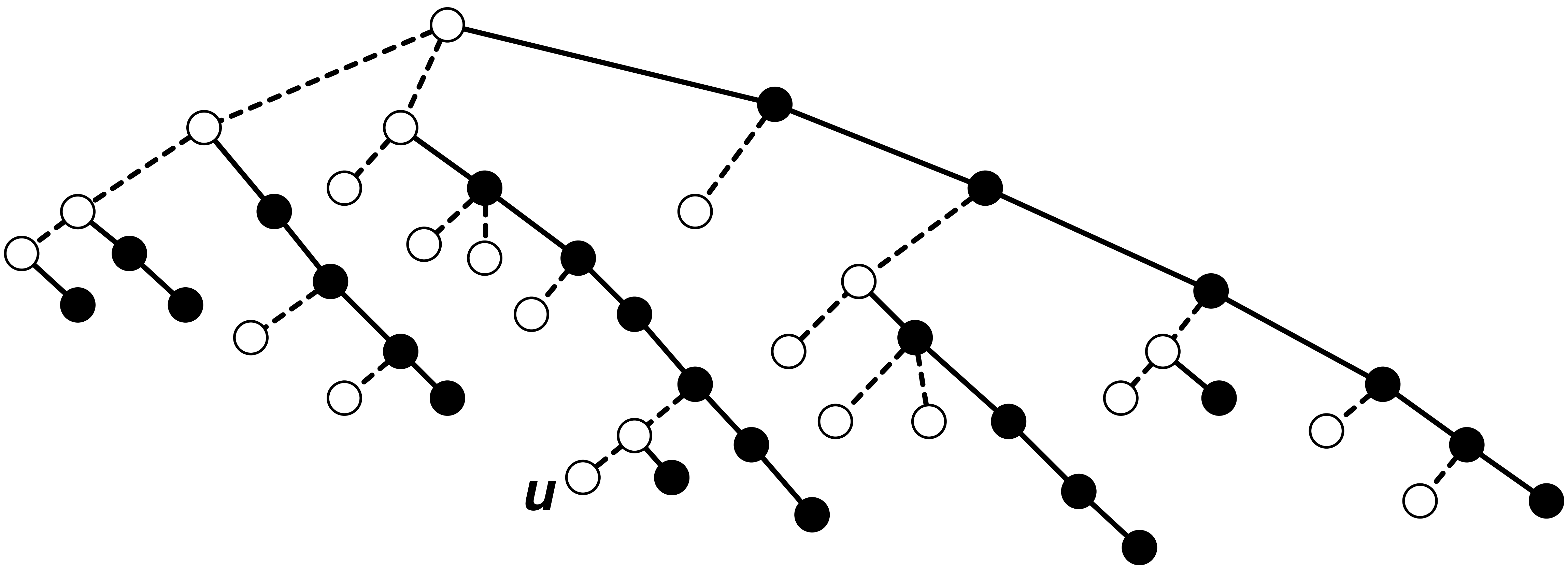
Light Depth := #Light edges



Light Depth := #Light edges

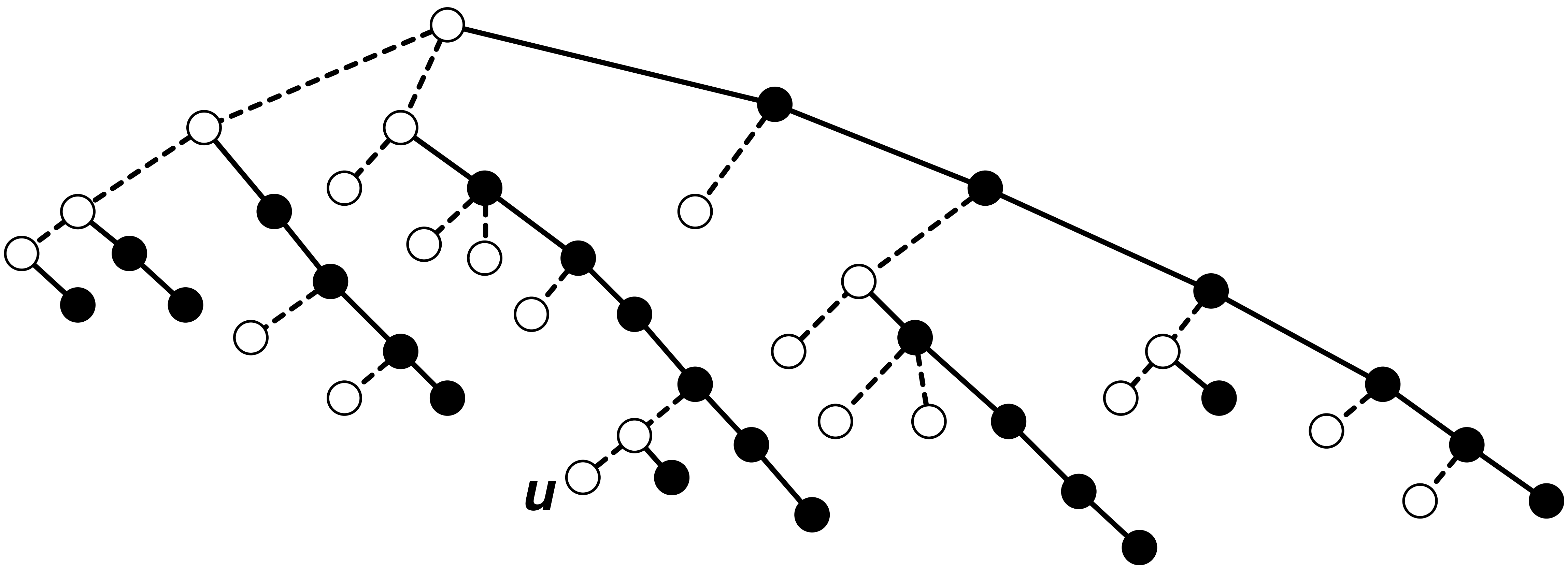


Light Depth := #Light edges



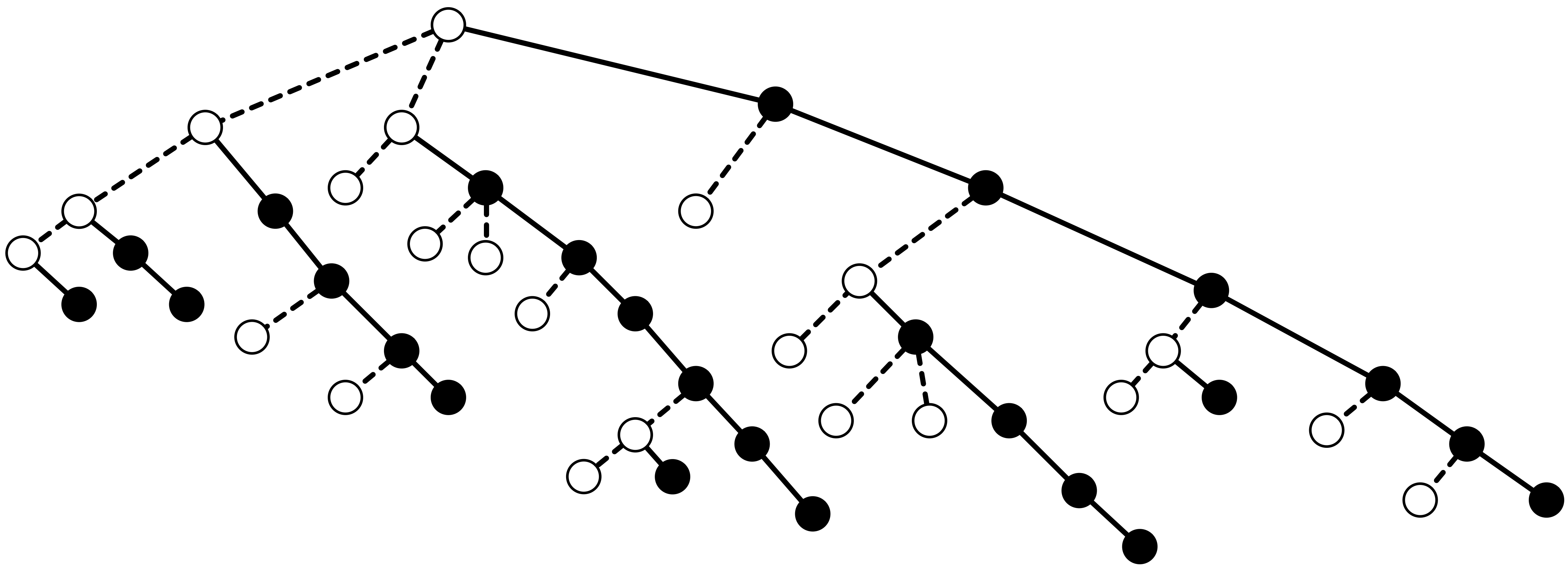
light-depth(u) = 3

Light Depth := #Light edges

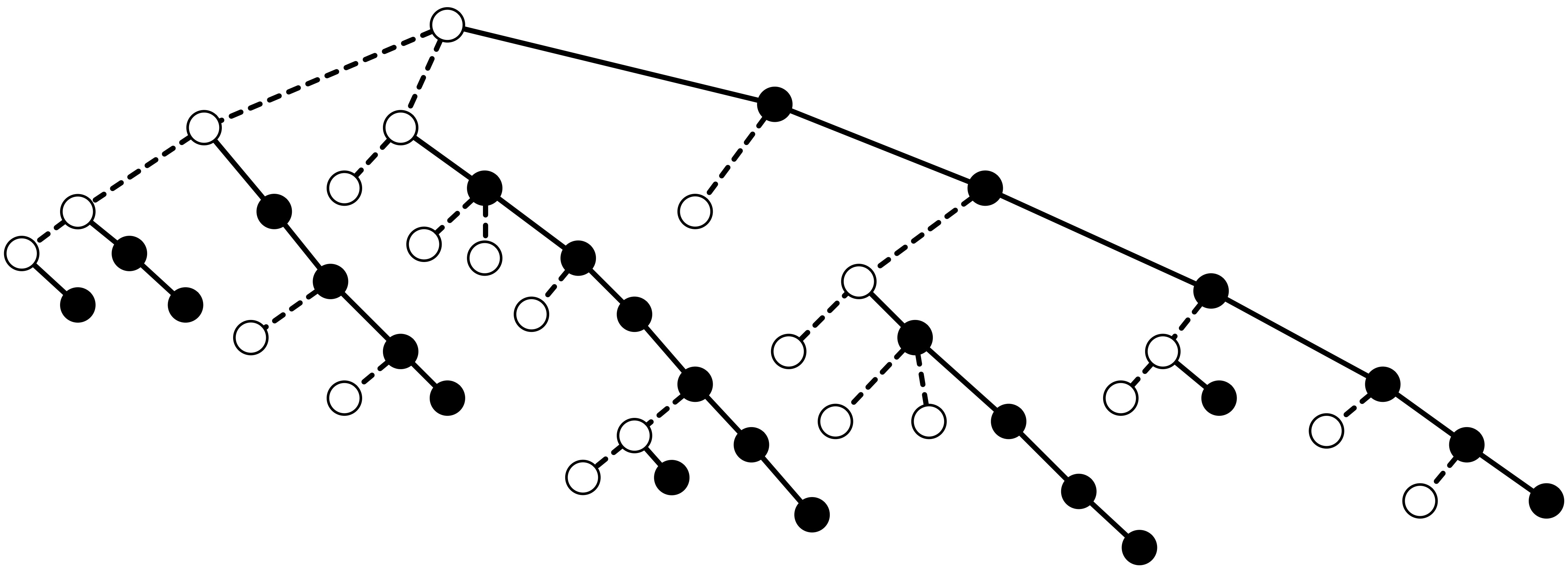


light-depth(u) = 3

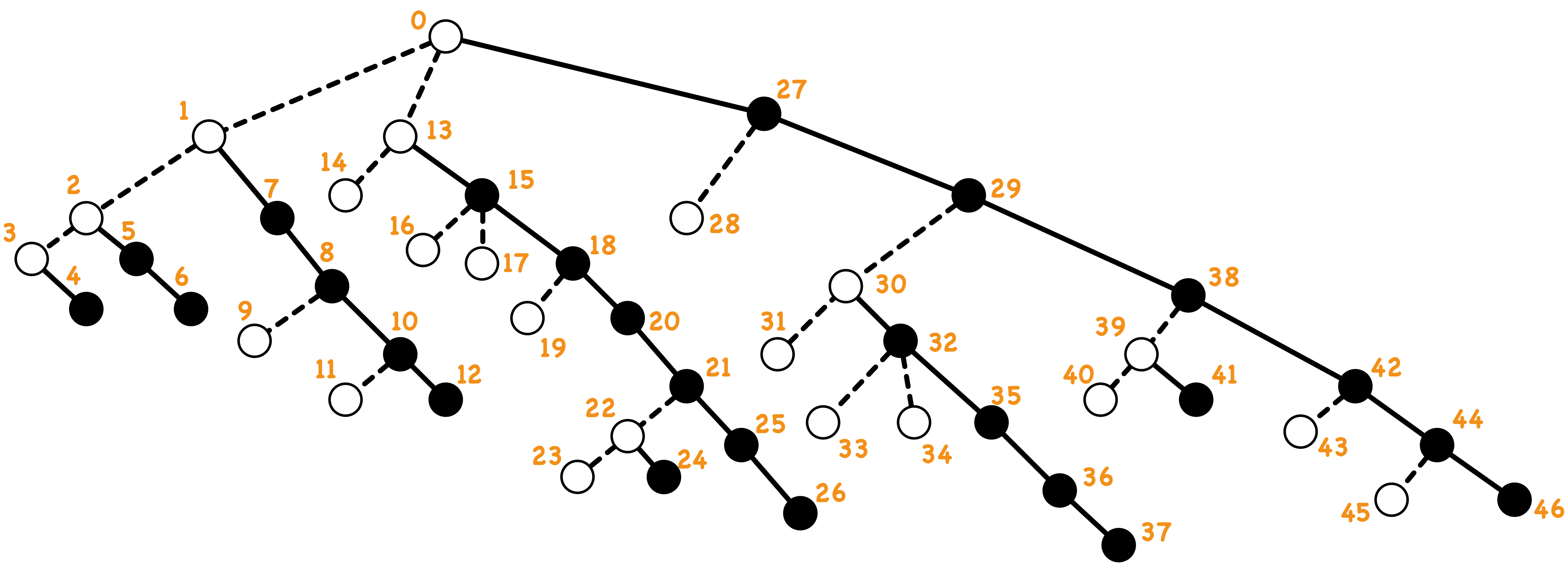
light-depth < logn



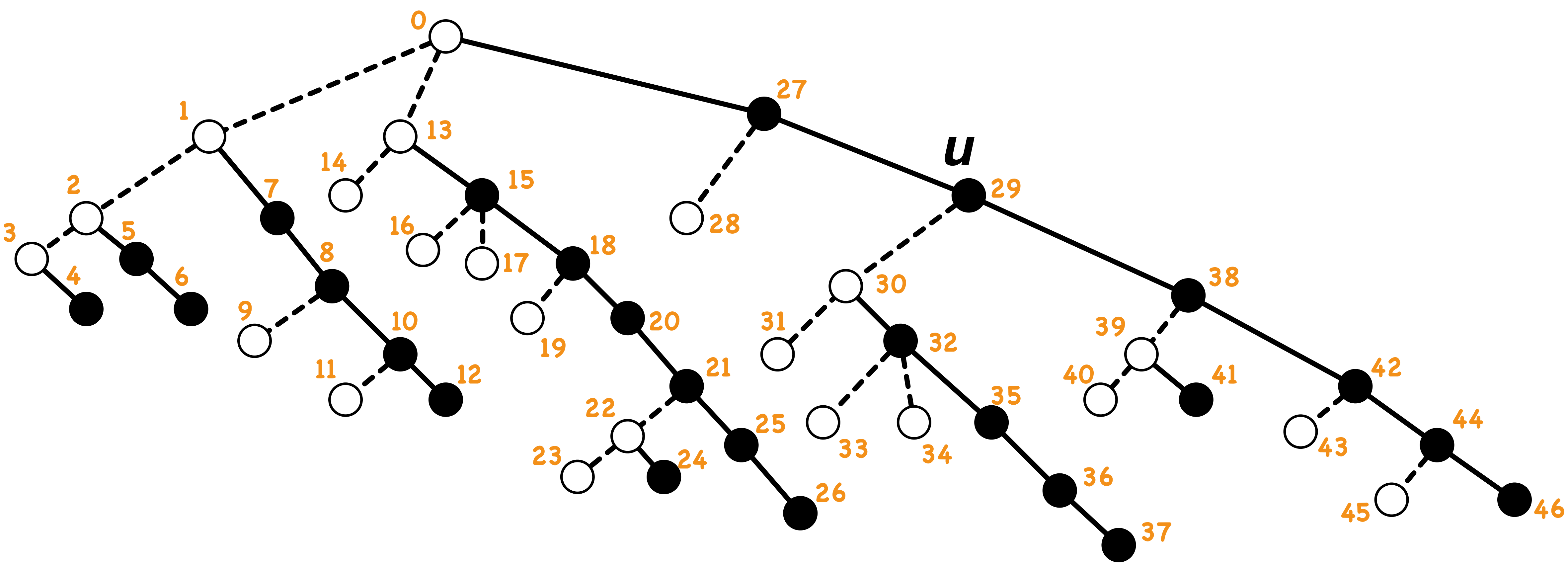
PreOrder Numbers



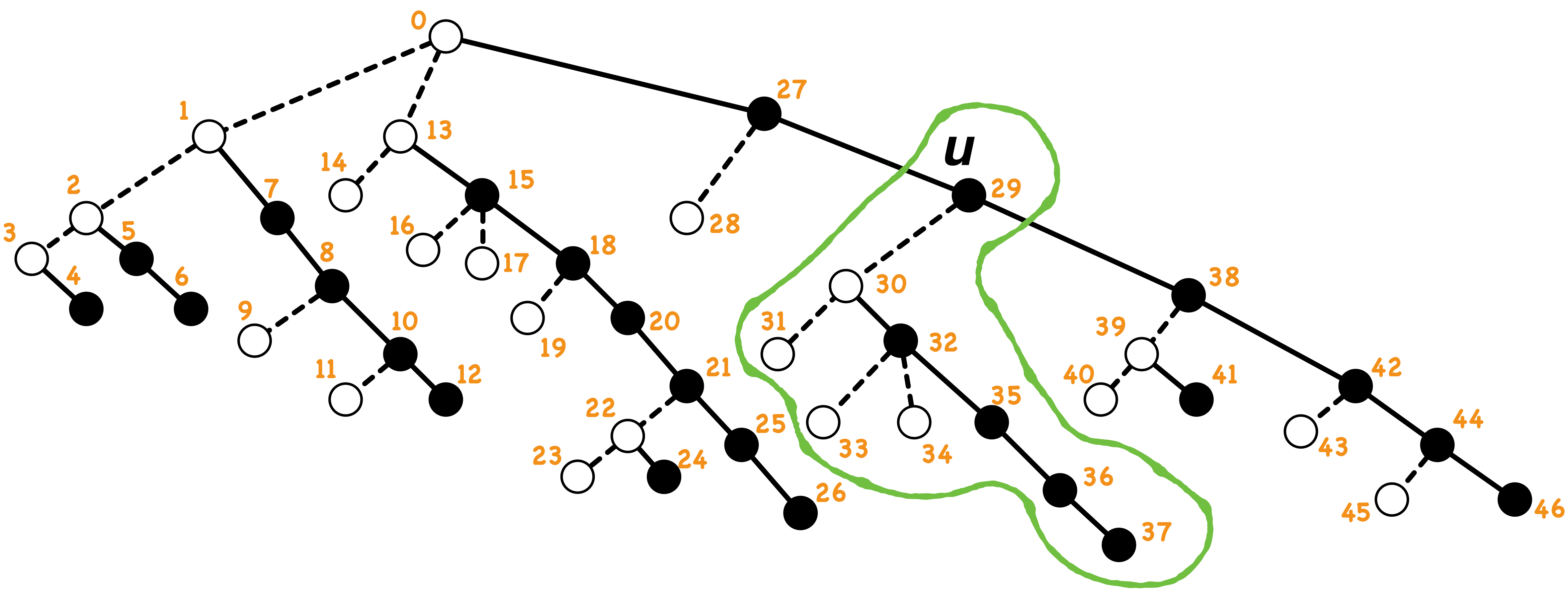
PreOrder Numbers



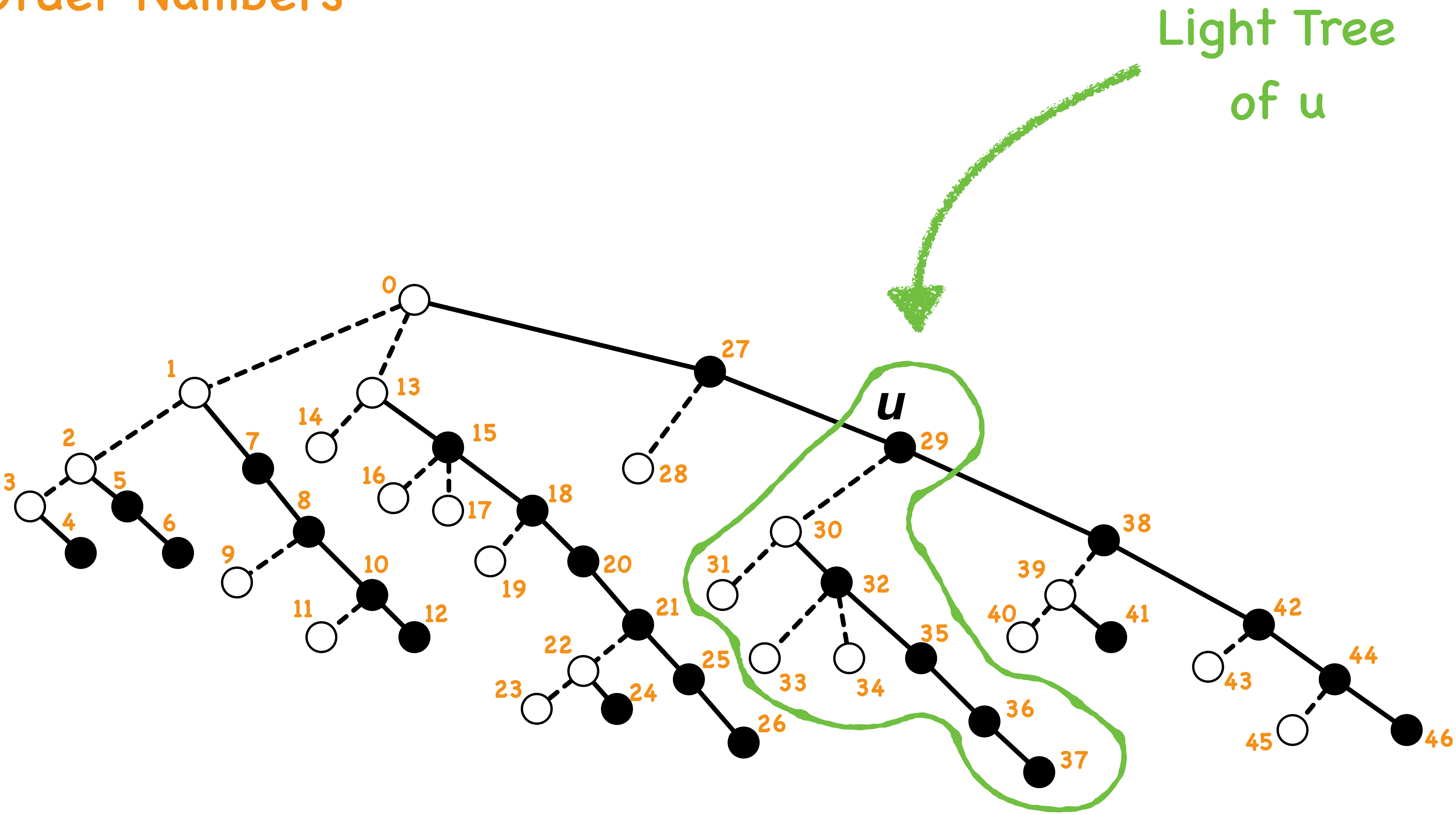
PreOrder Numbers

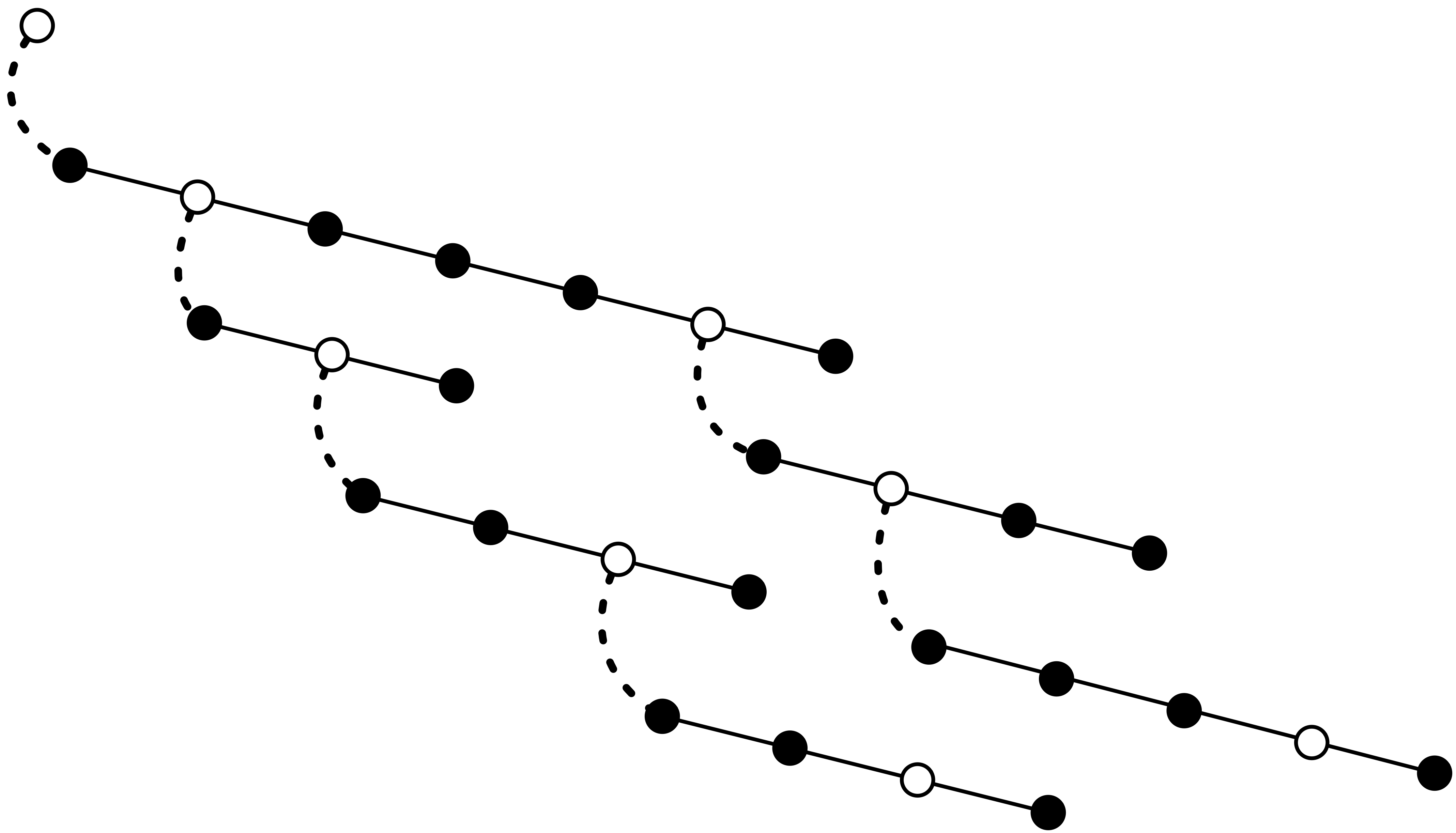


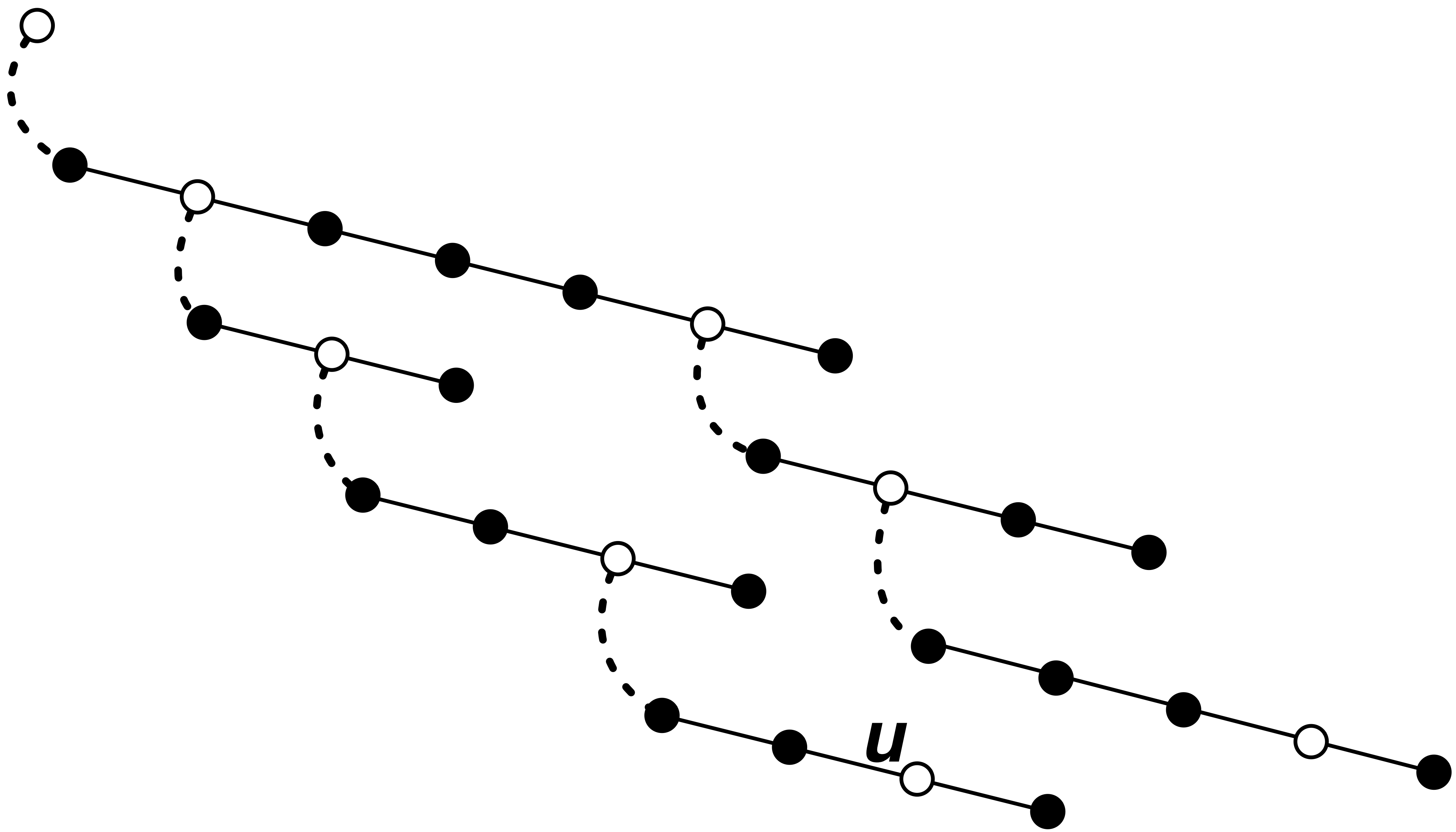
PreOrder Numbers



PreOrder Numbers



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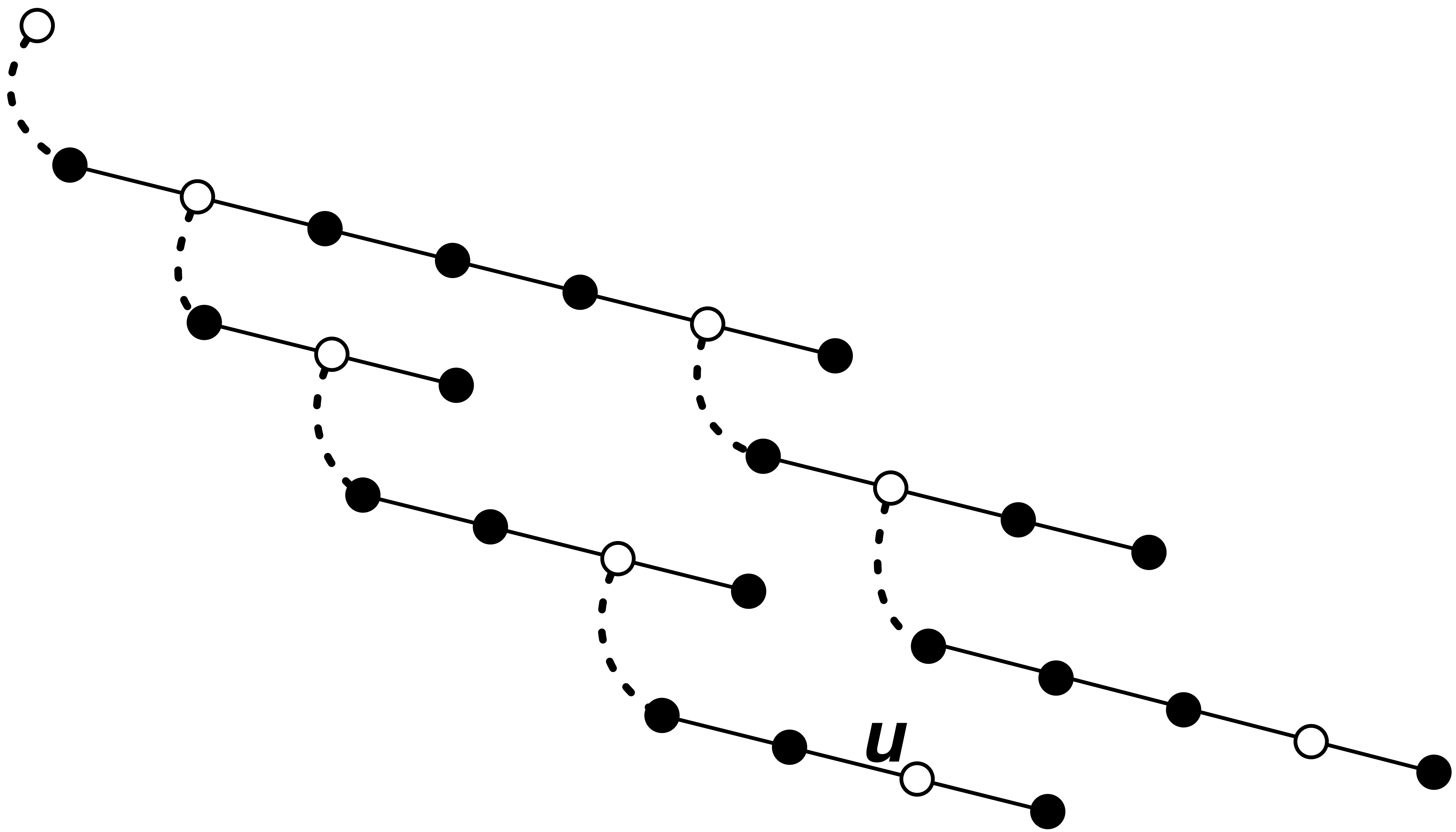
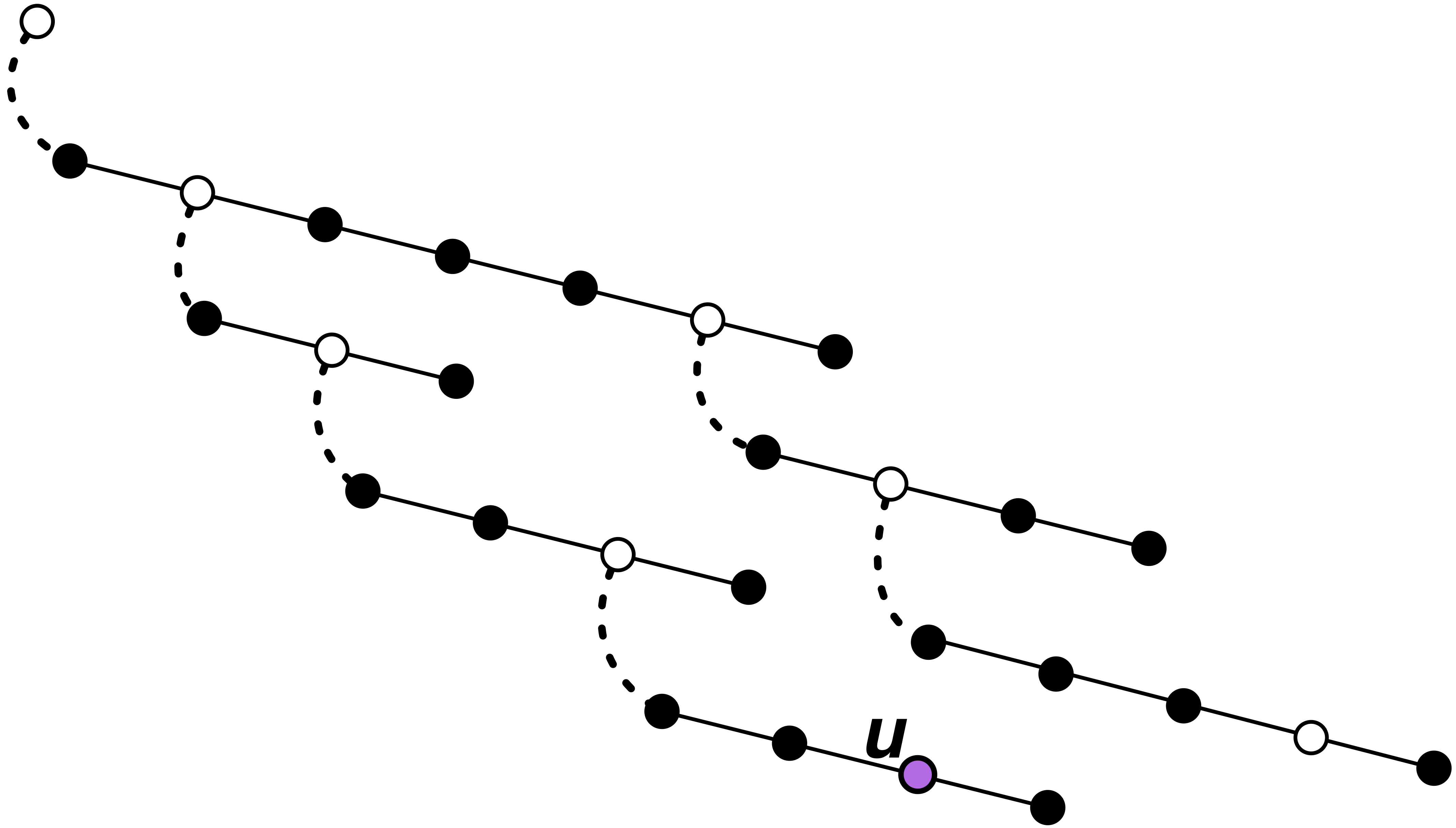
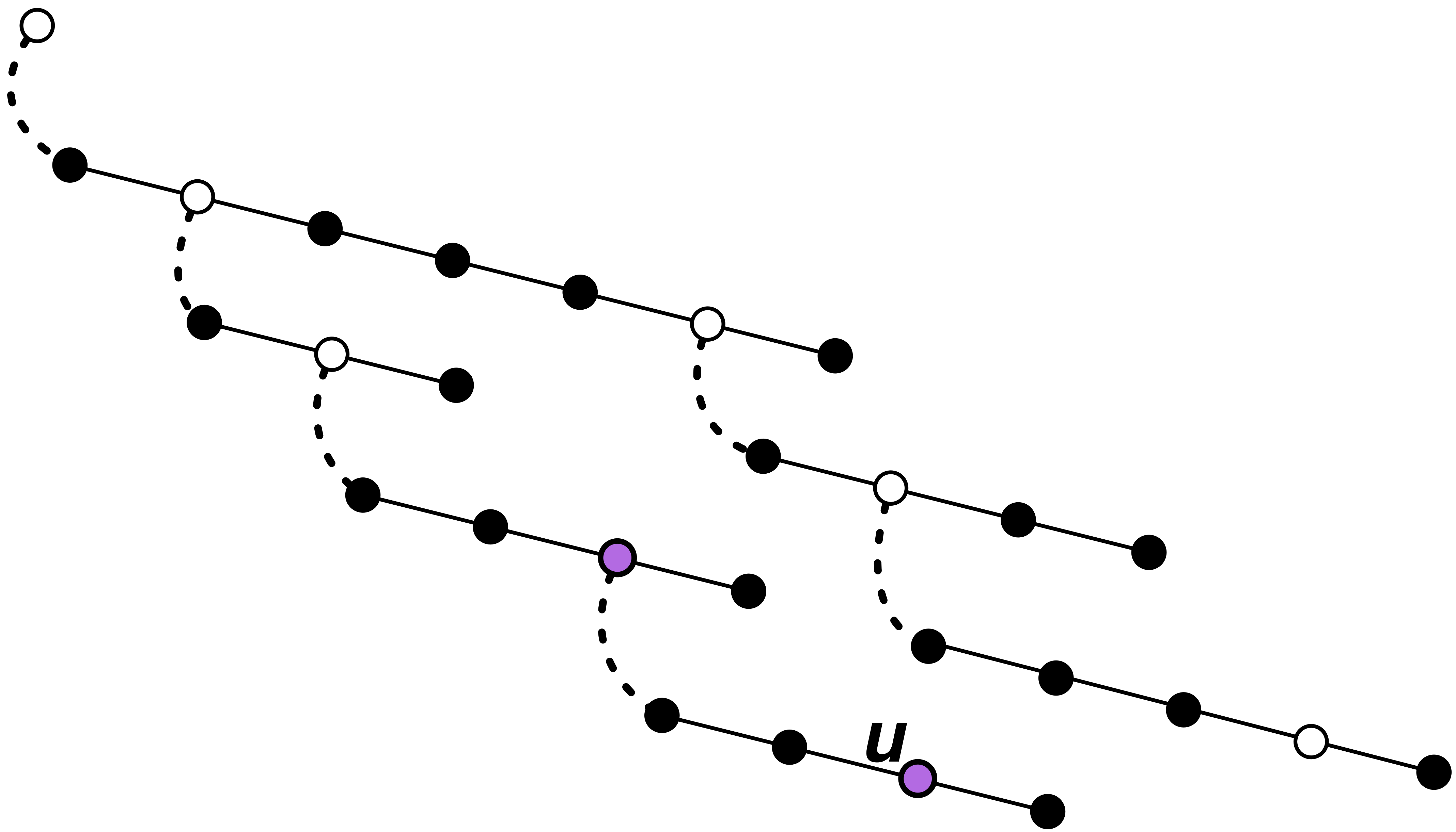
Significant Ancestors

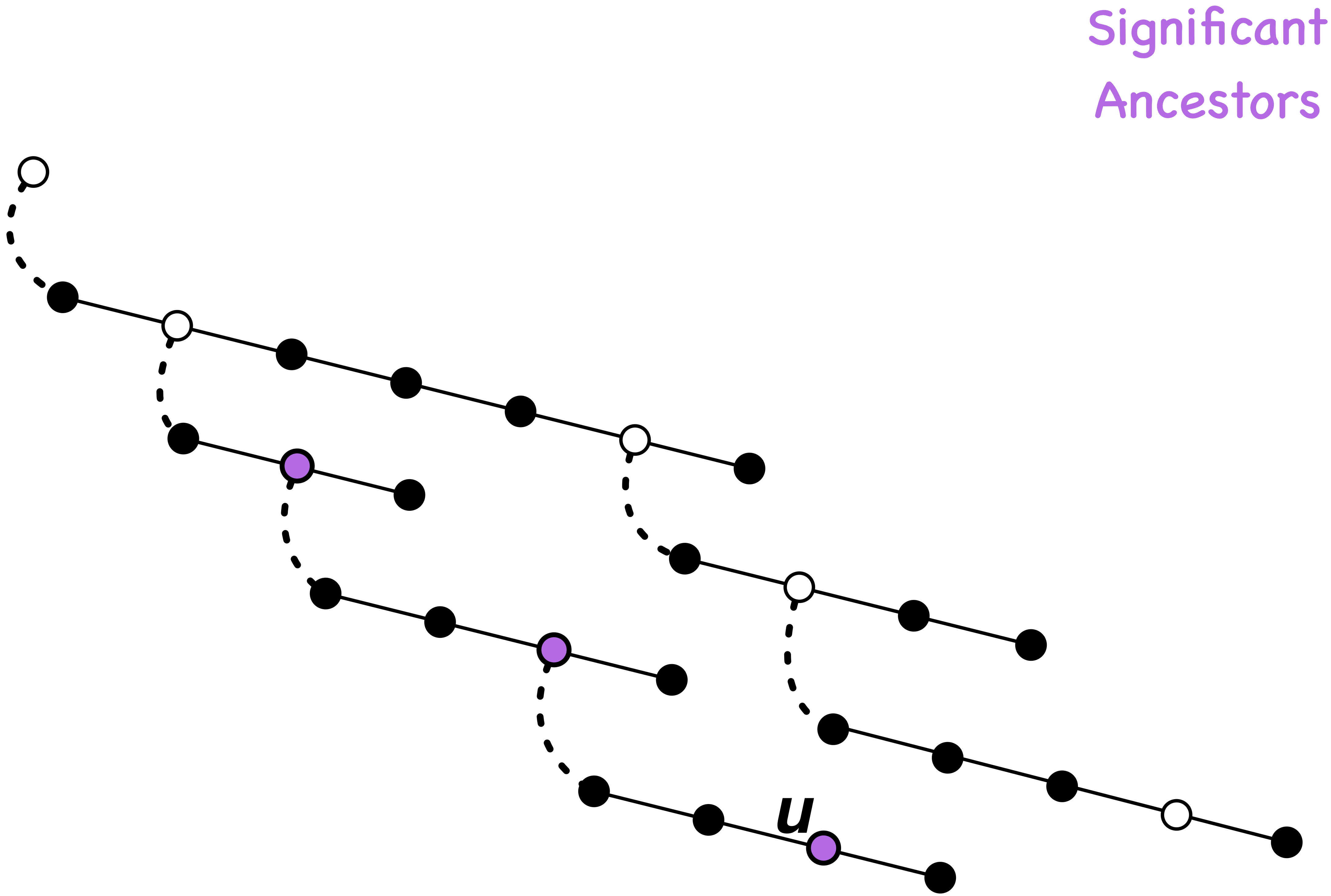


Significant Ancestors

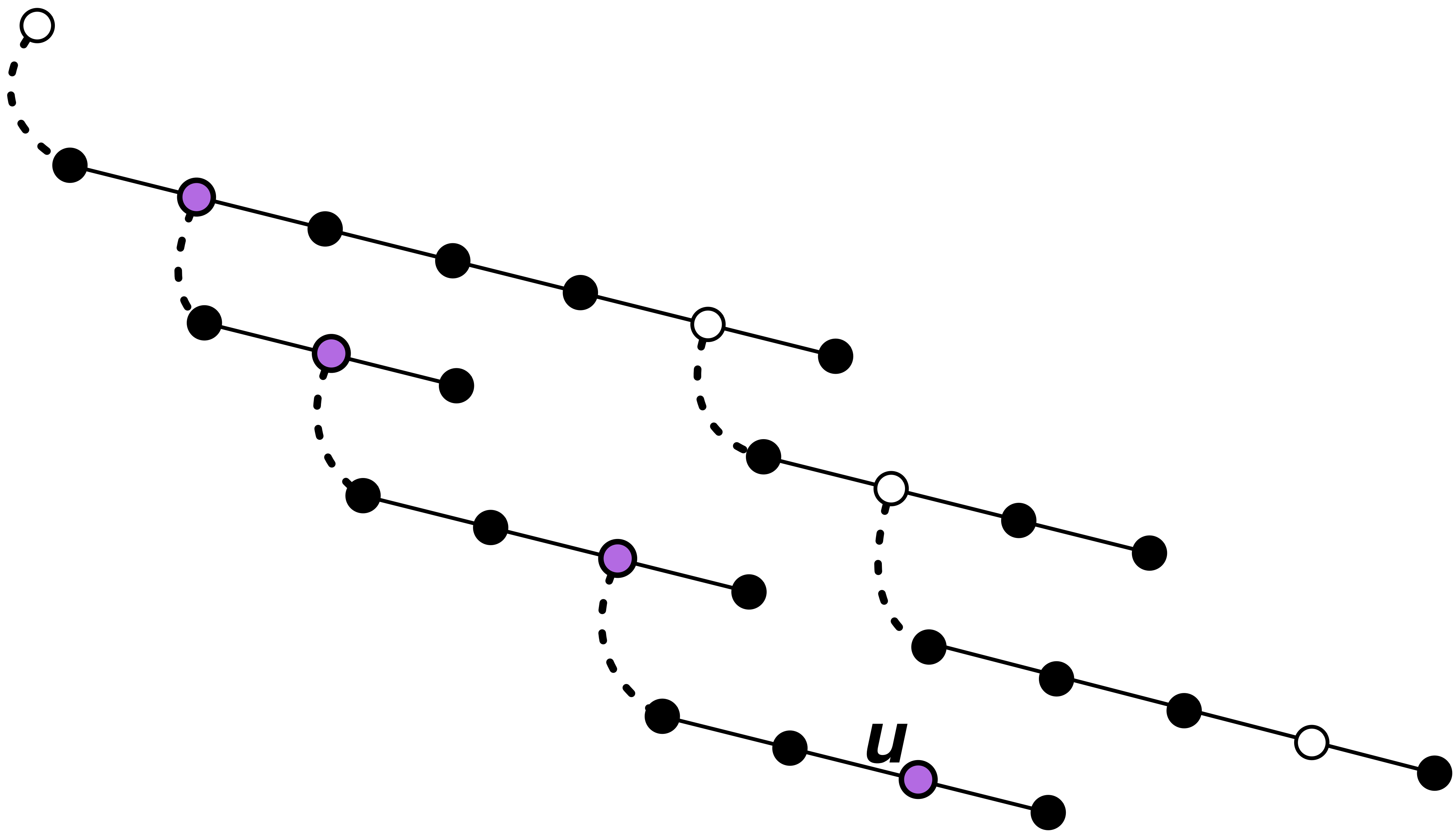


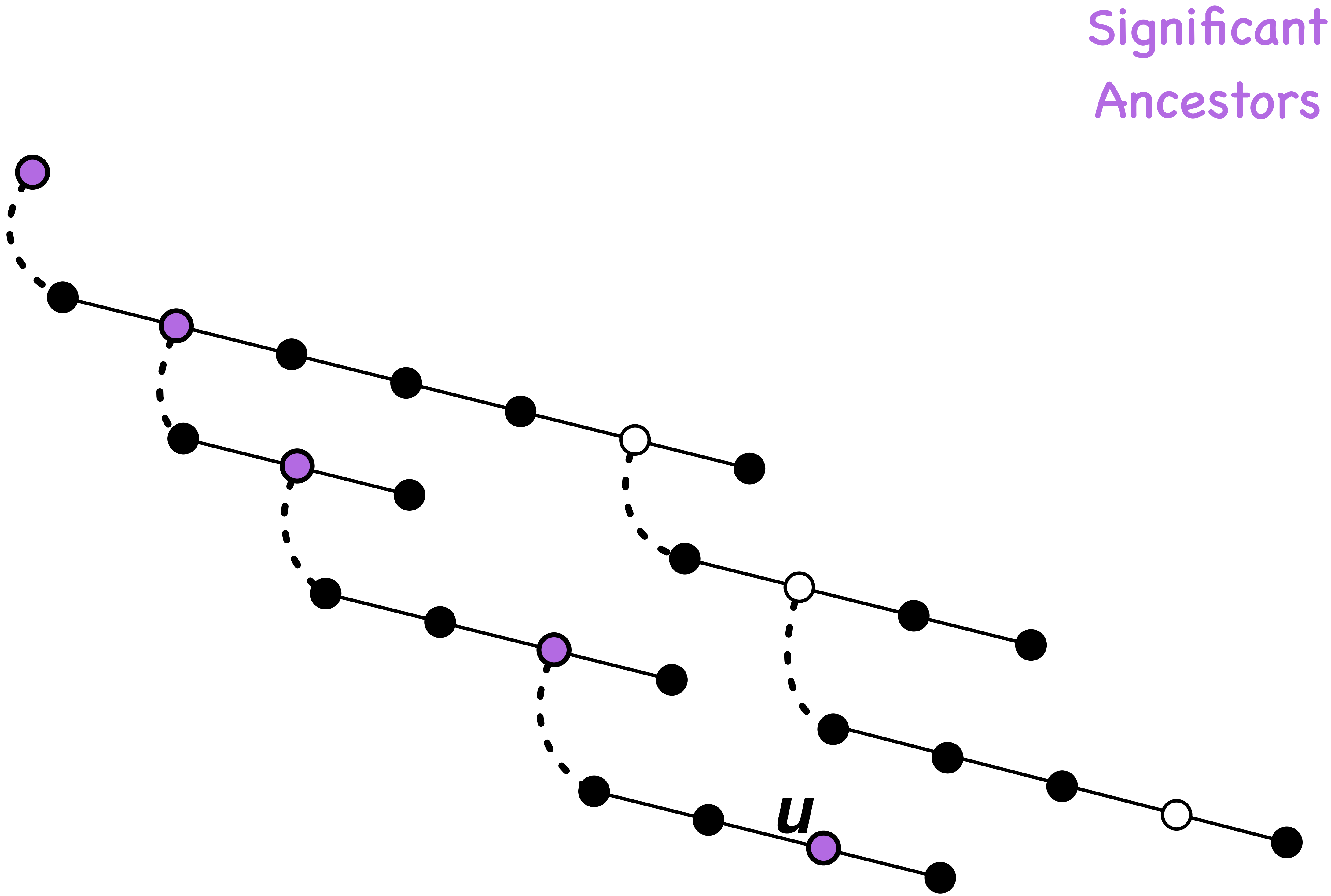
Significant Ancestors

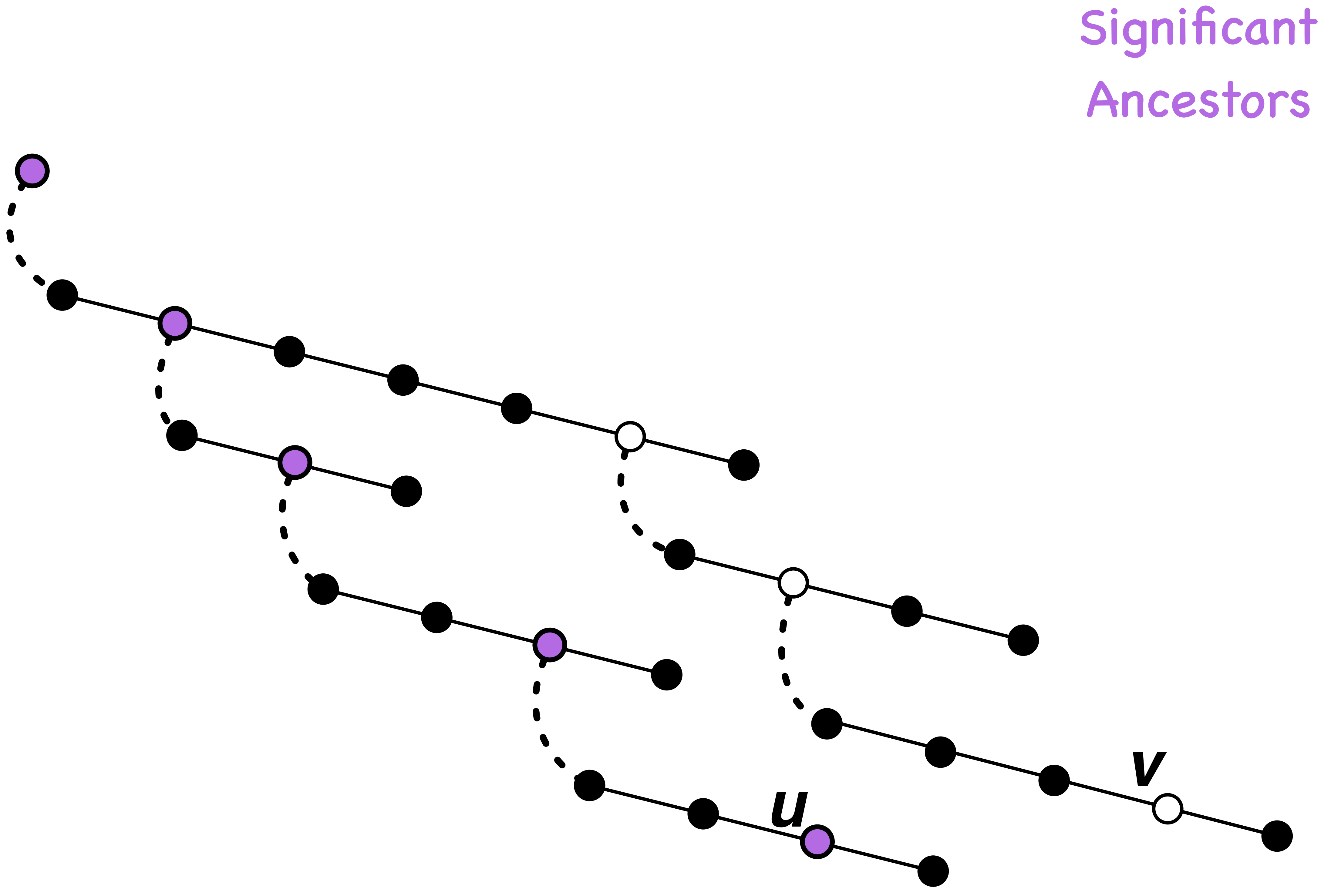



Significant Ancestors
u

Significant Ancestors




Significant
Ancestors
u


Significant
Ancestors
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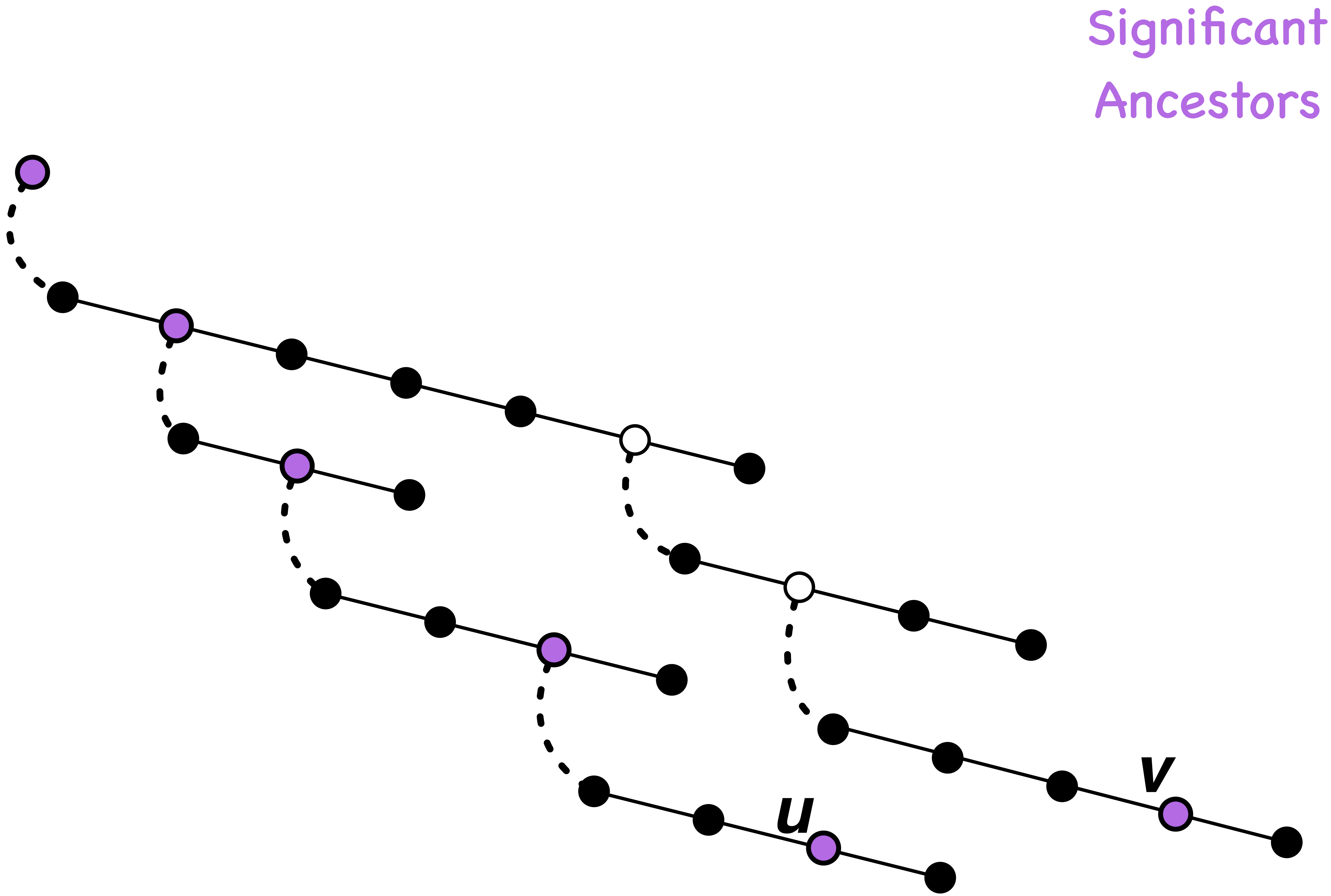
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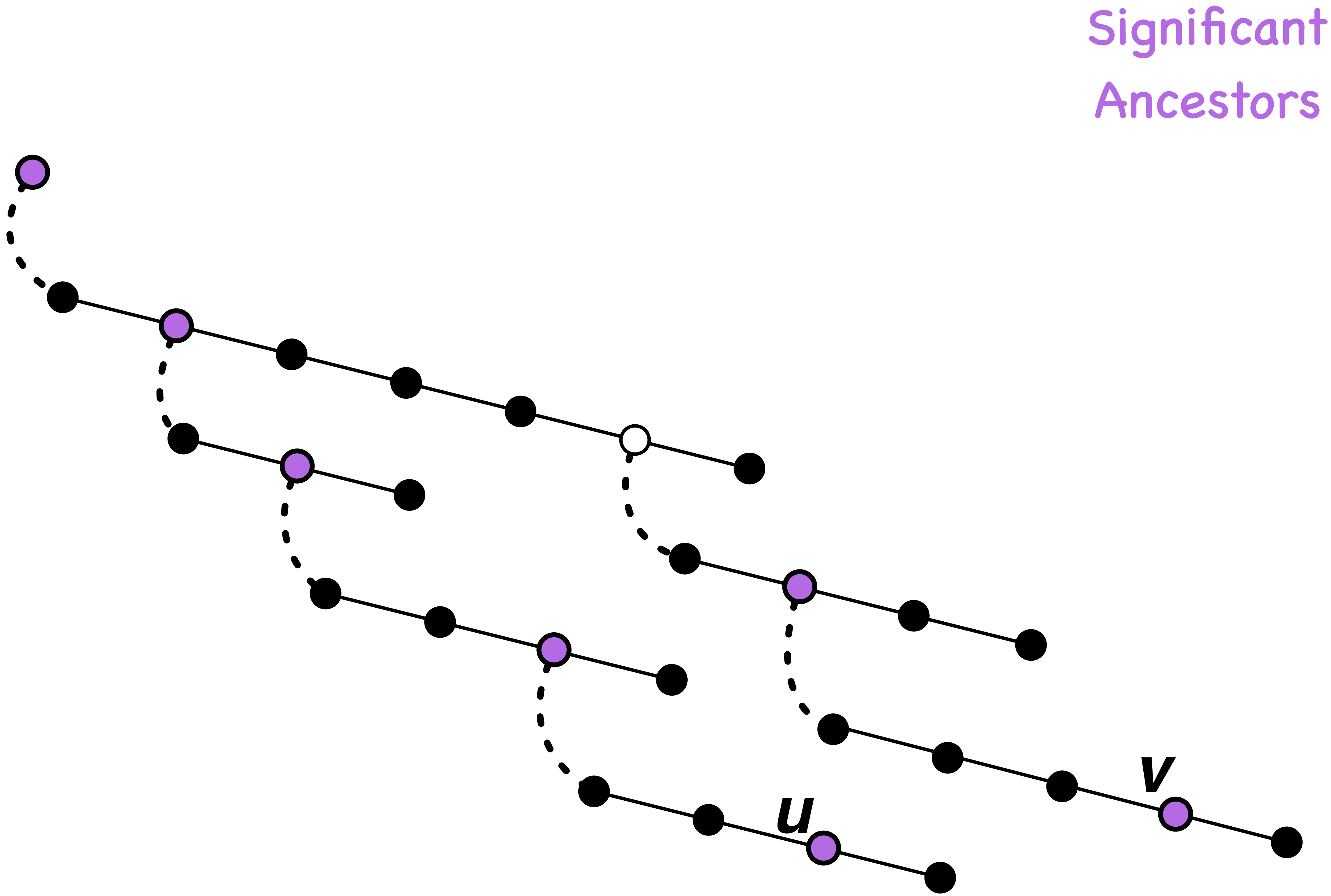
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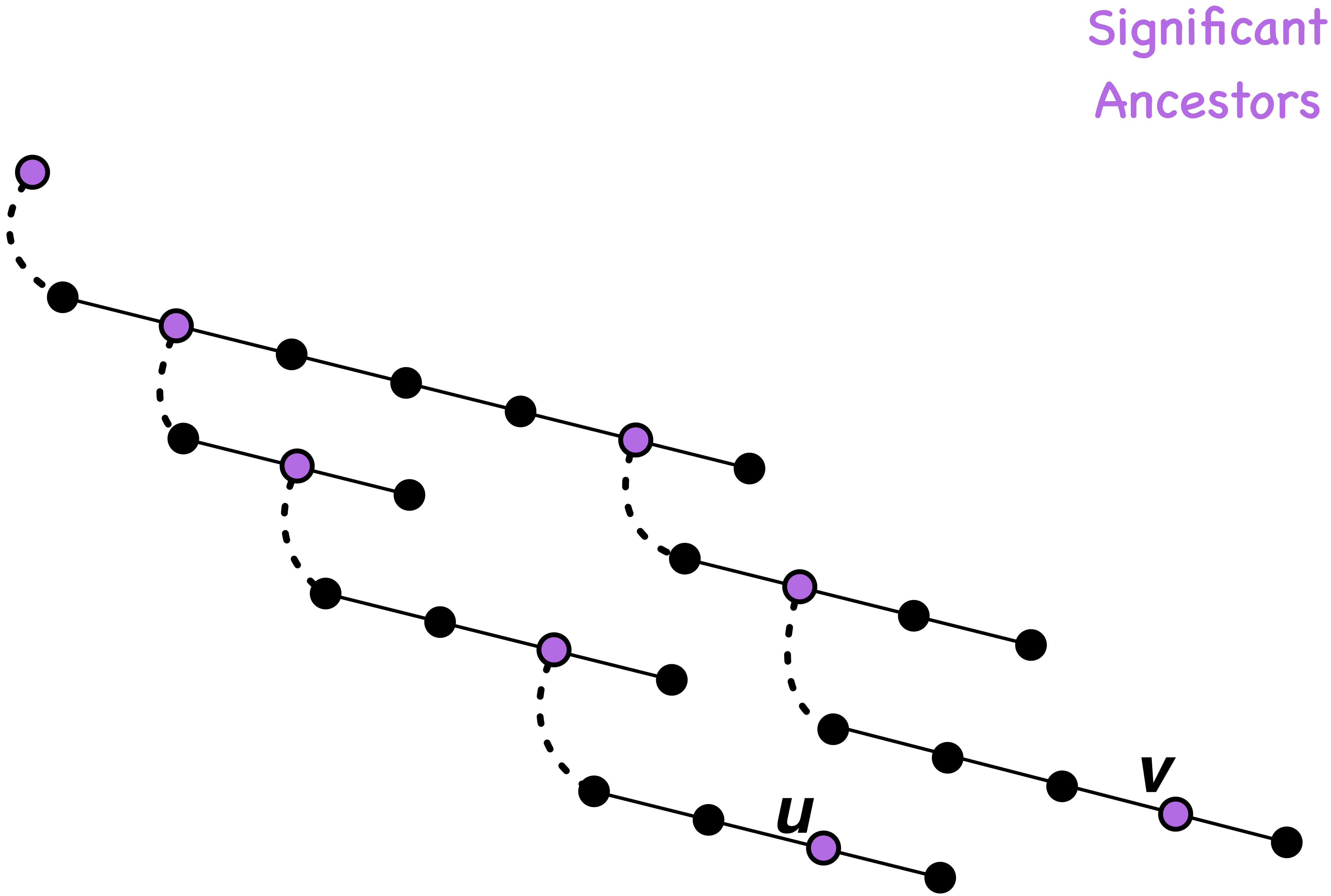
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Ancestors
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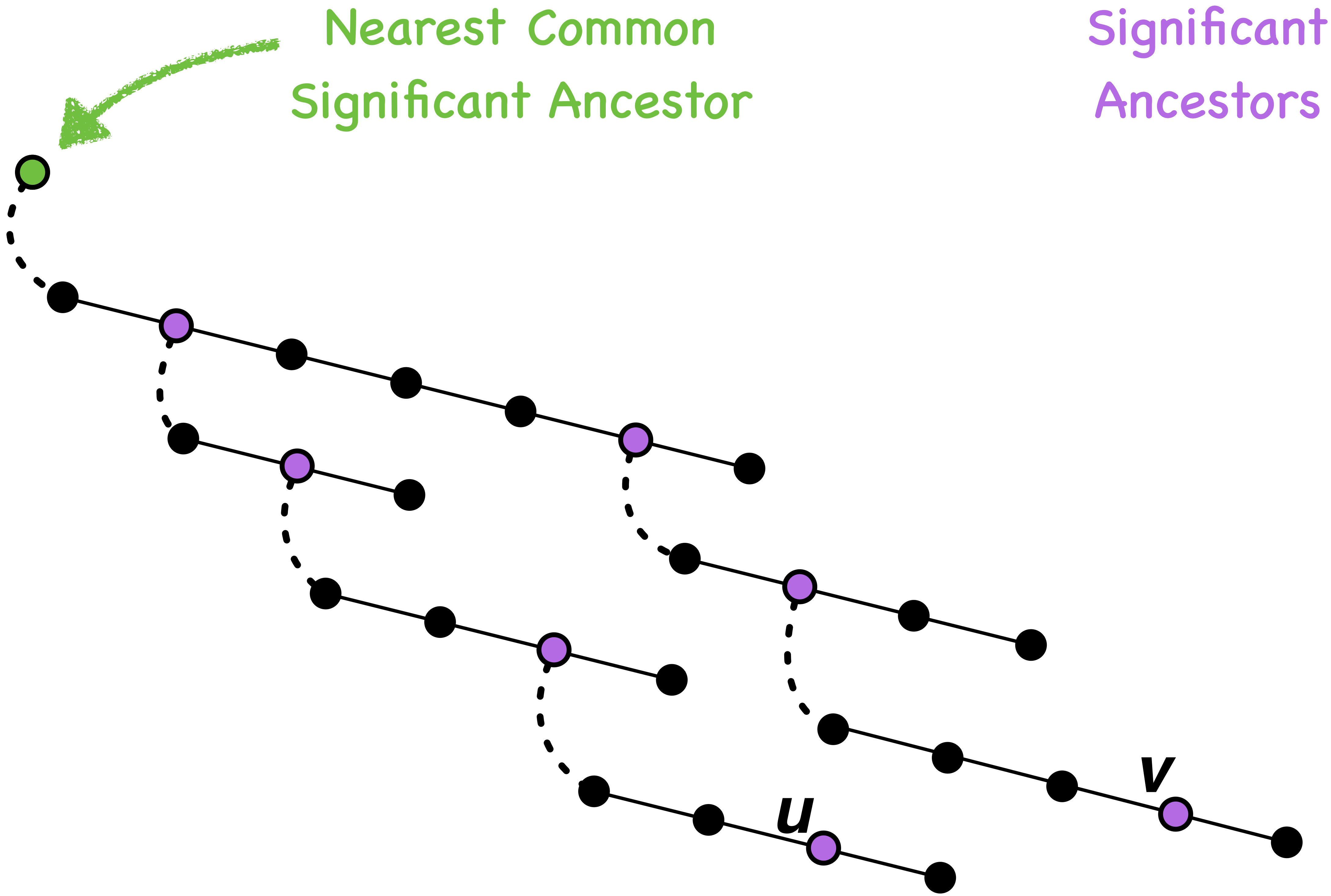

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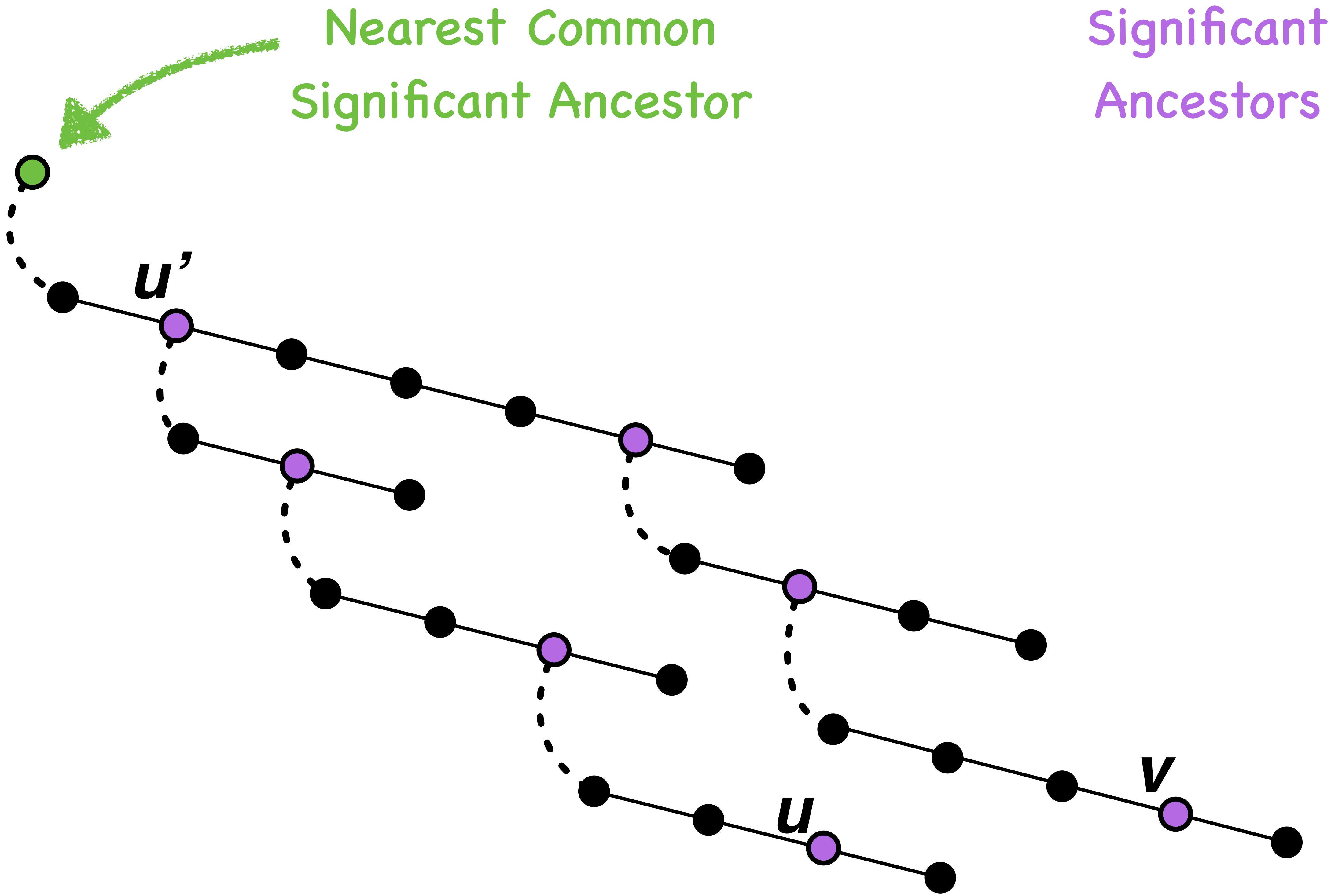

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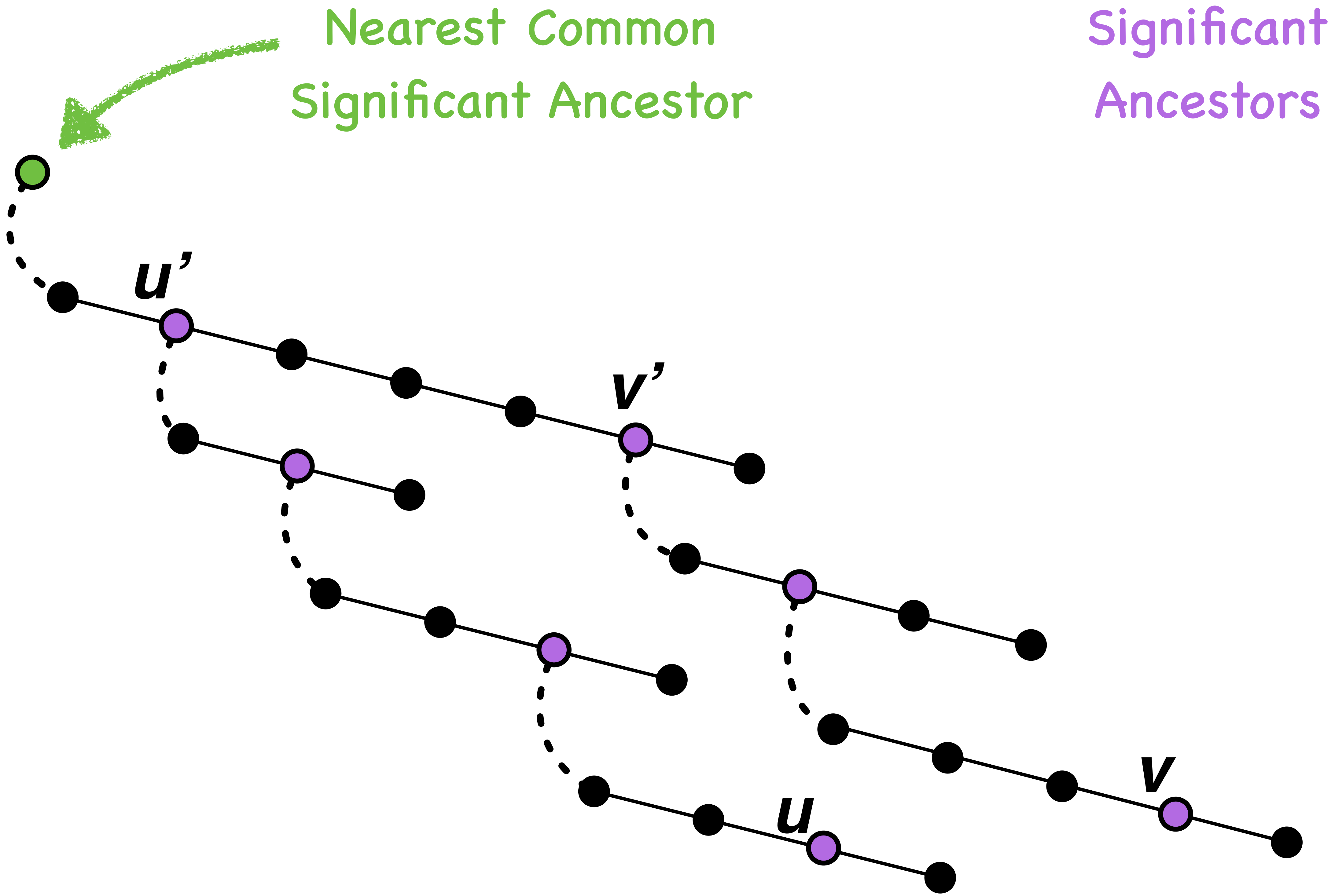

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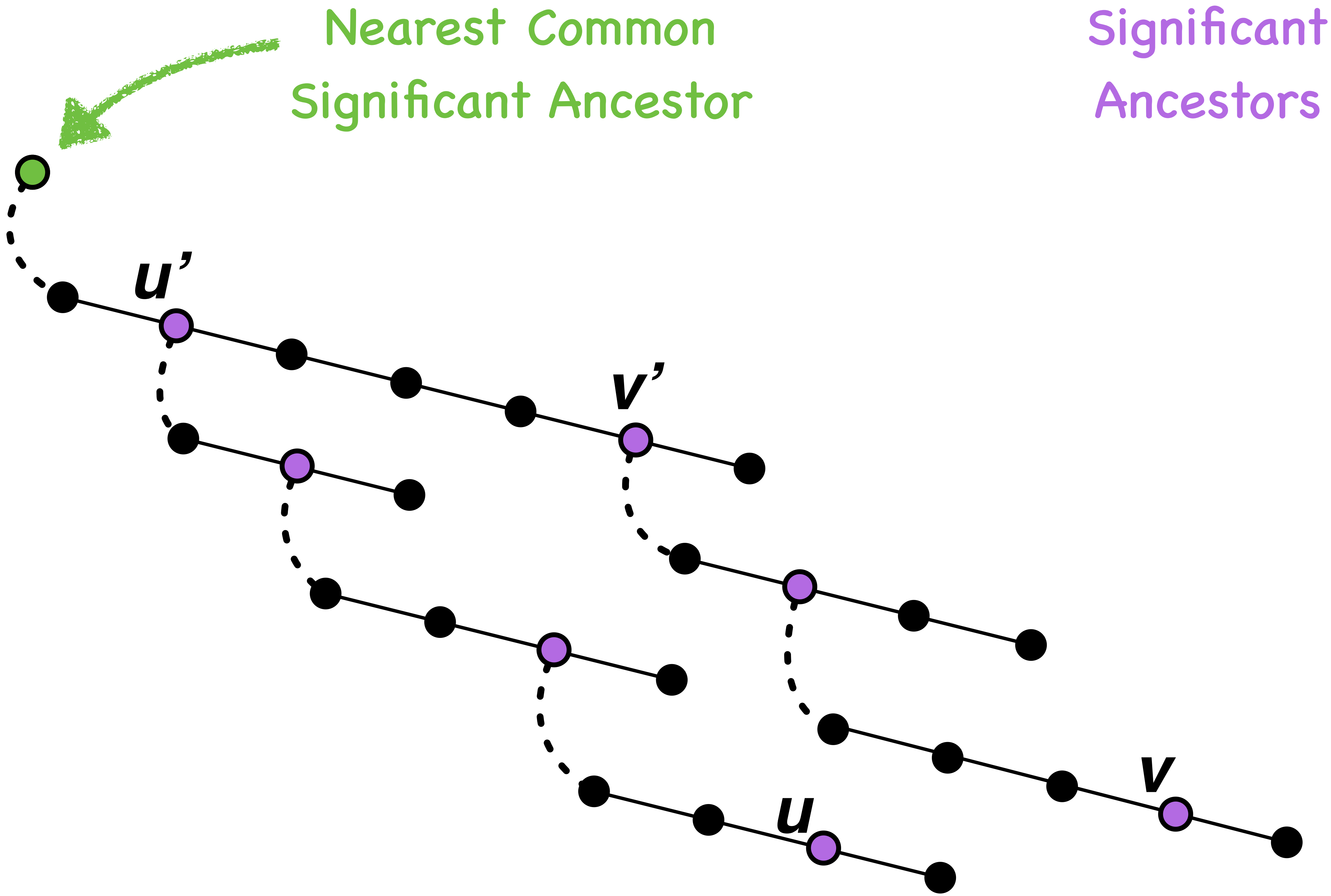

$$d(u,v) = d(u,u') + d(u',v') + d(v',v)$$

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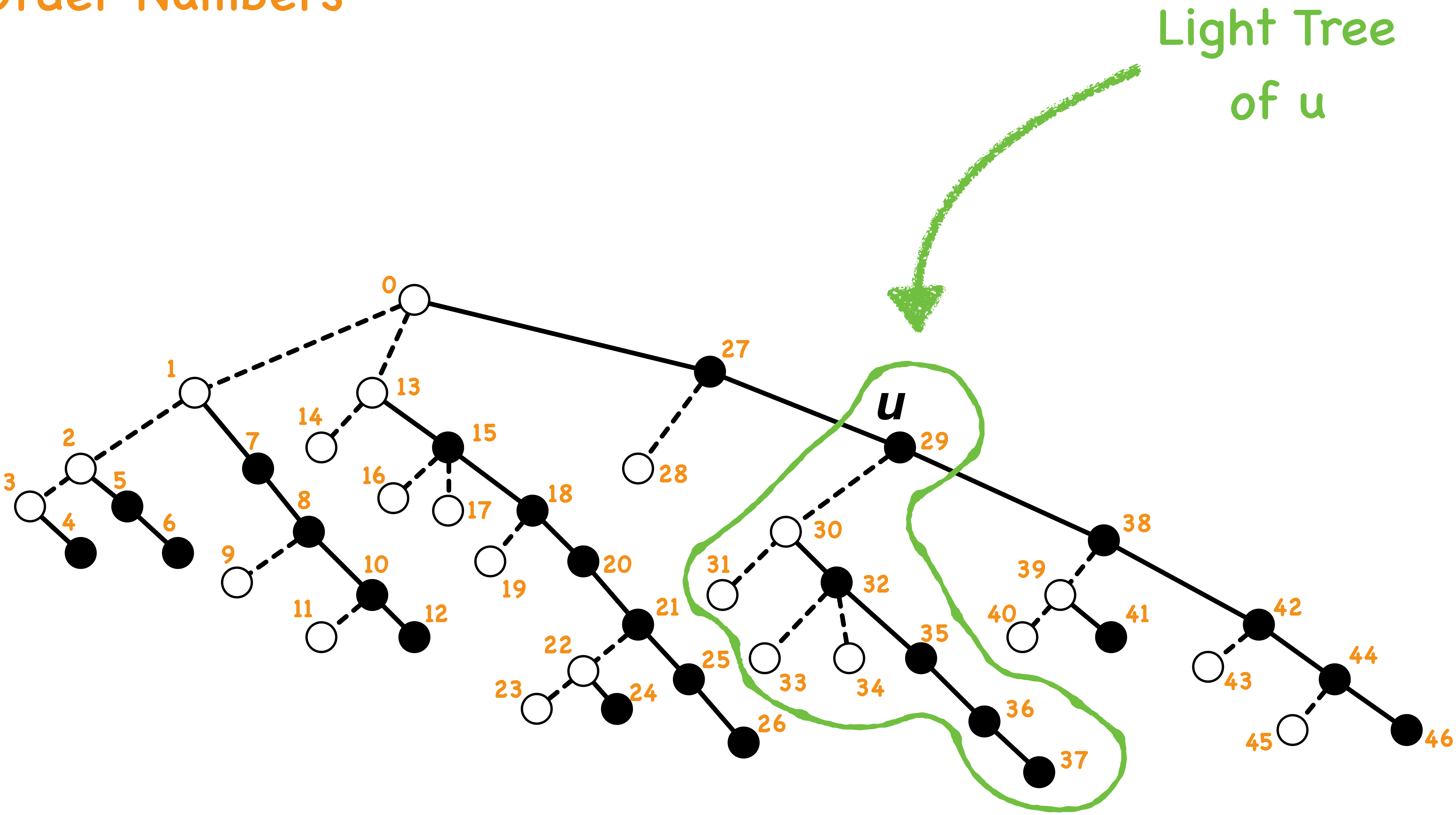

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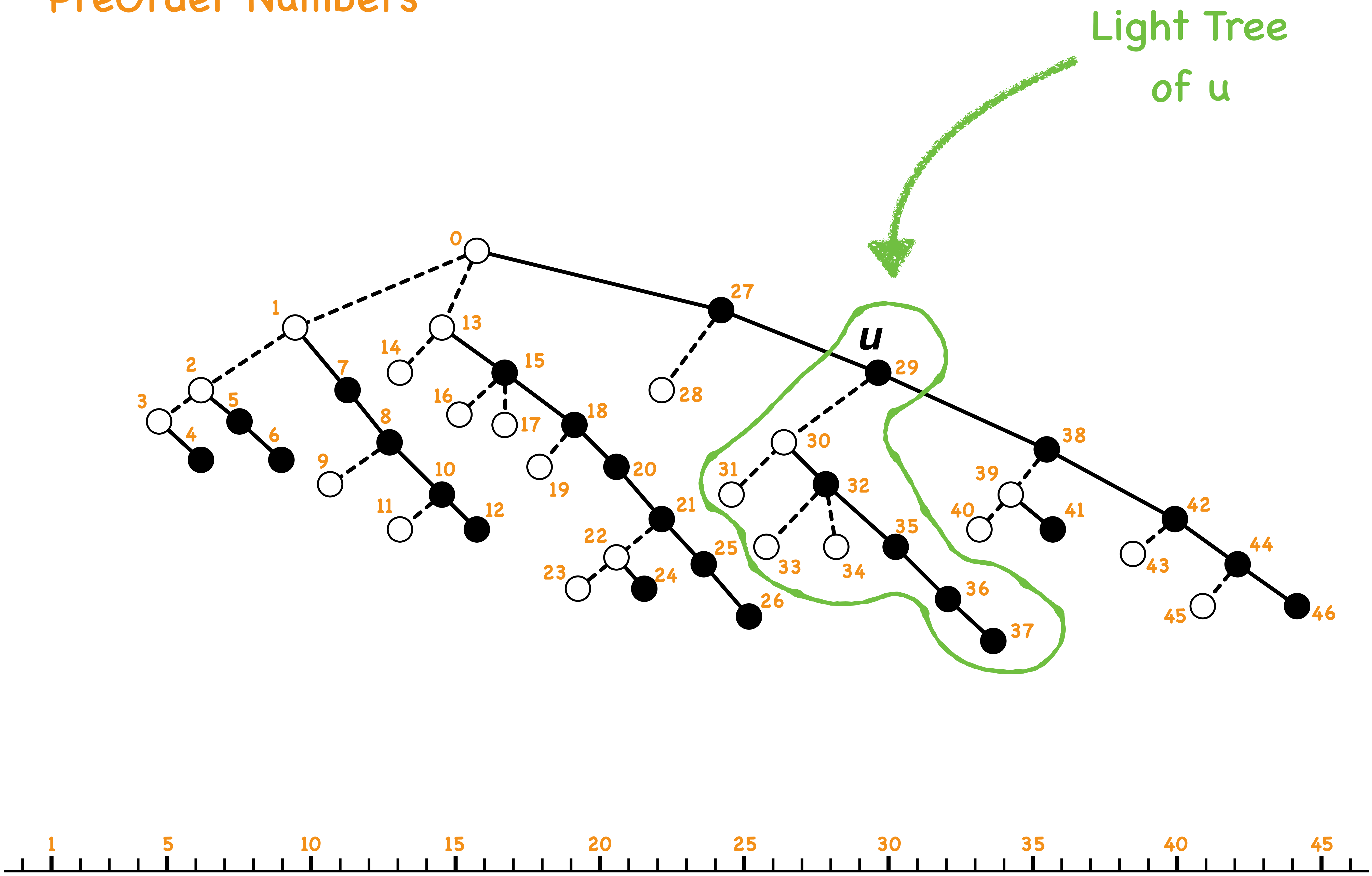

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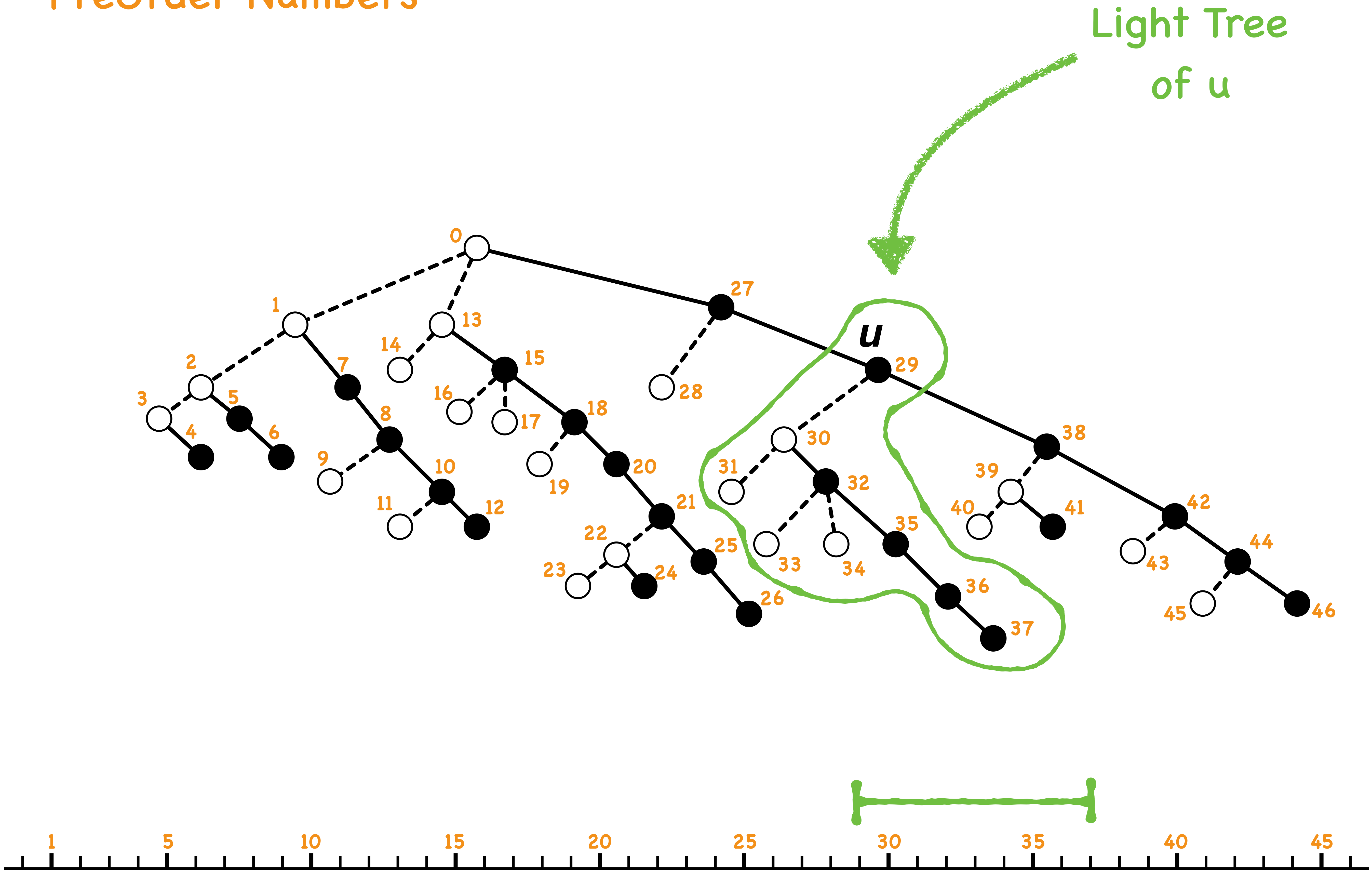

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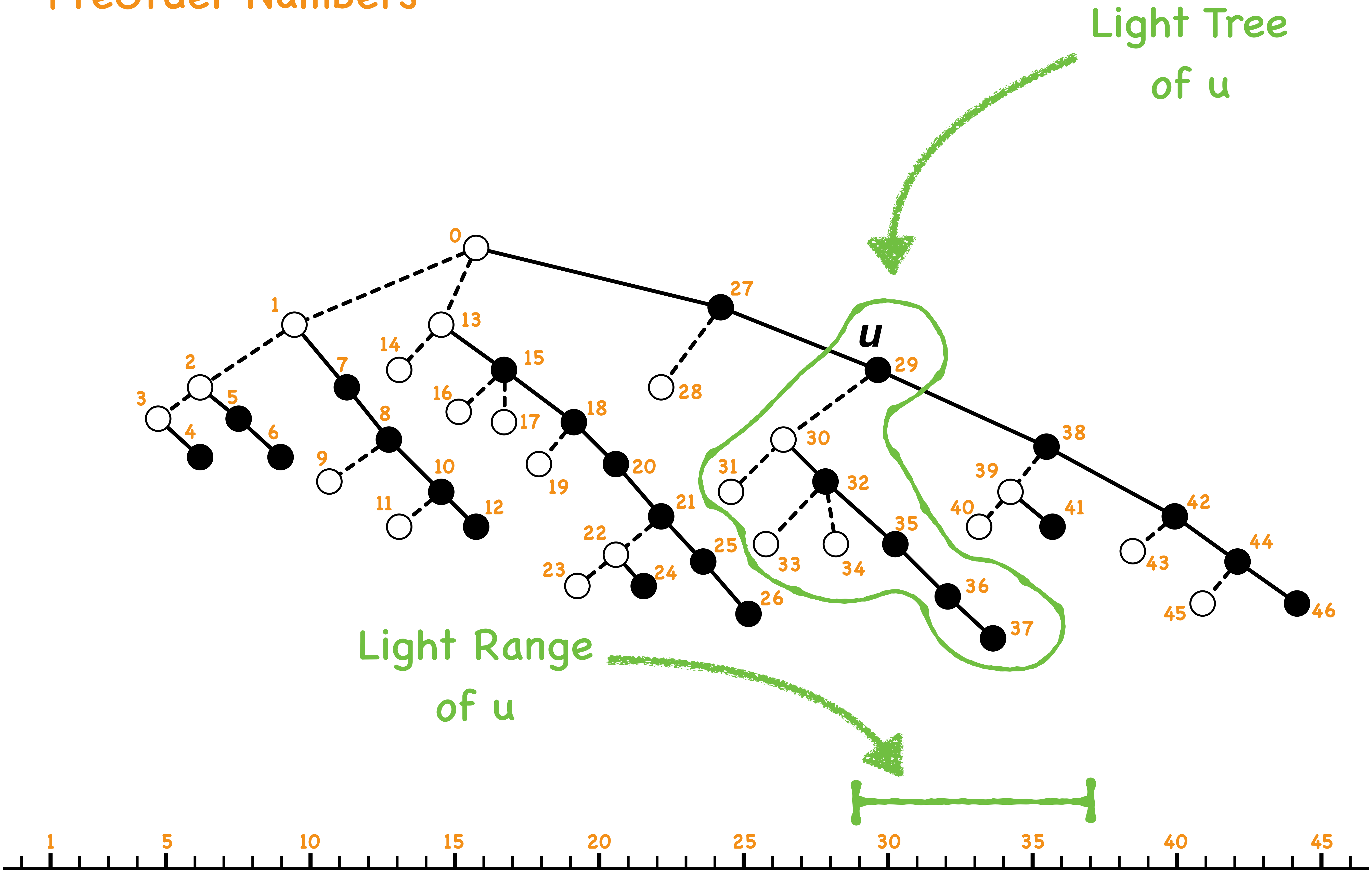

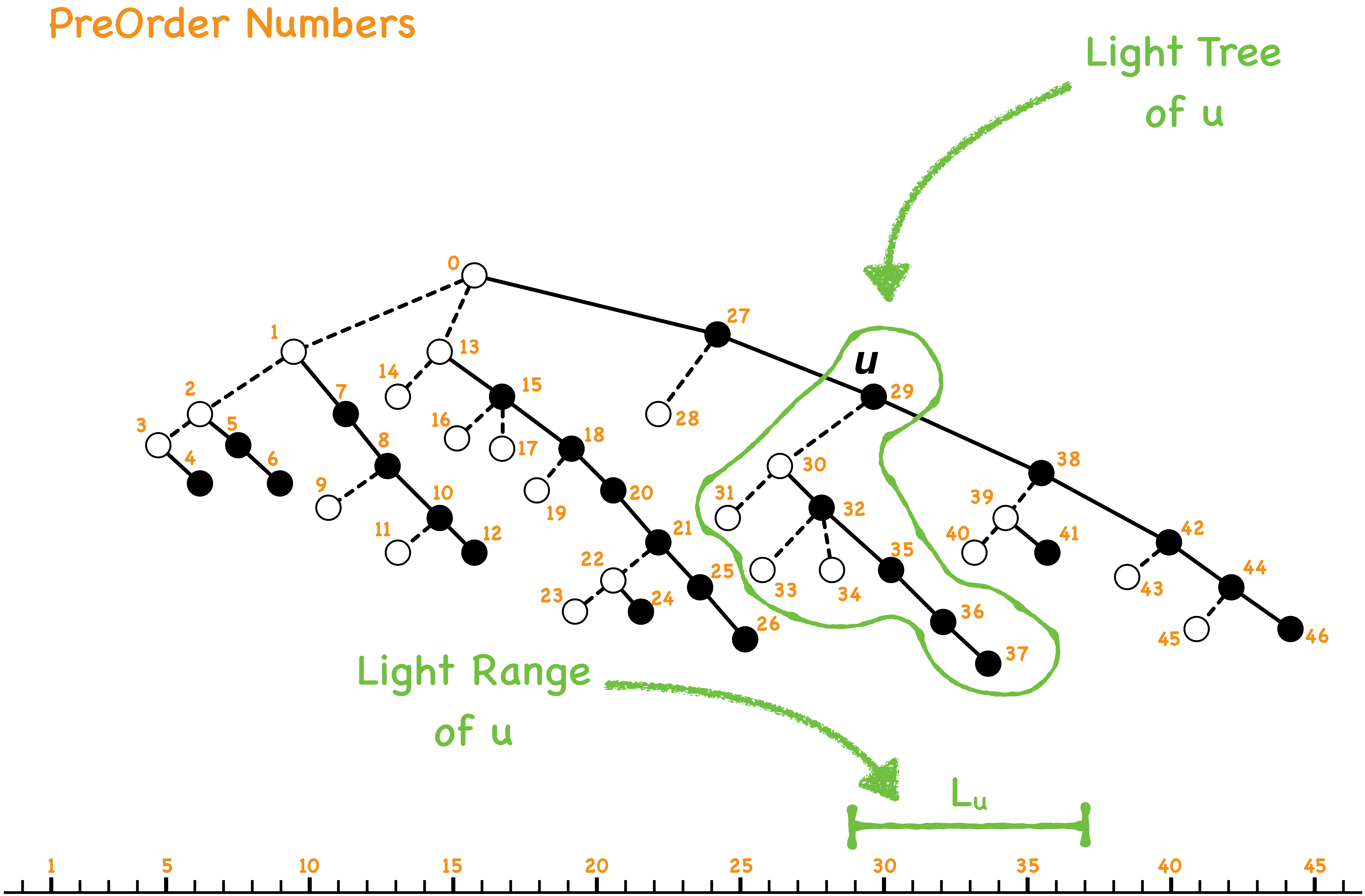

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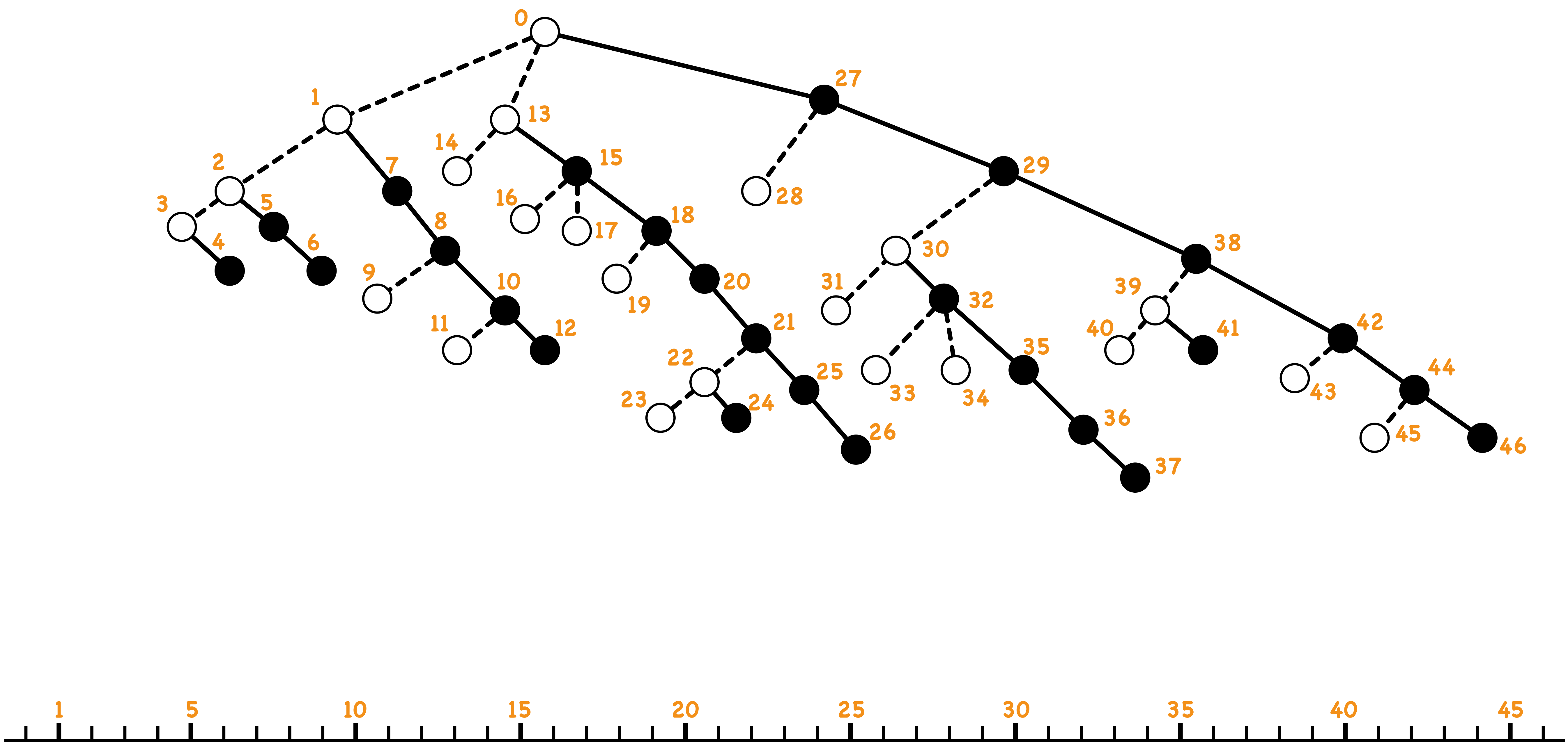

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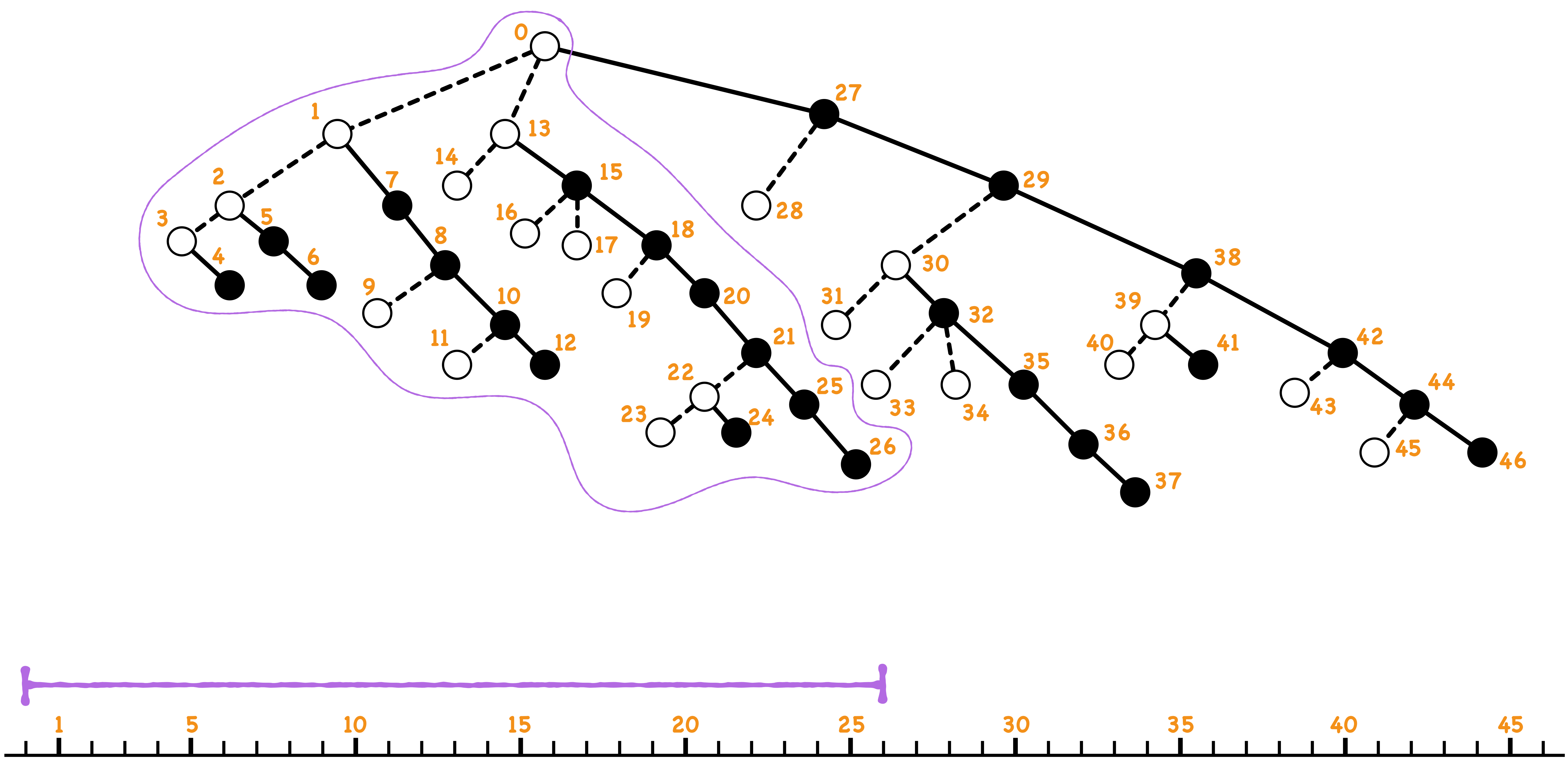

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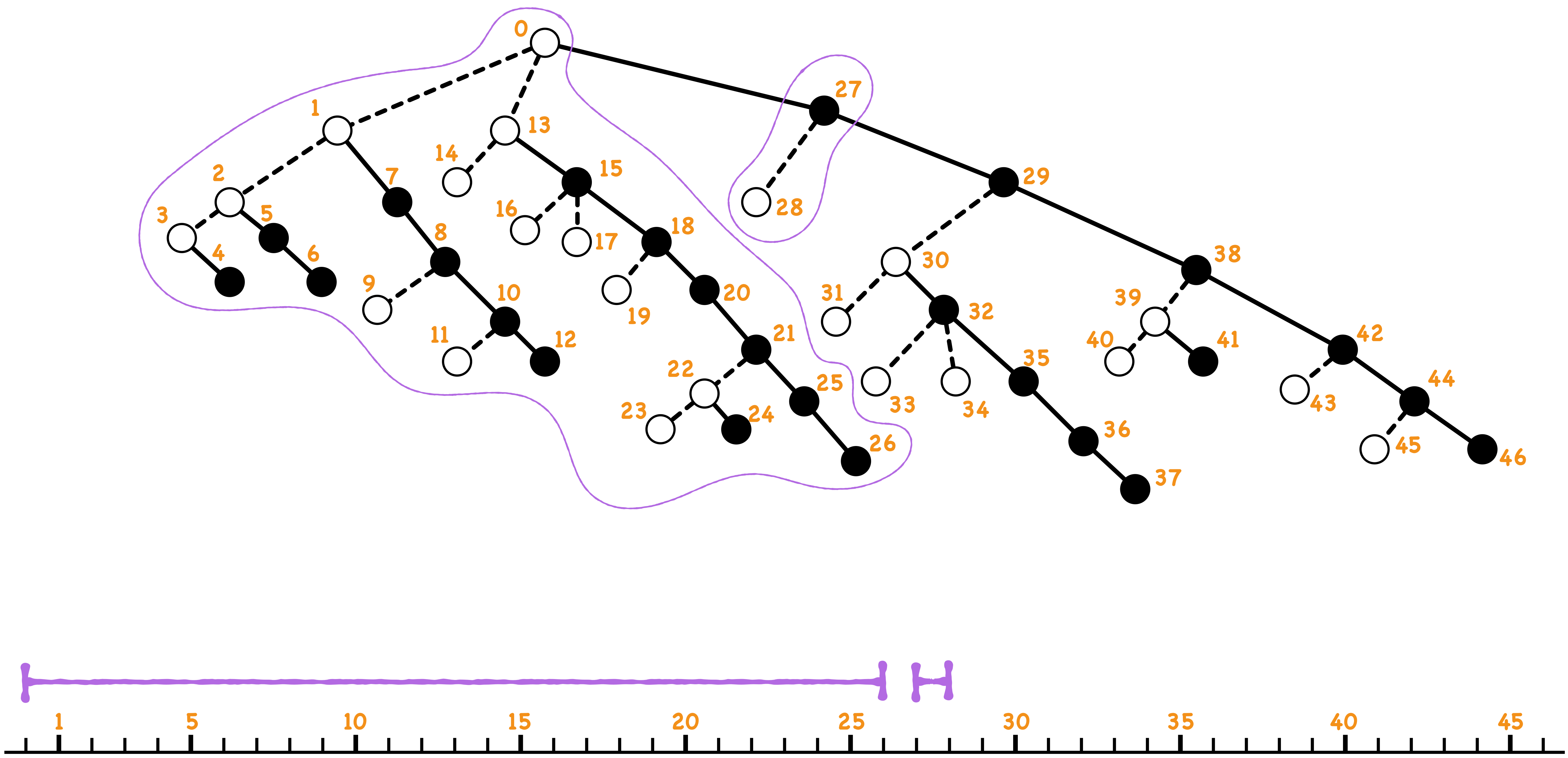
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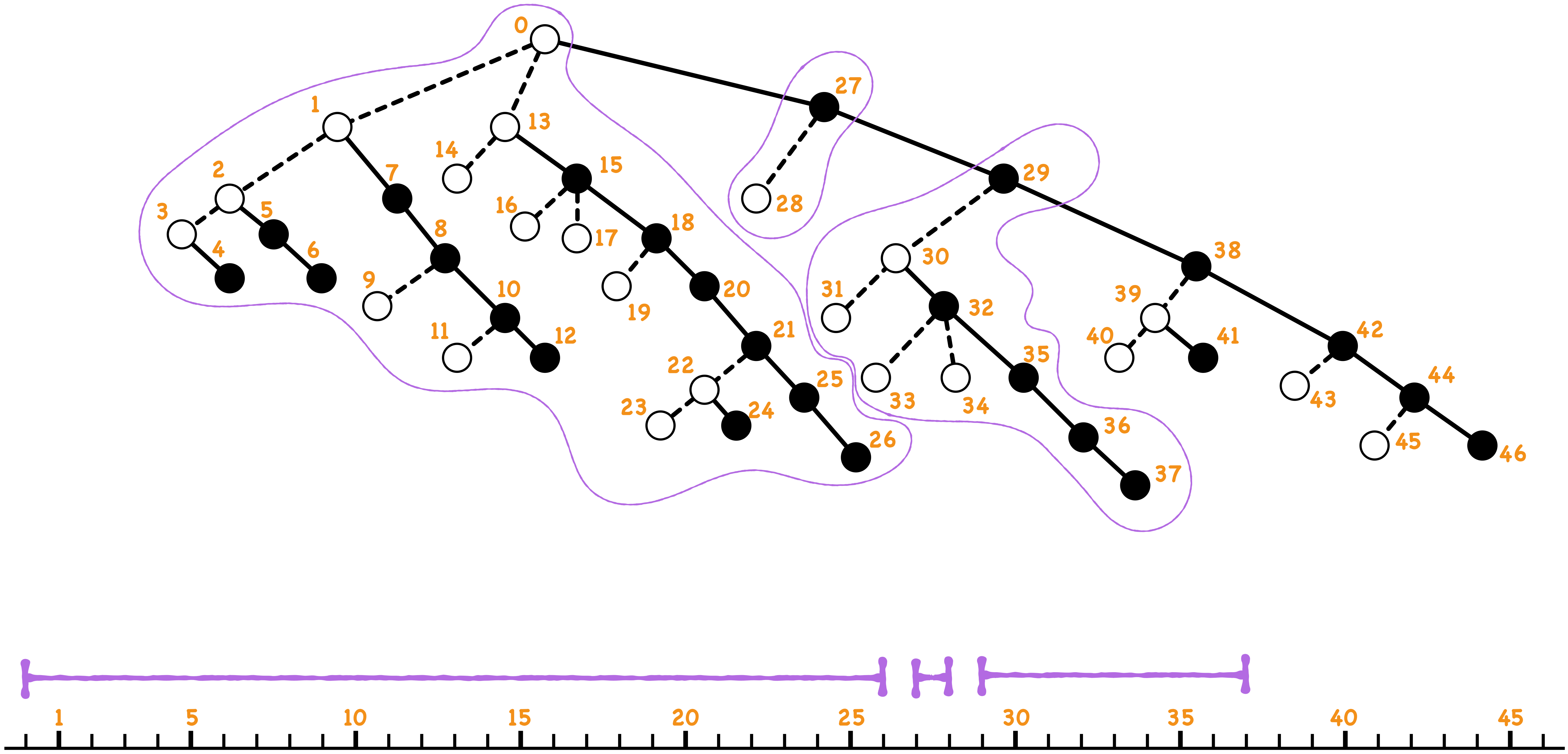

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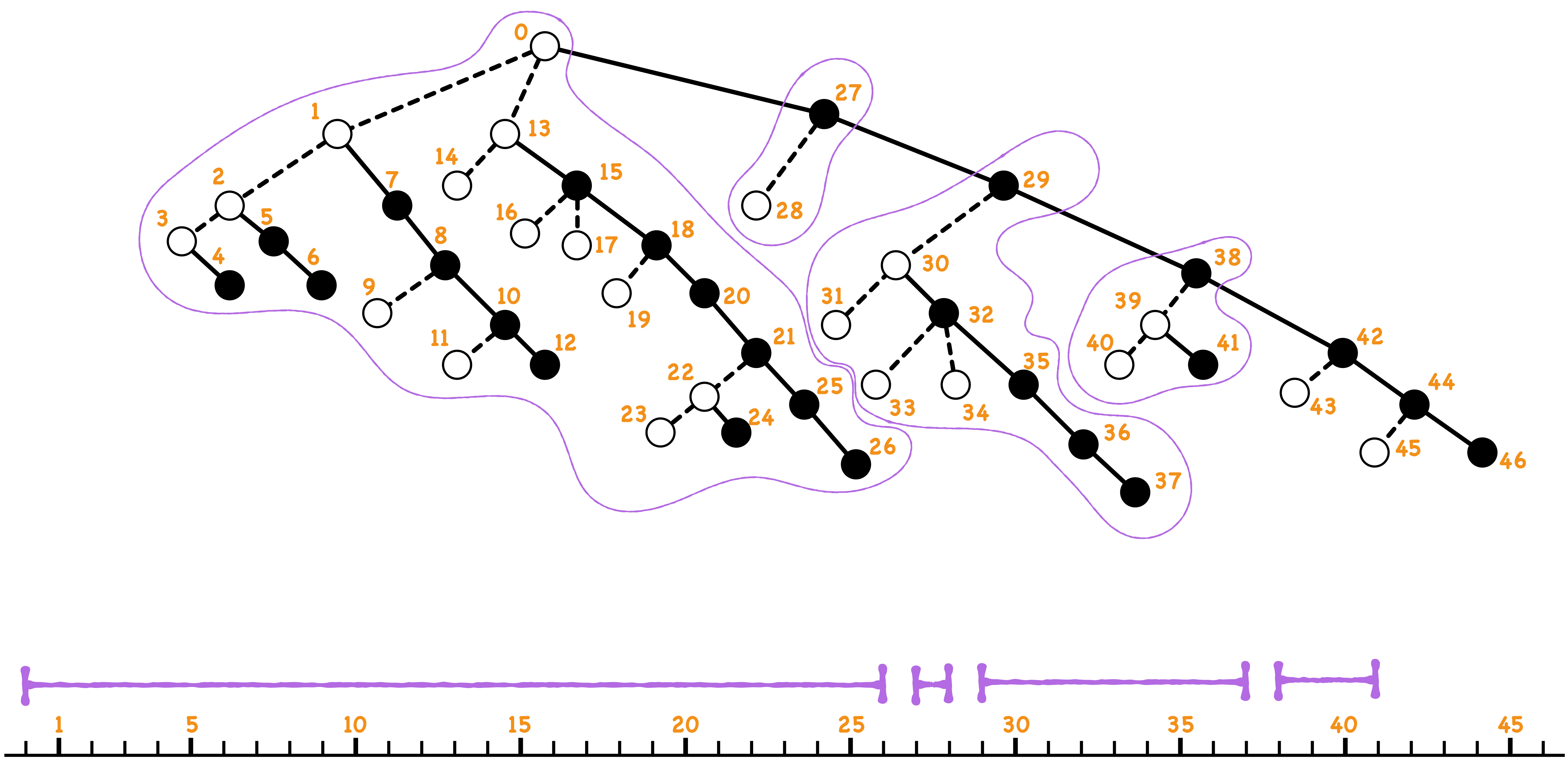

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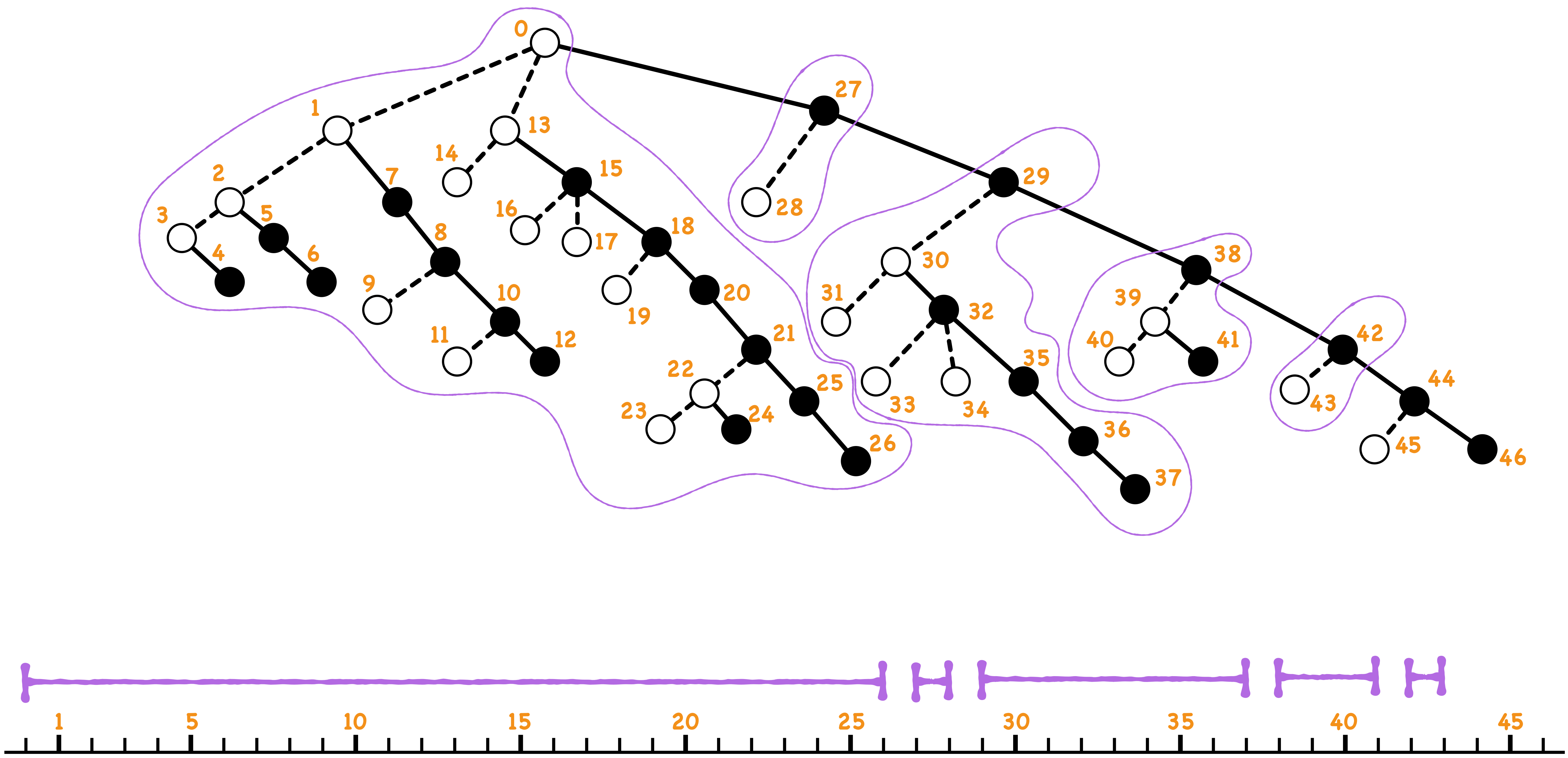

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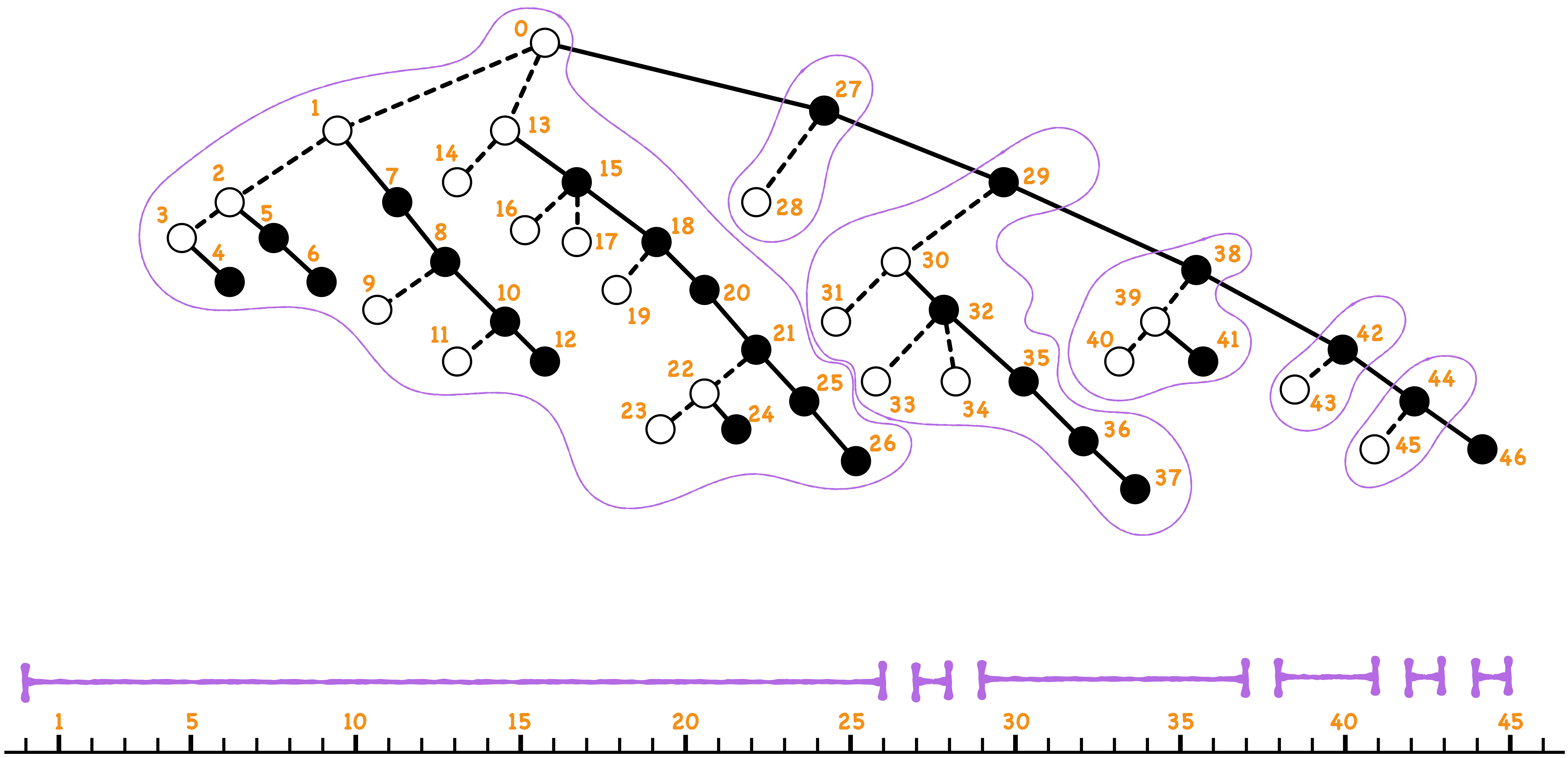

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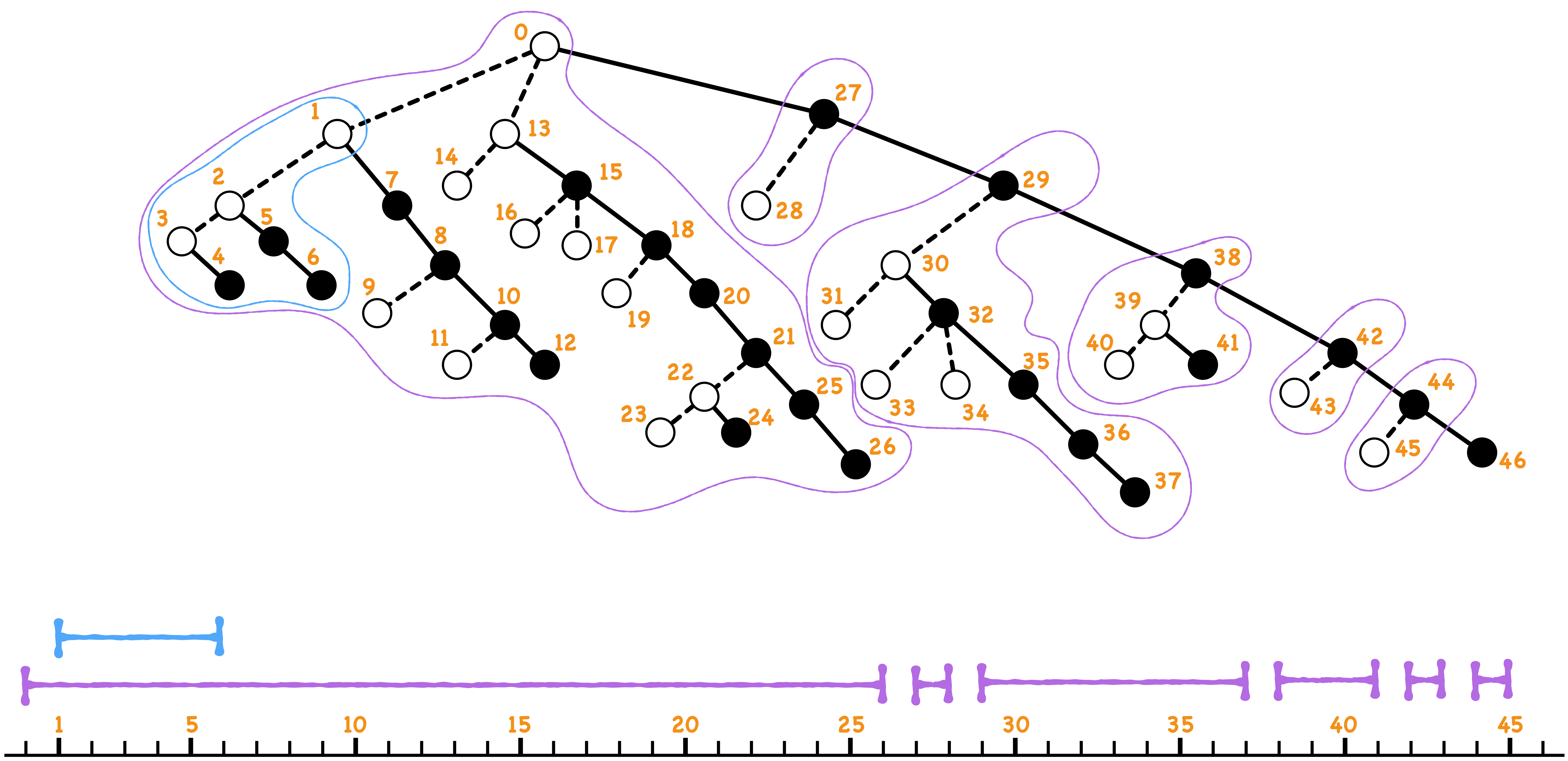

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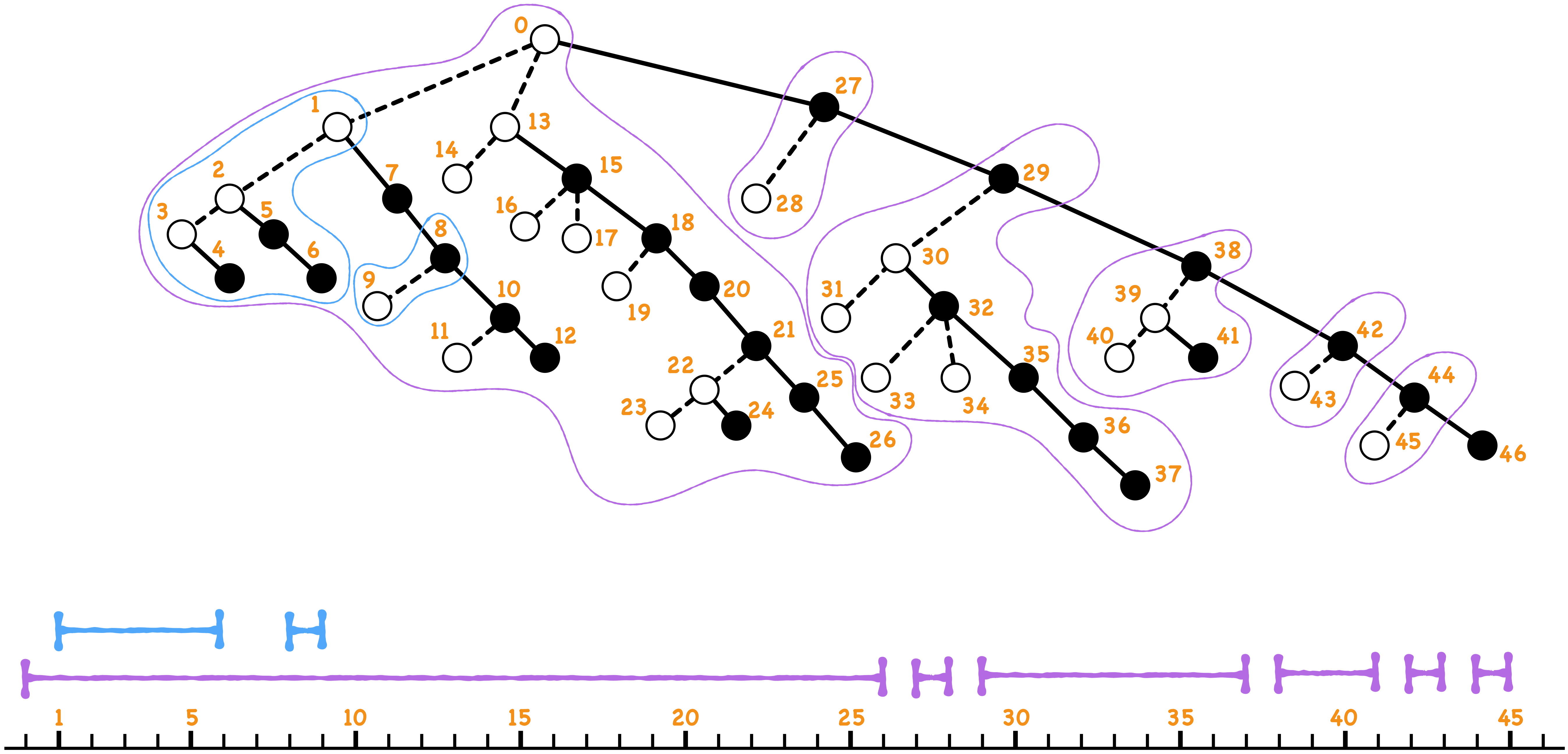

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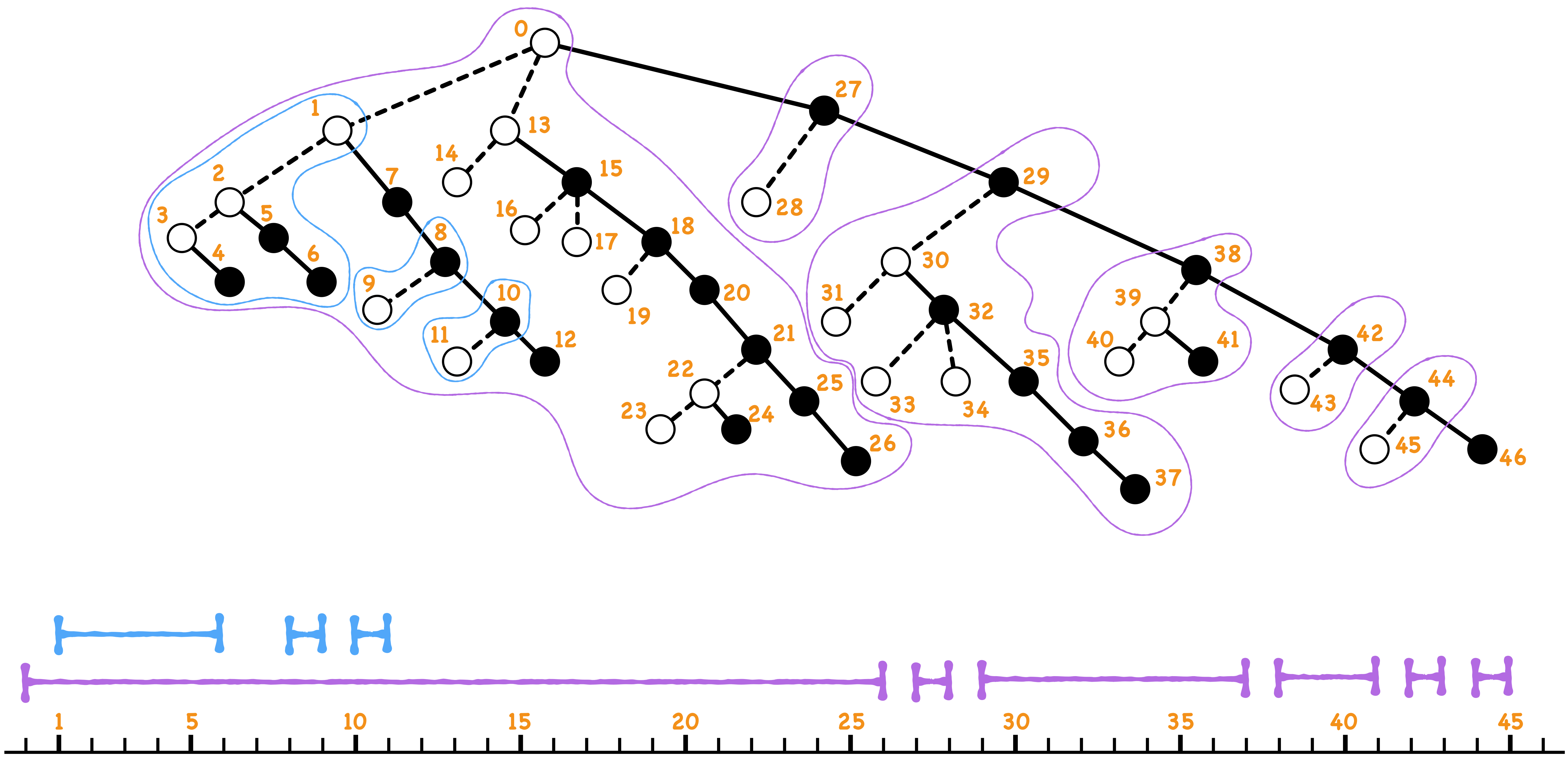

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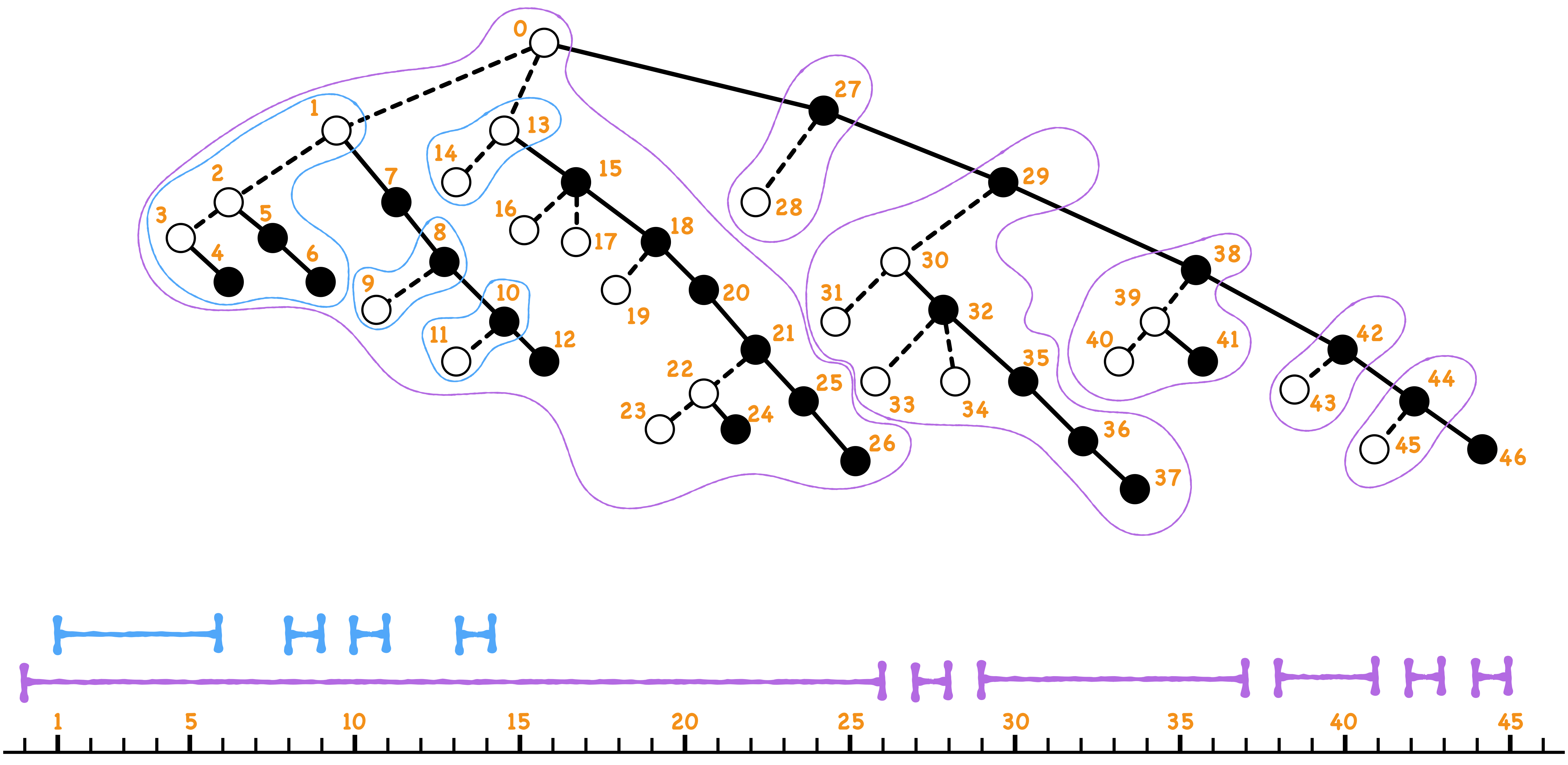

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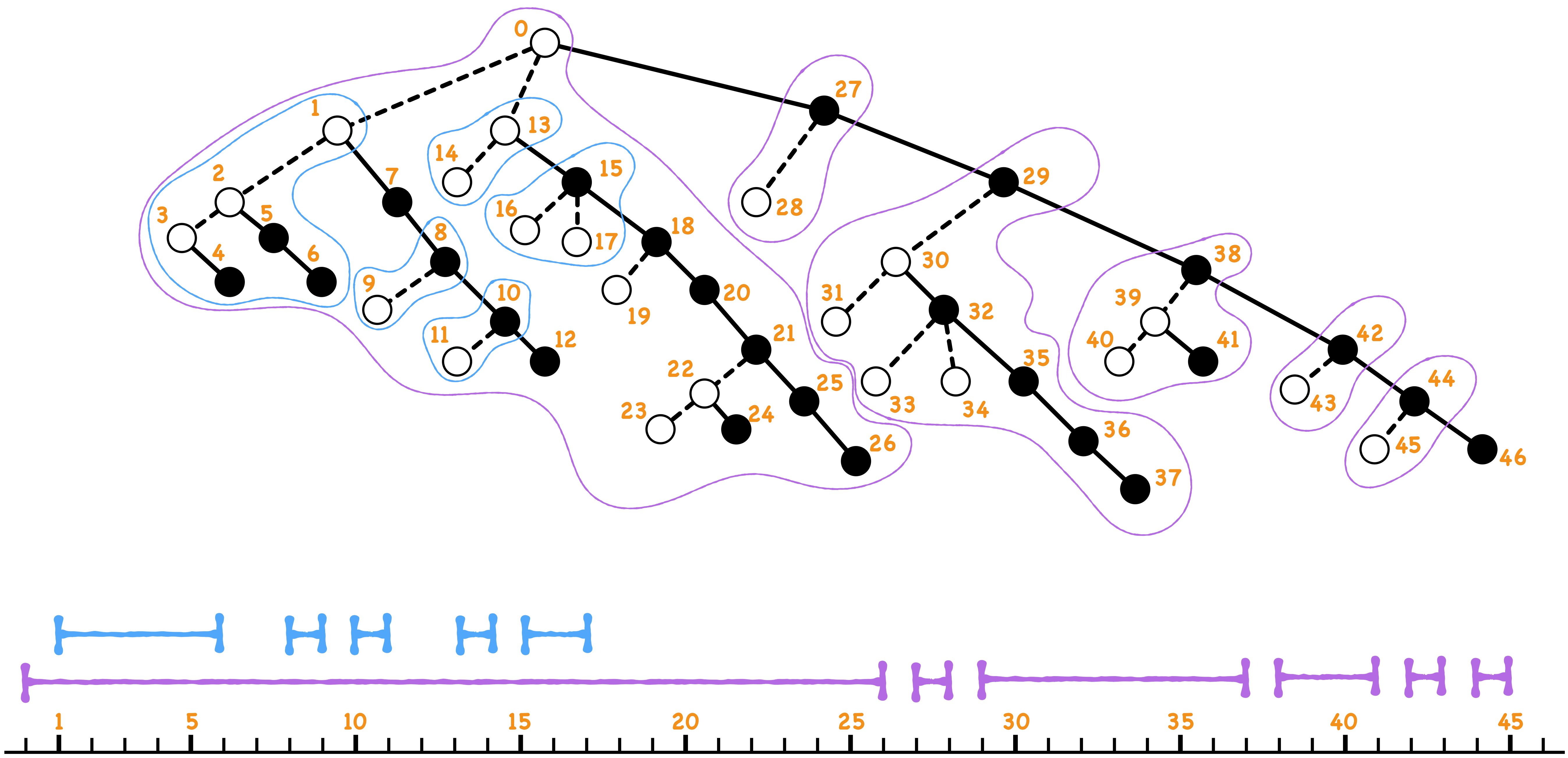

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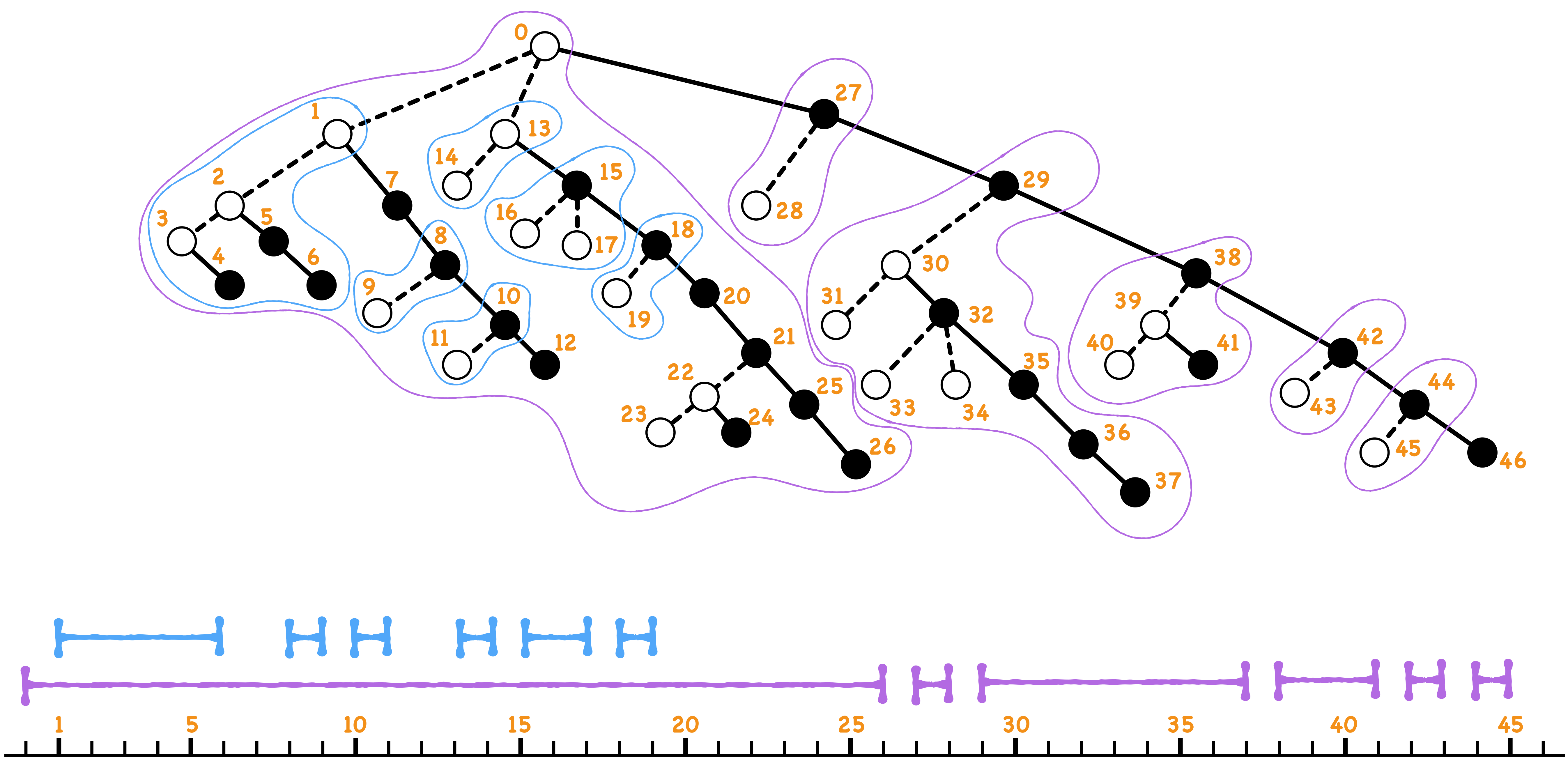

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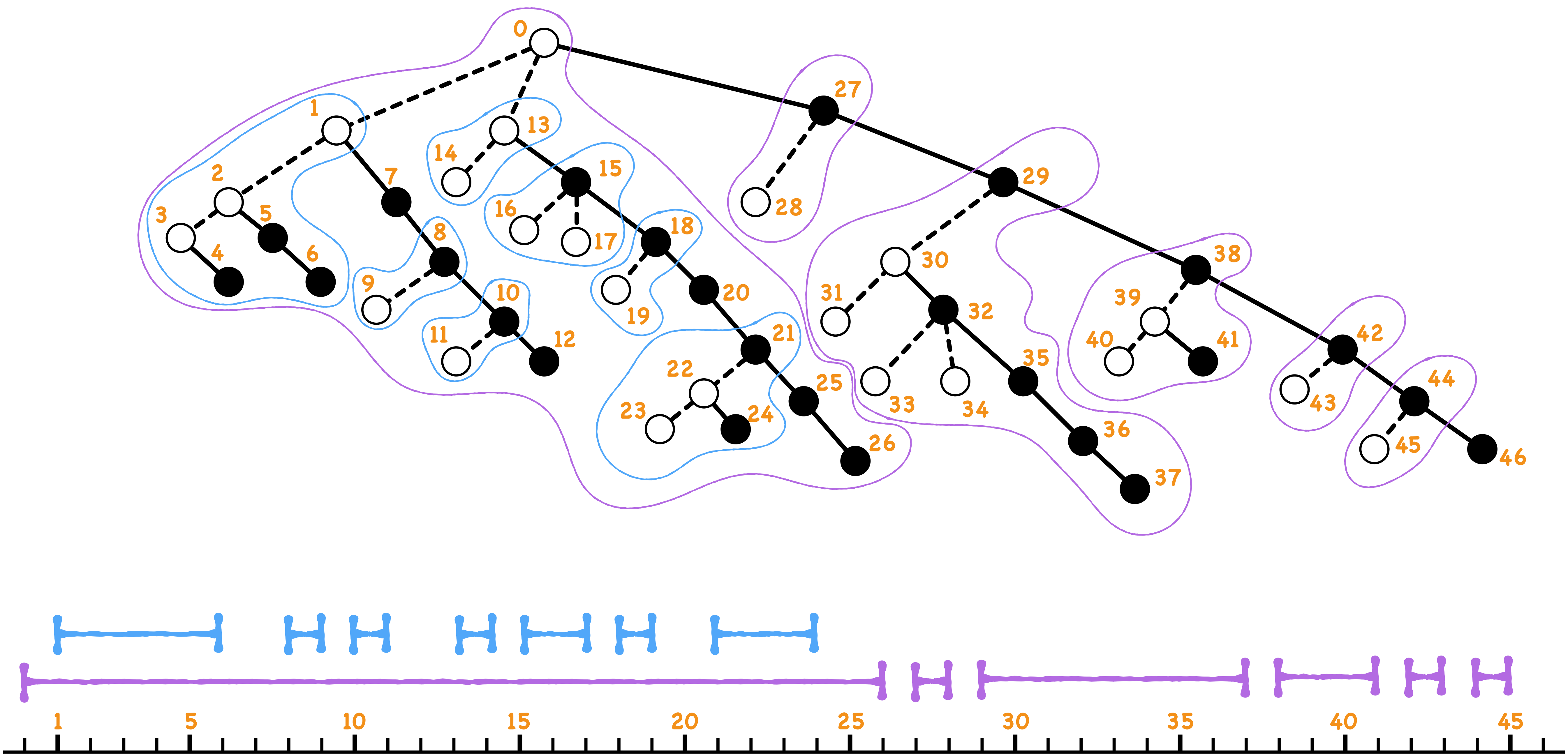
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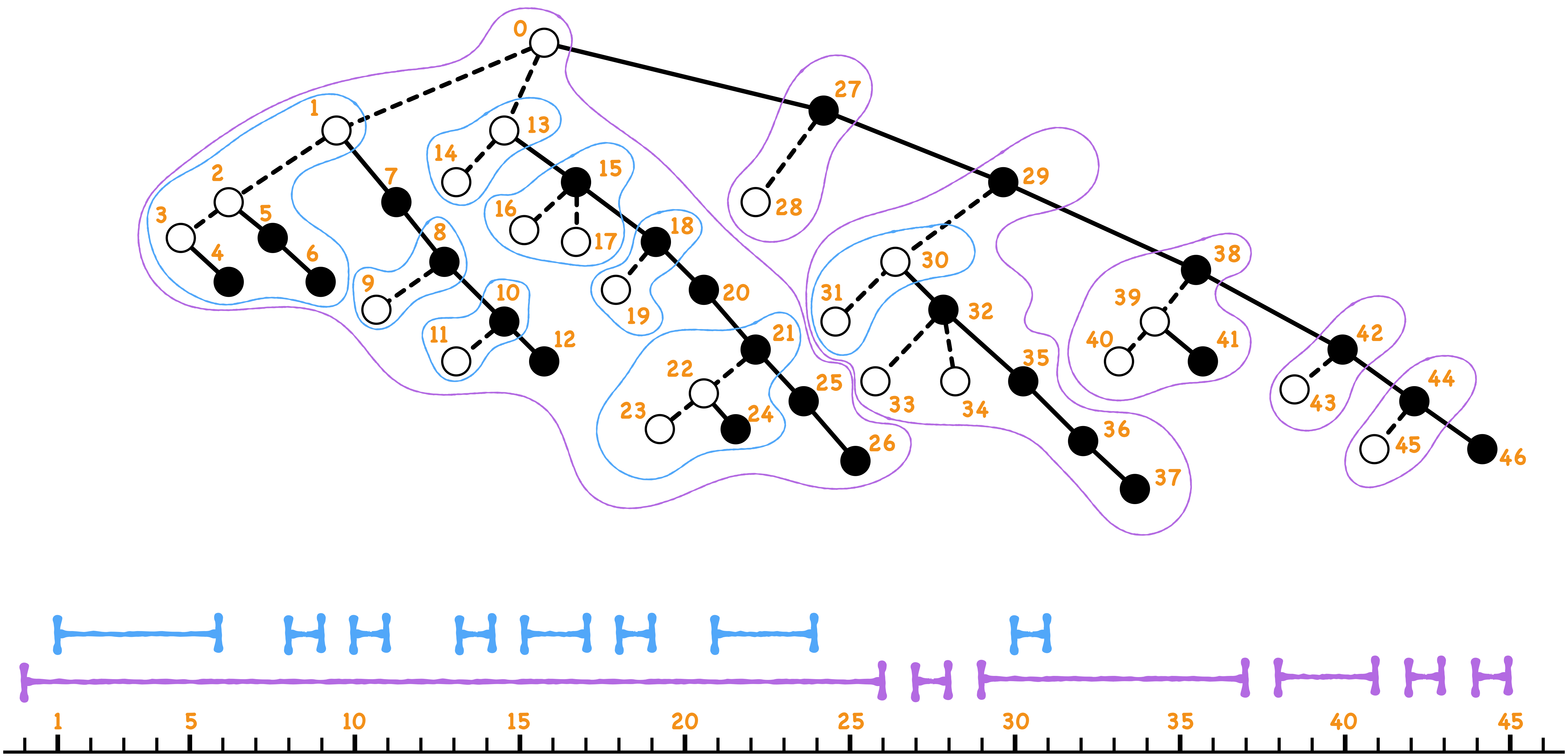
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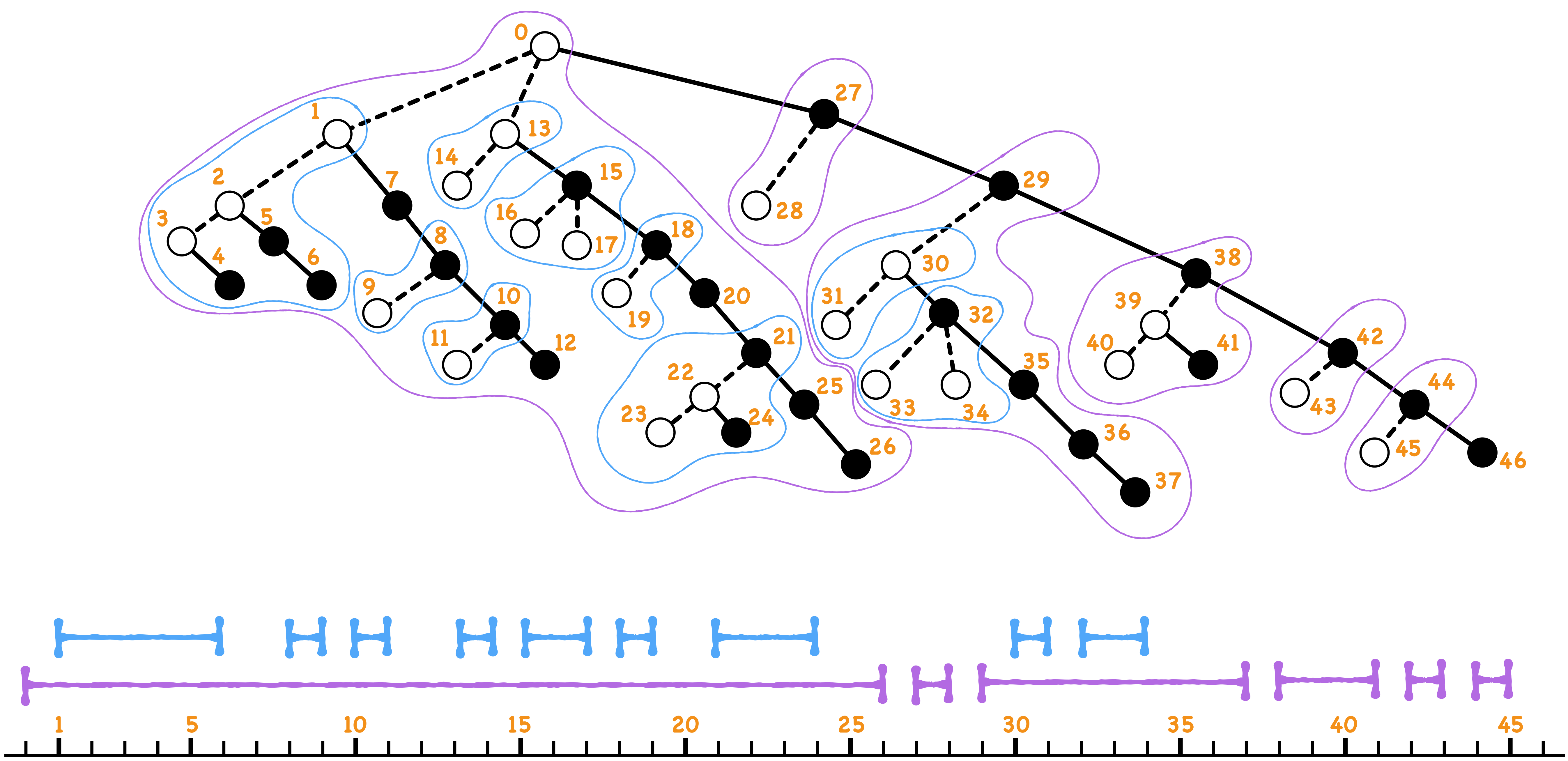
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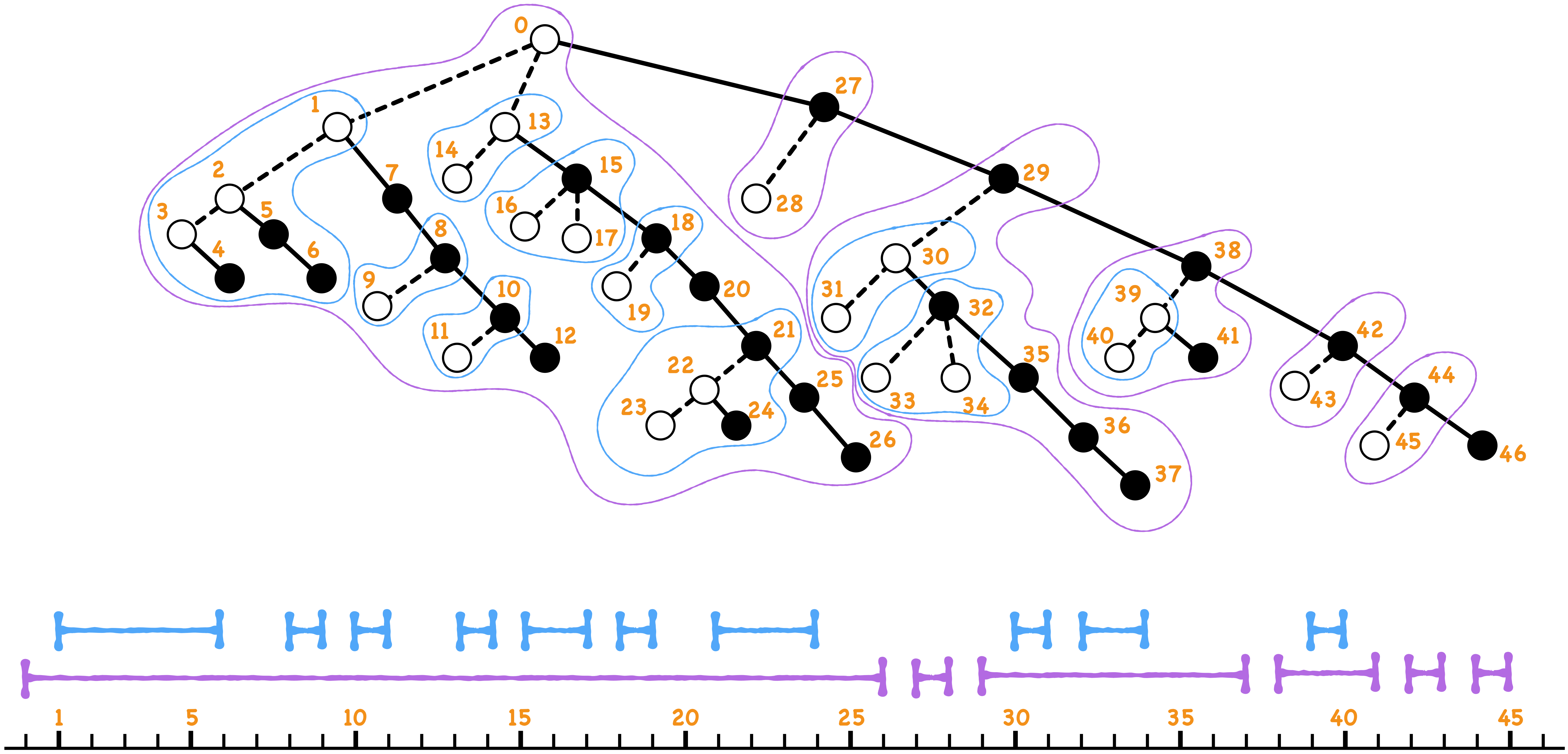

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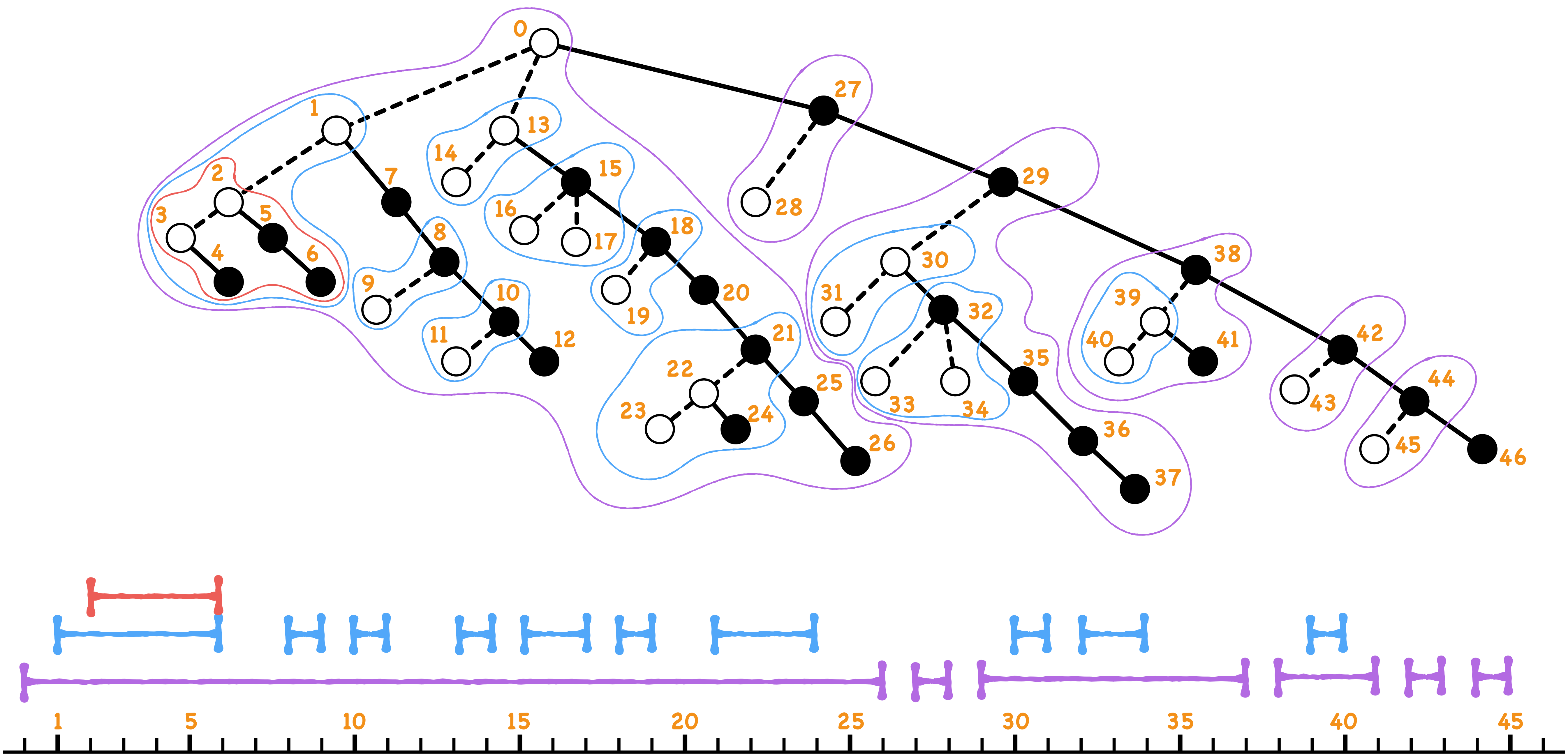
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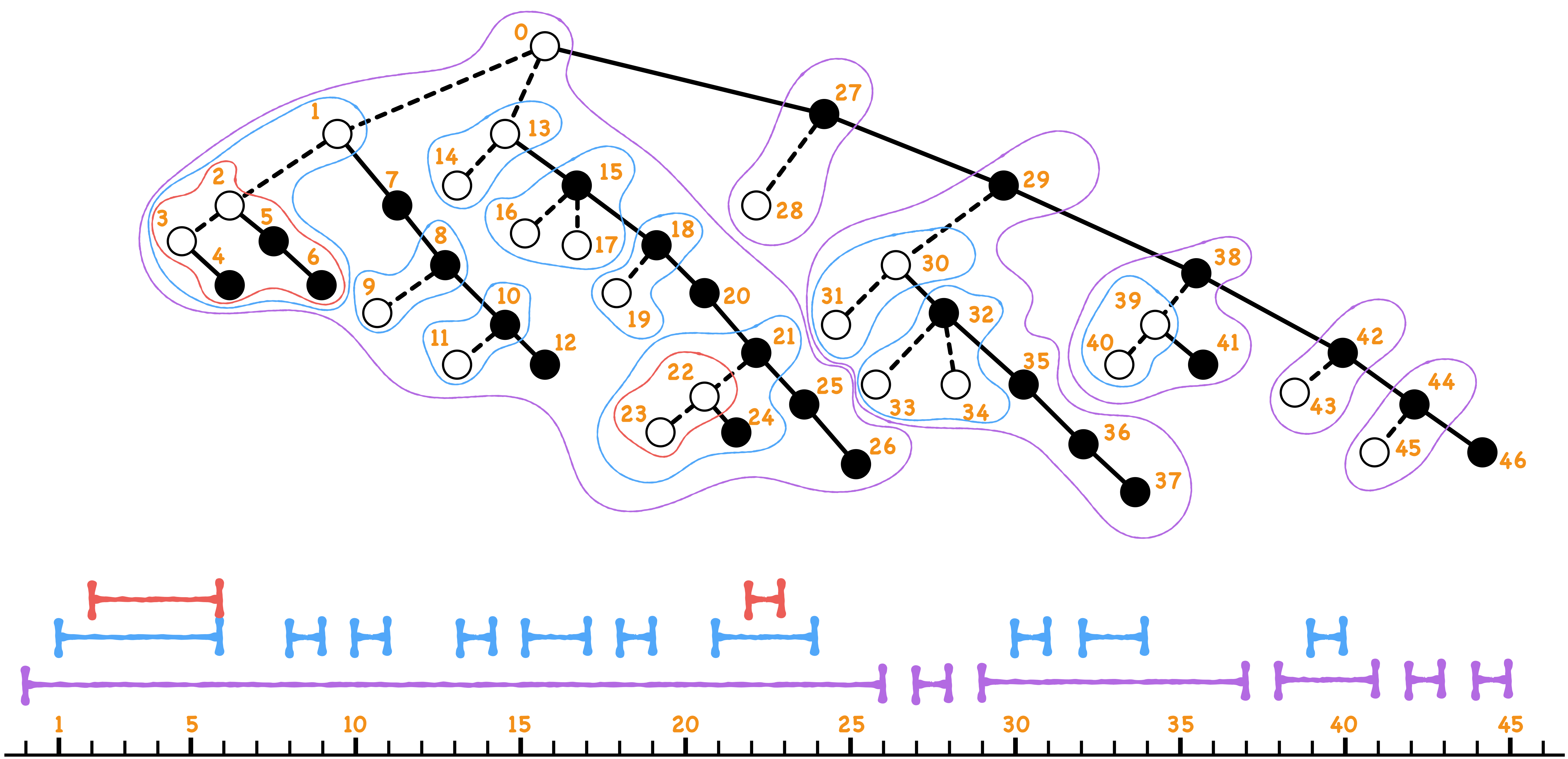

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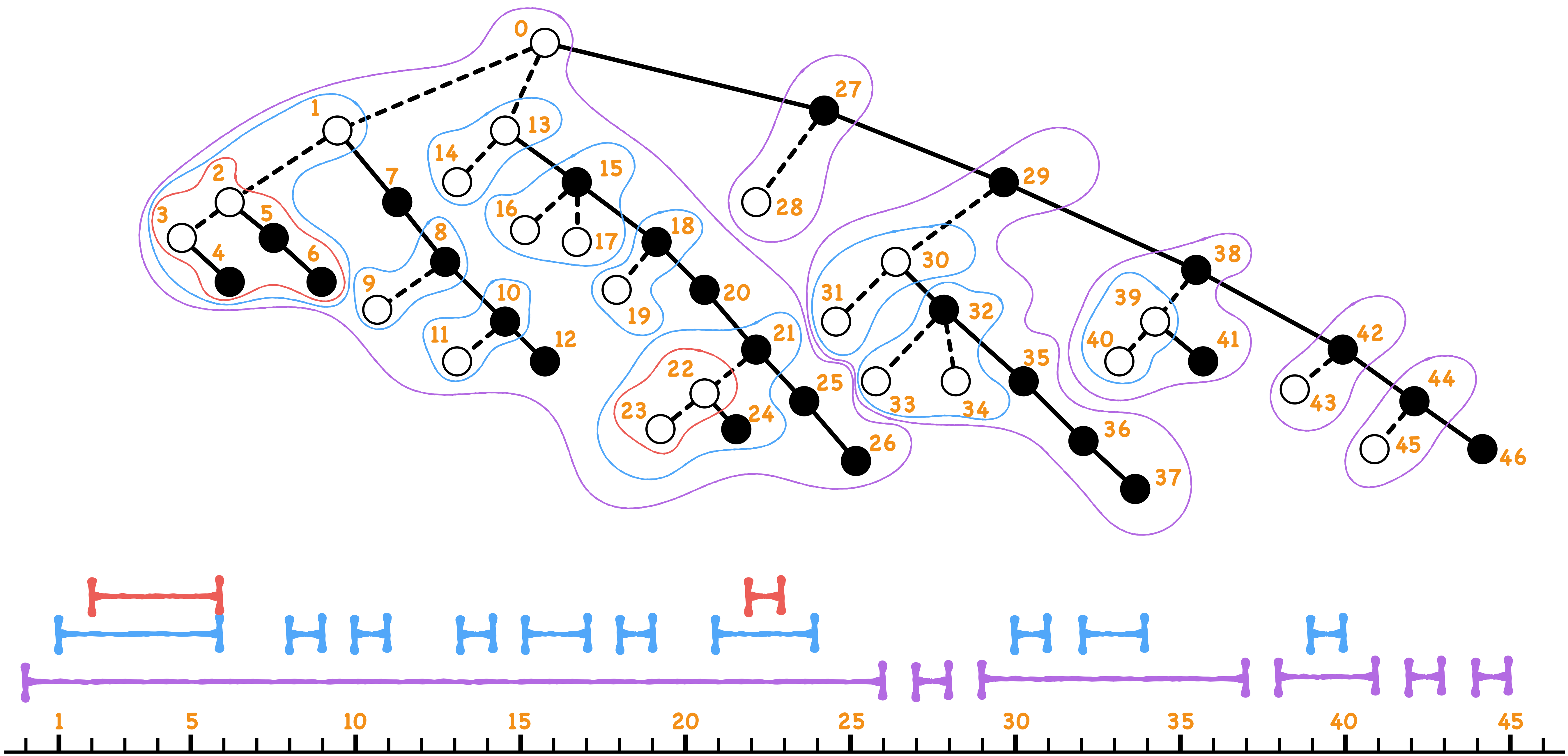

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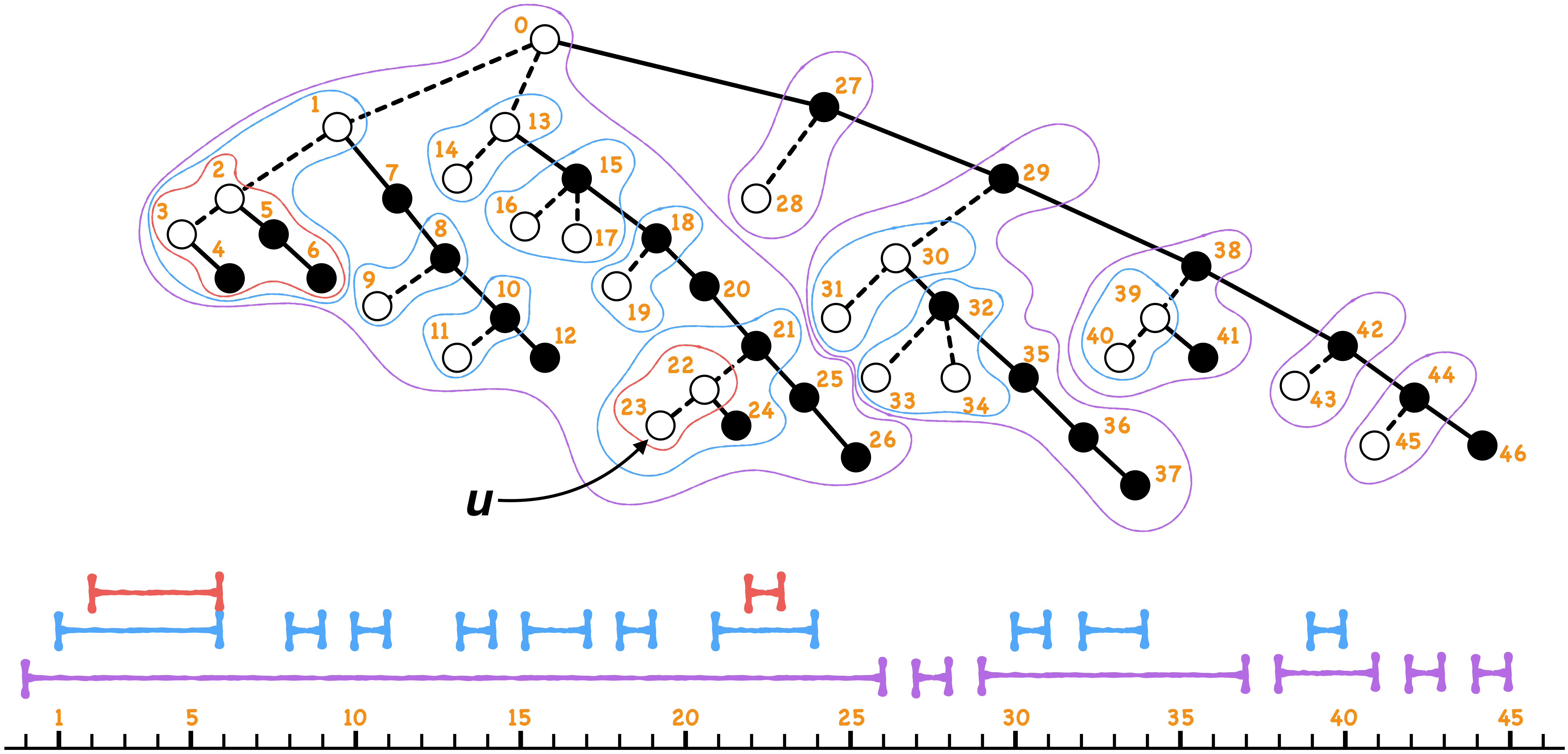

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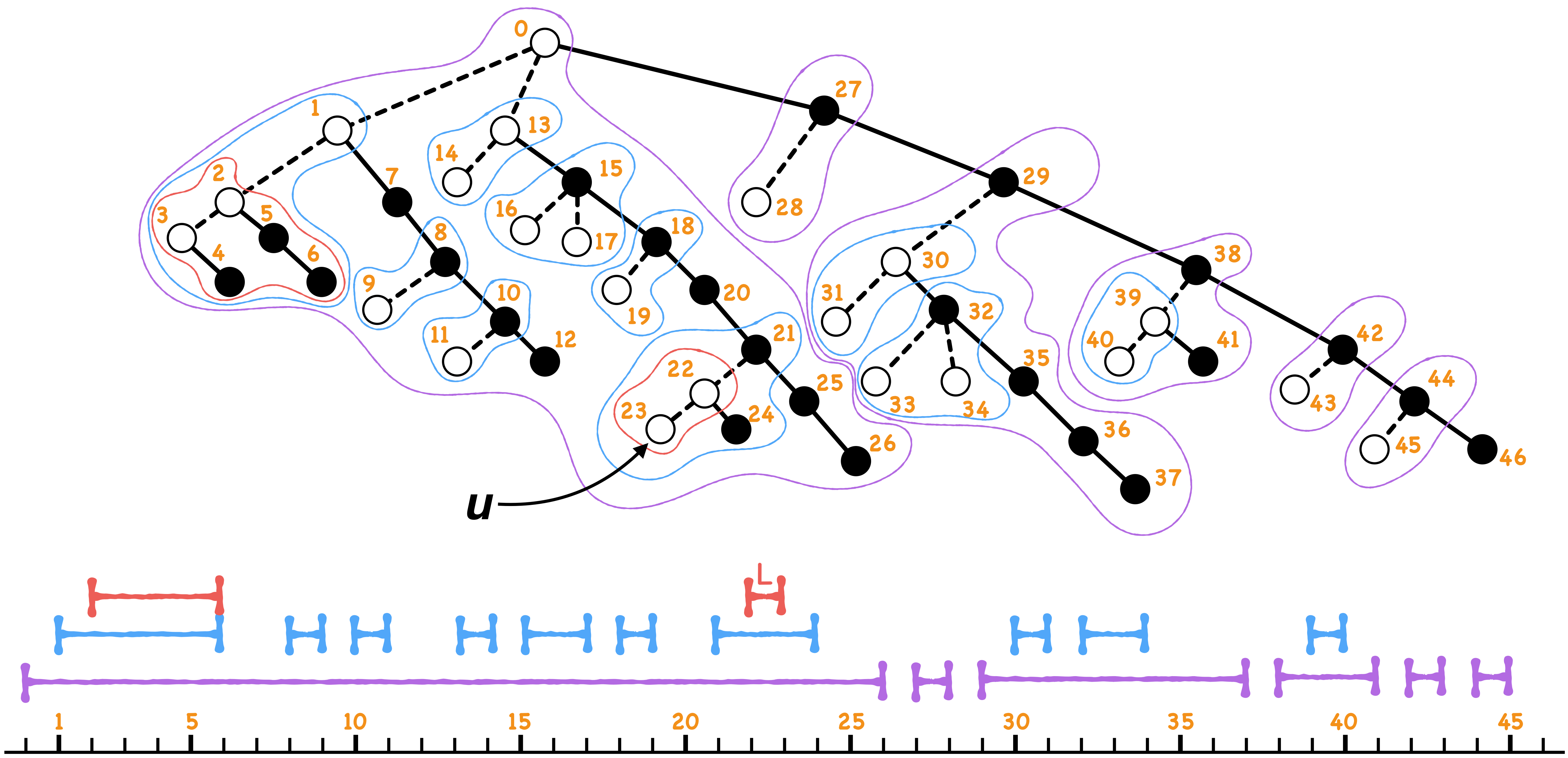

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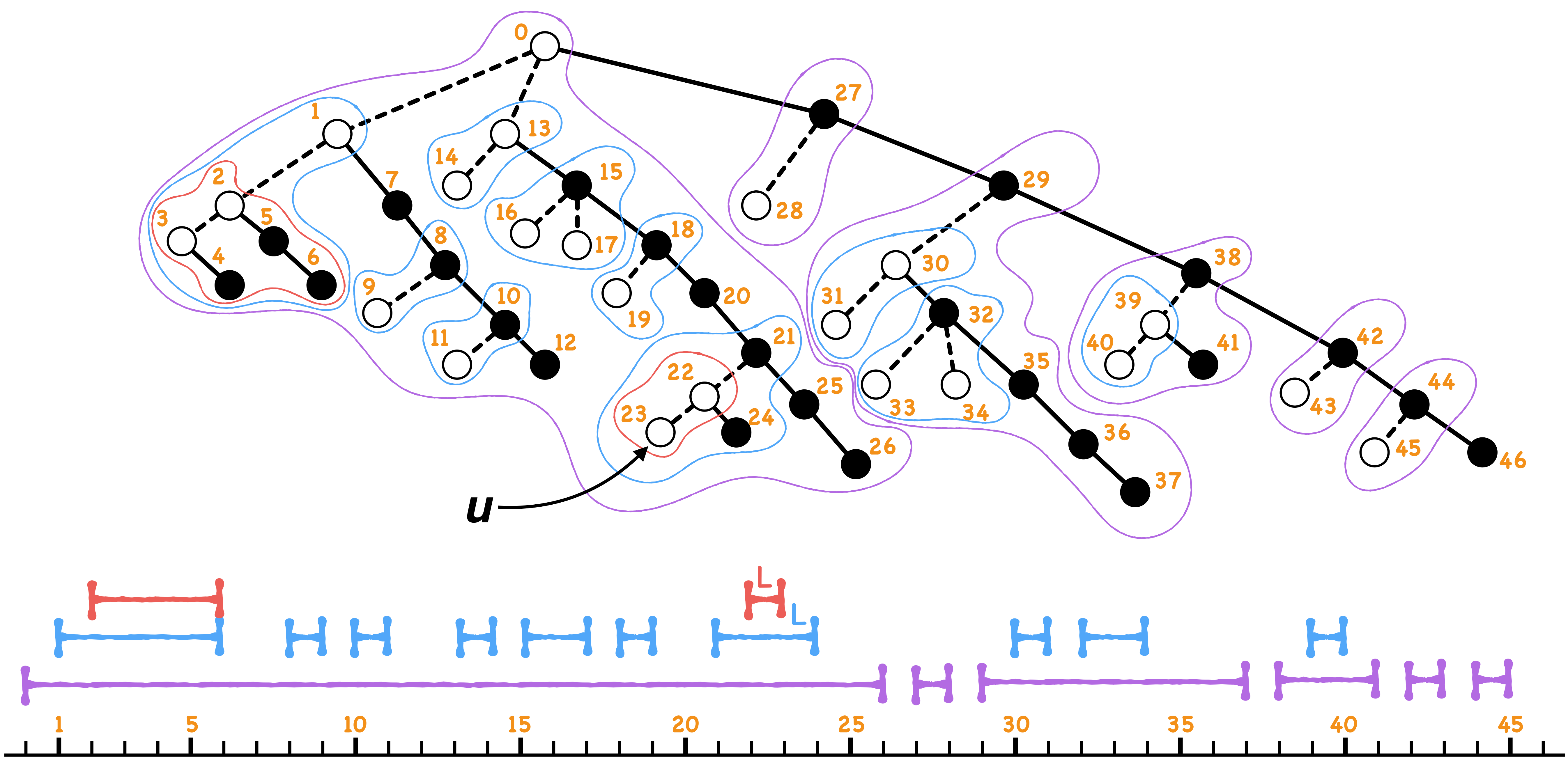

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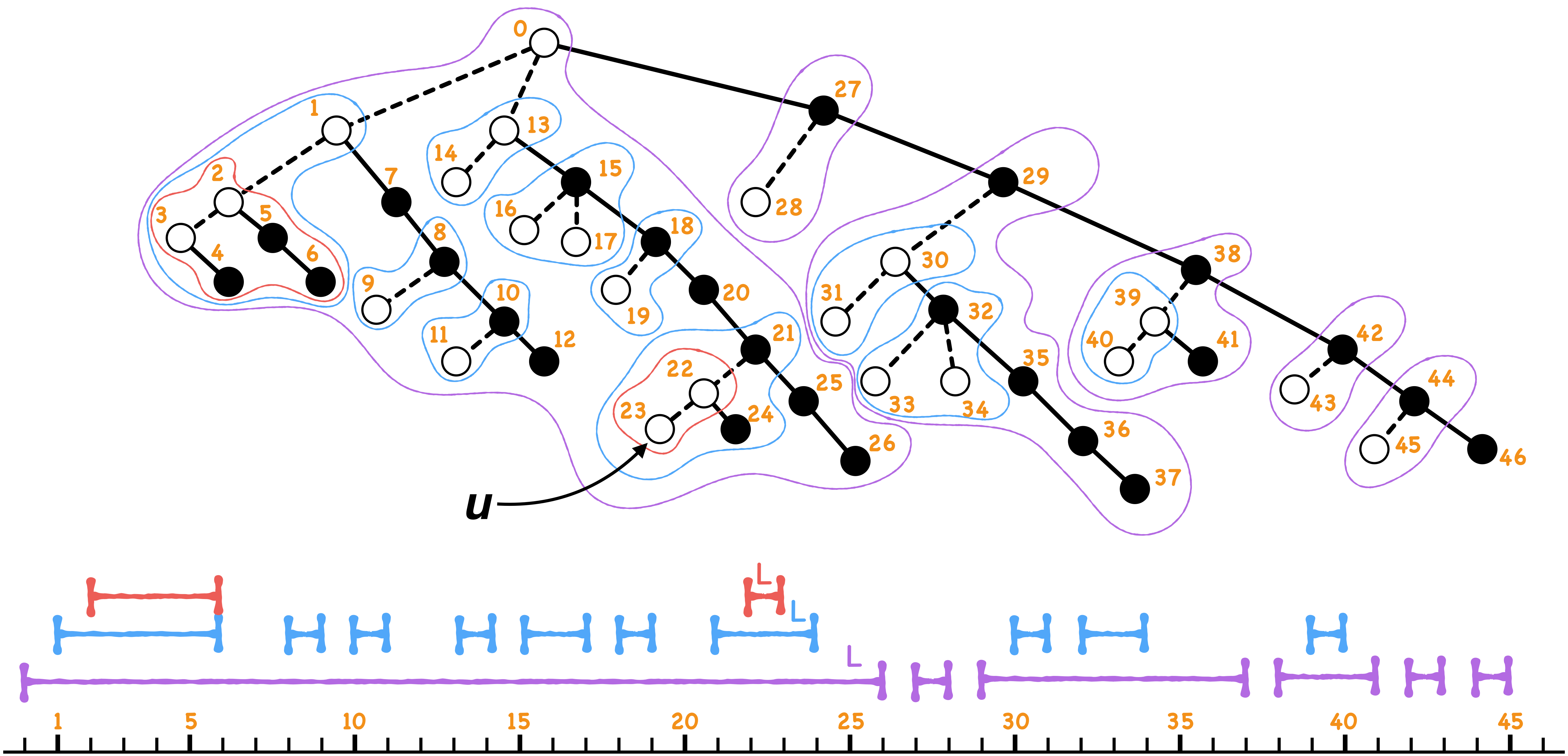

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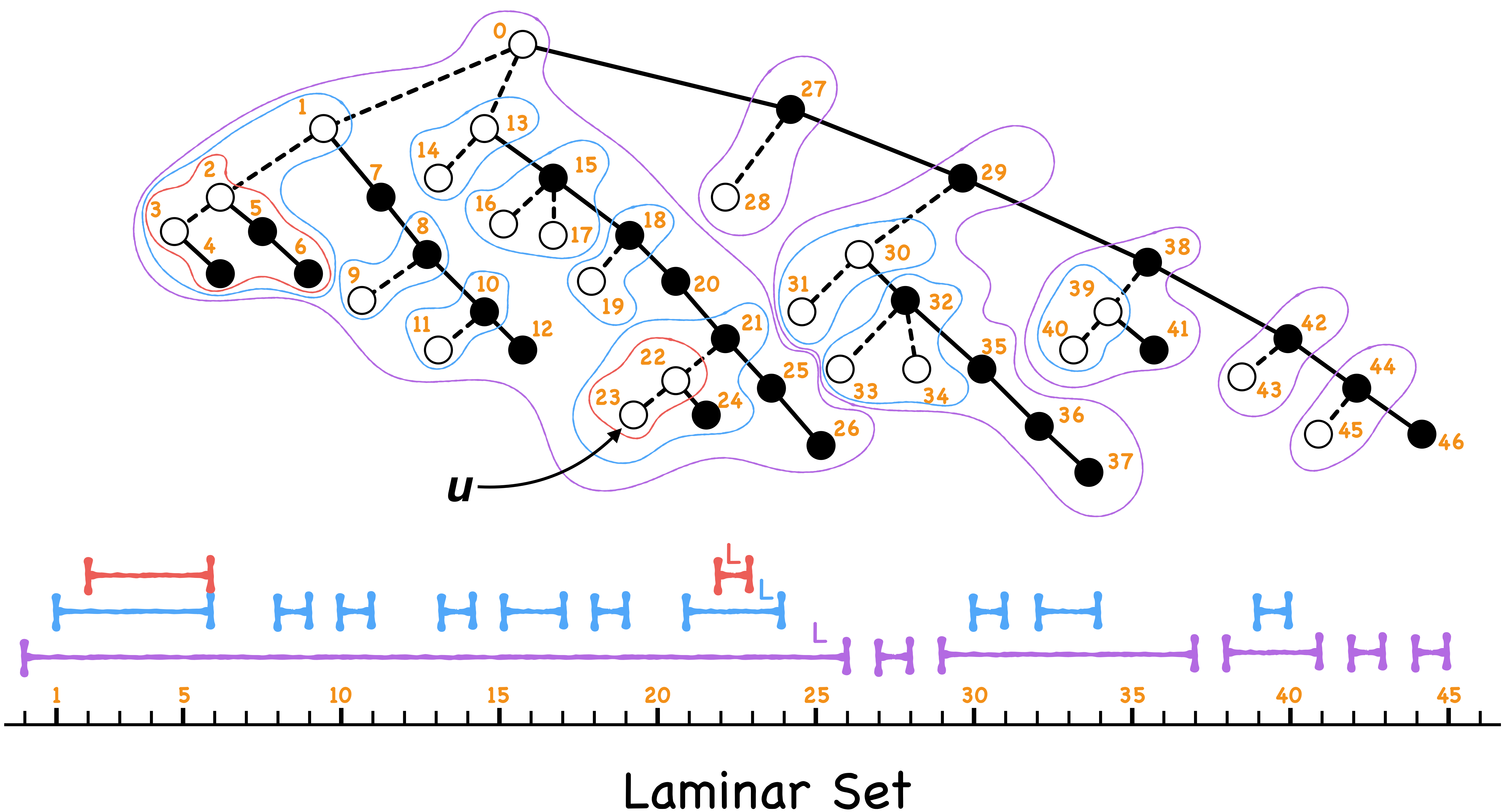

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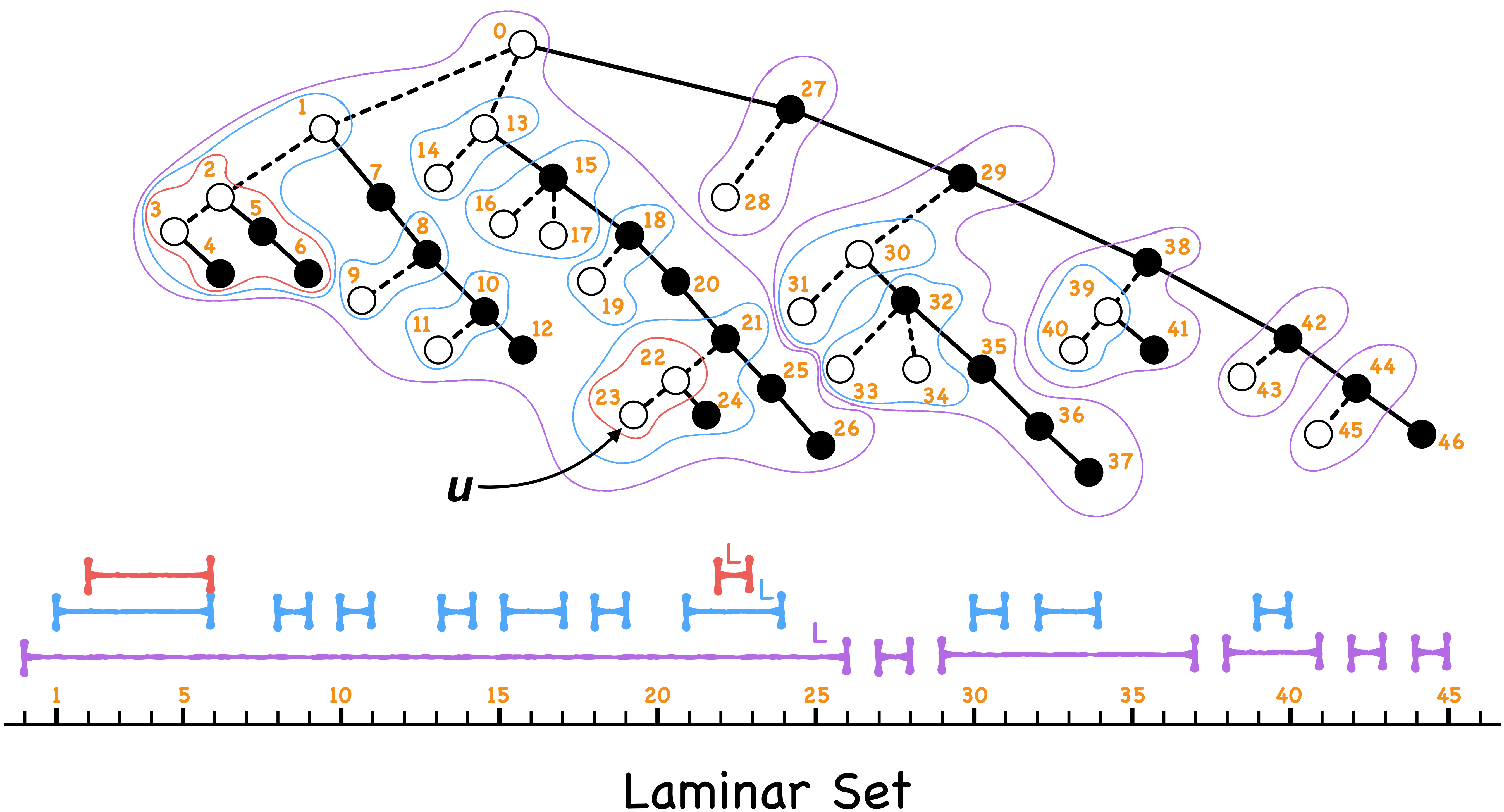

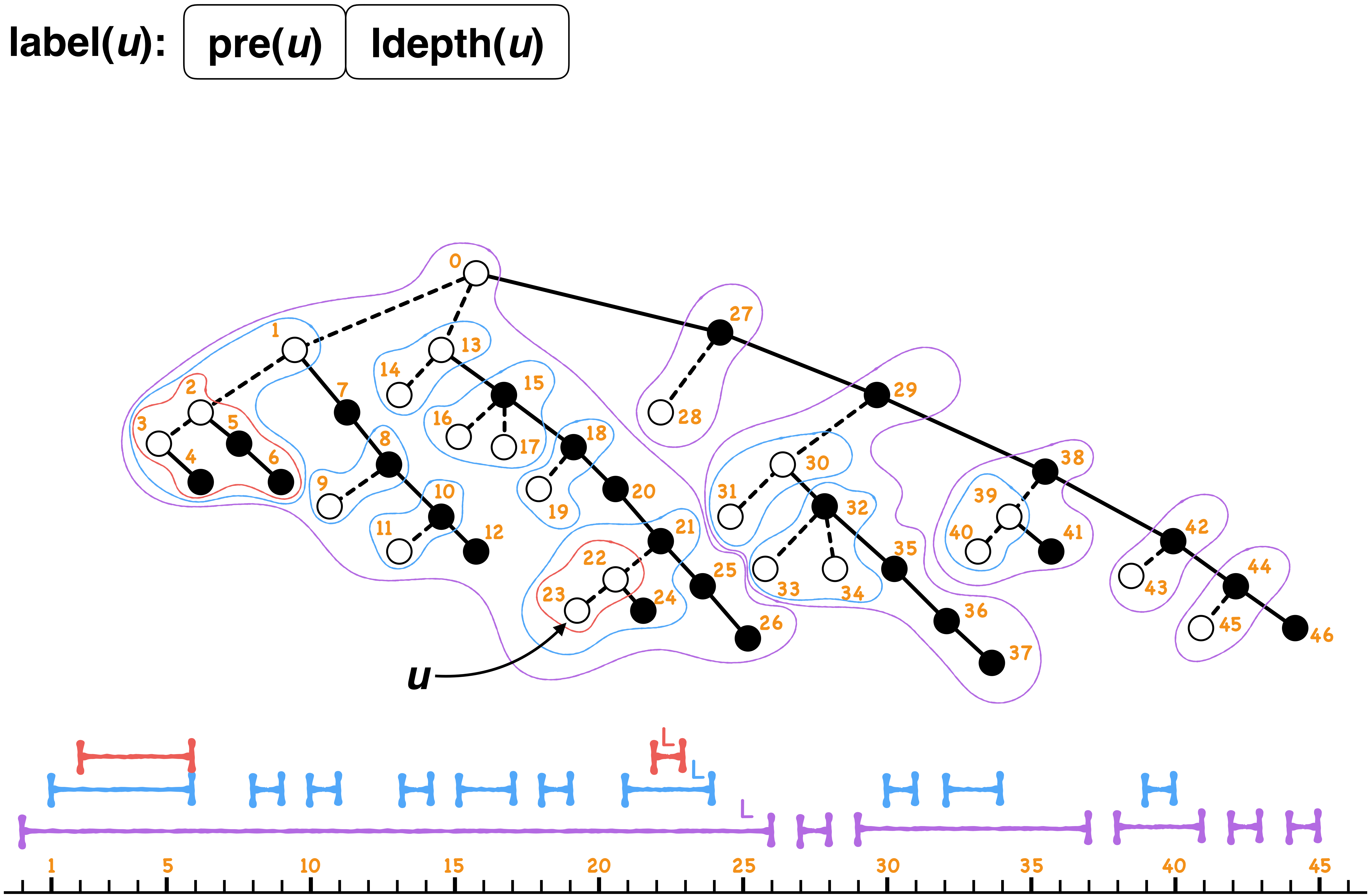

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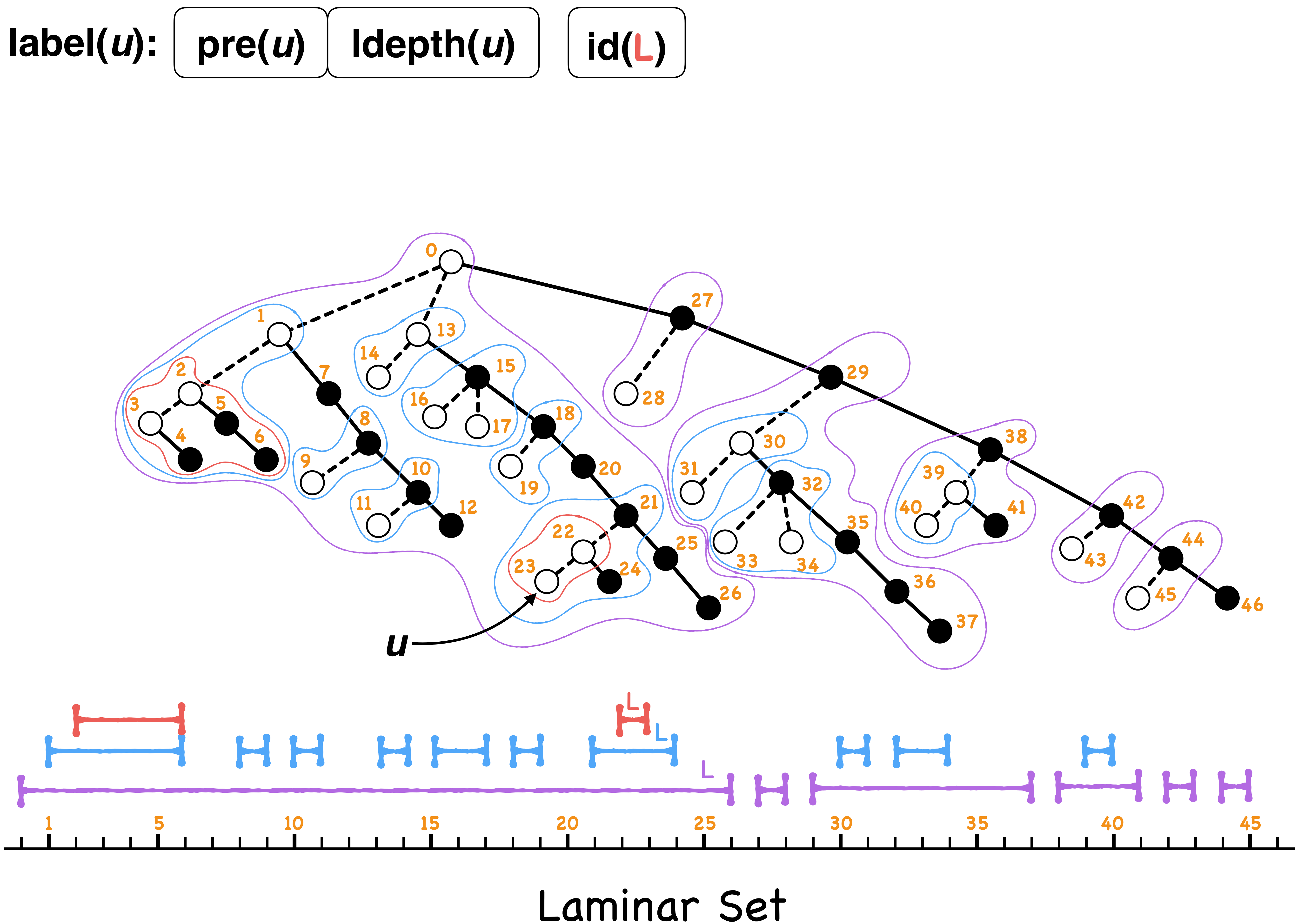
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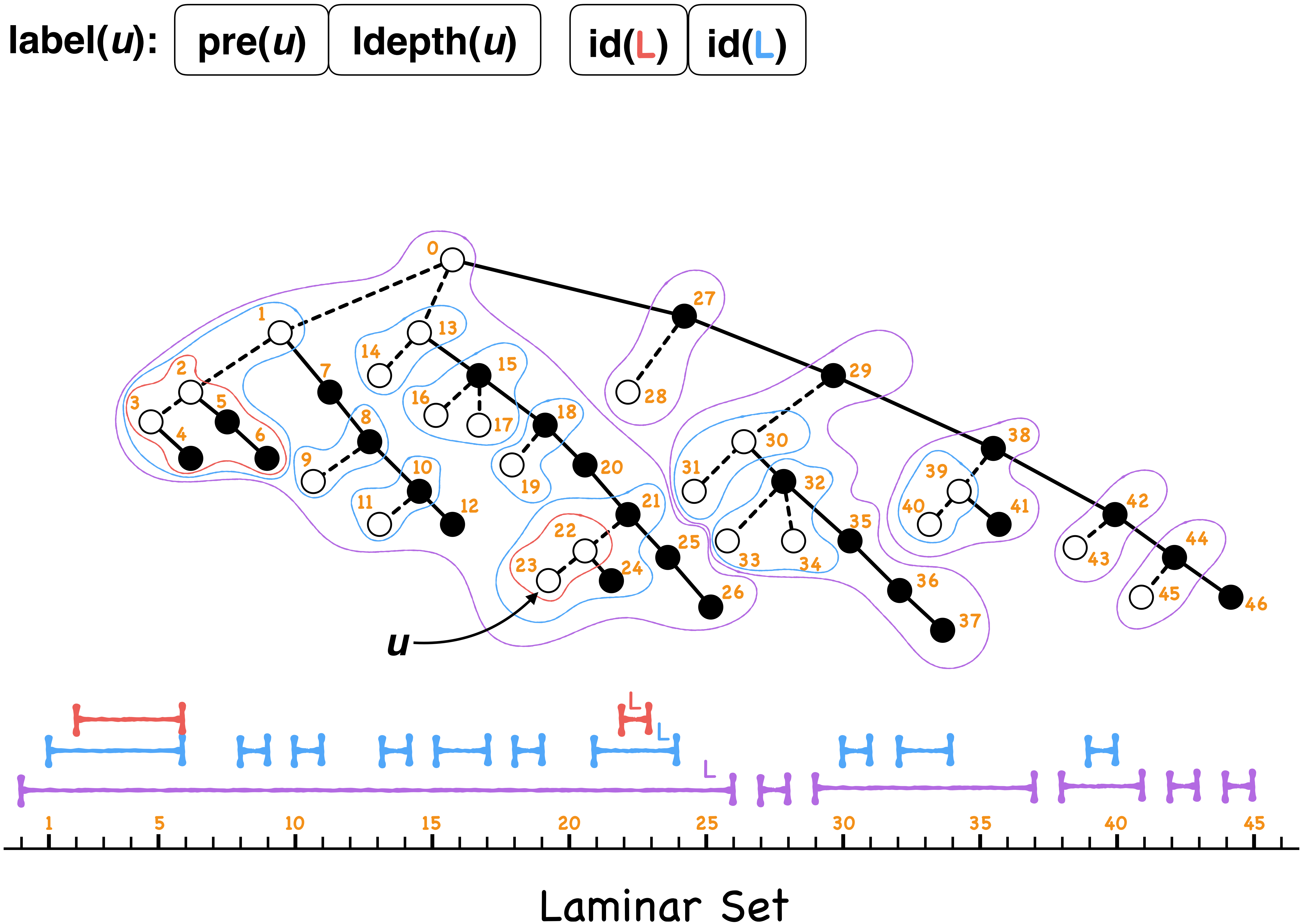

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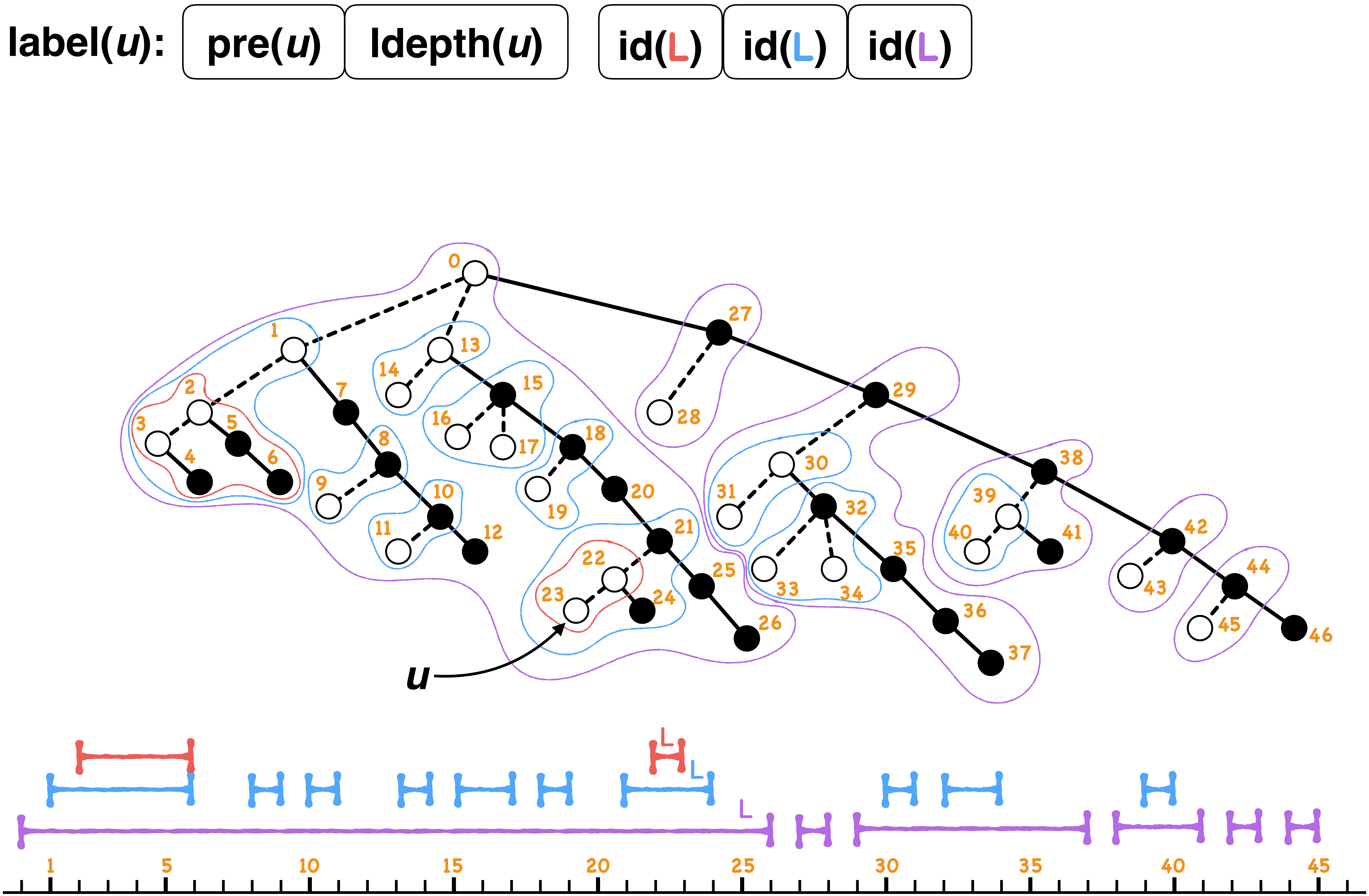

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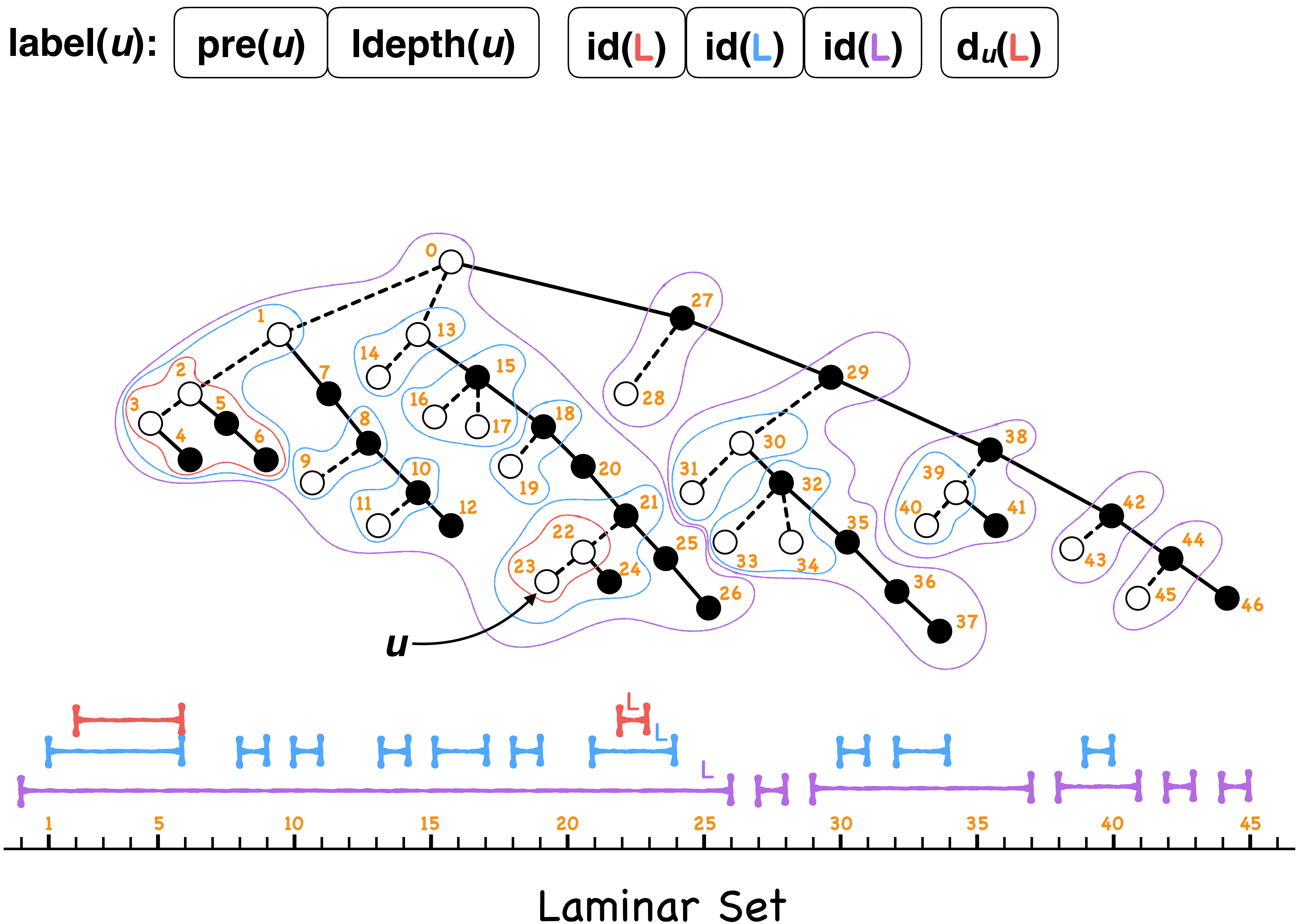
Laminar Set



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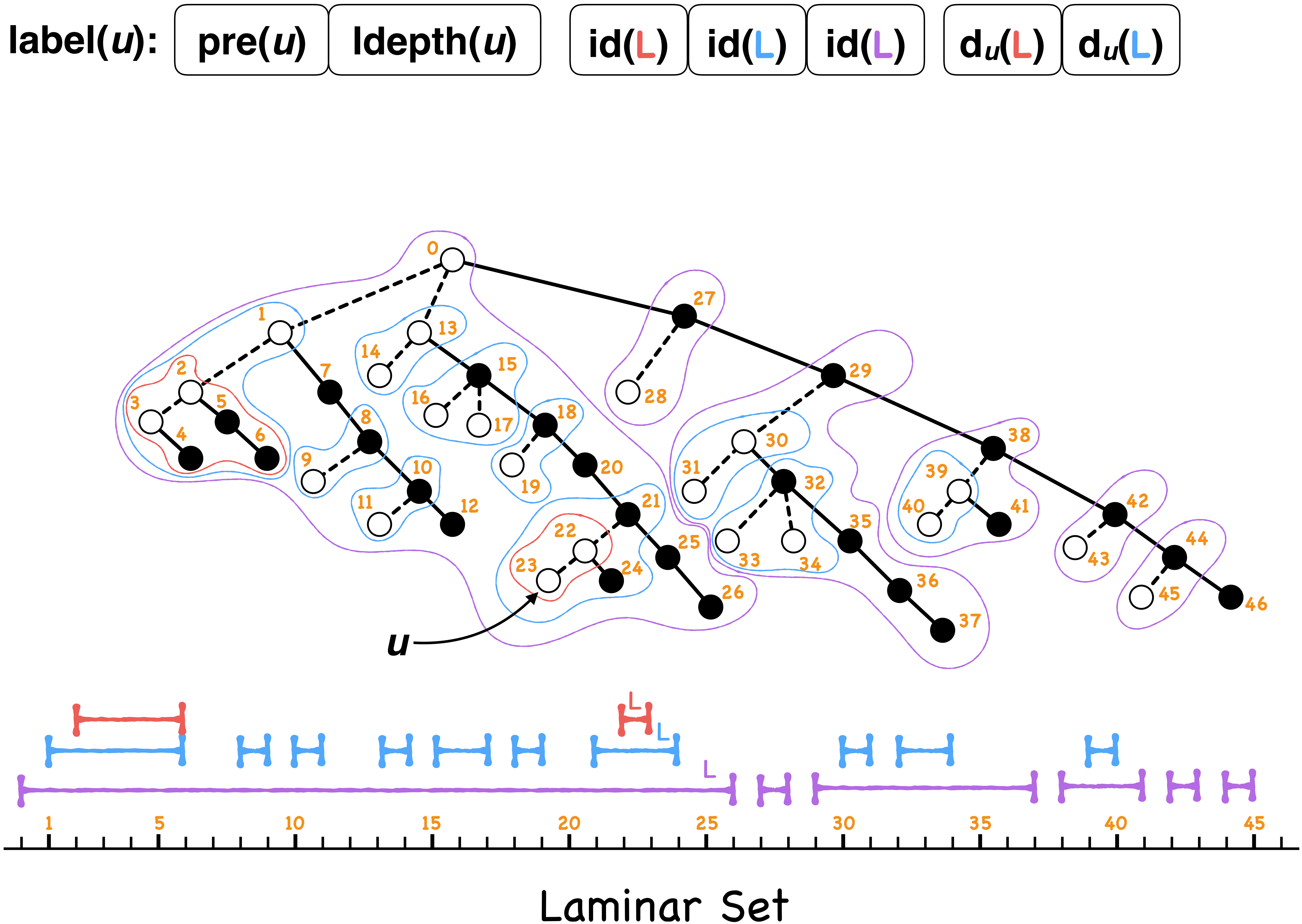

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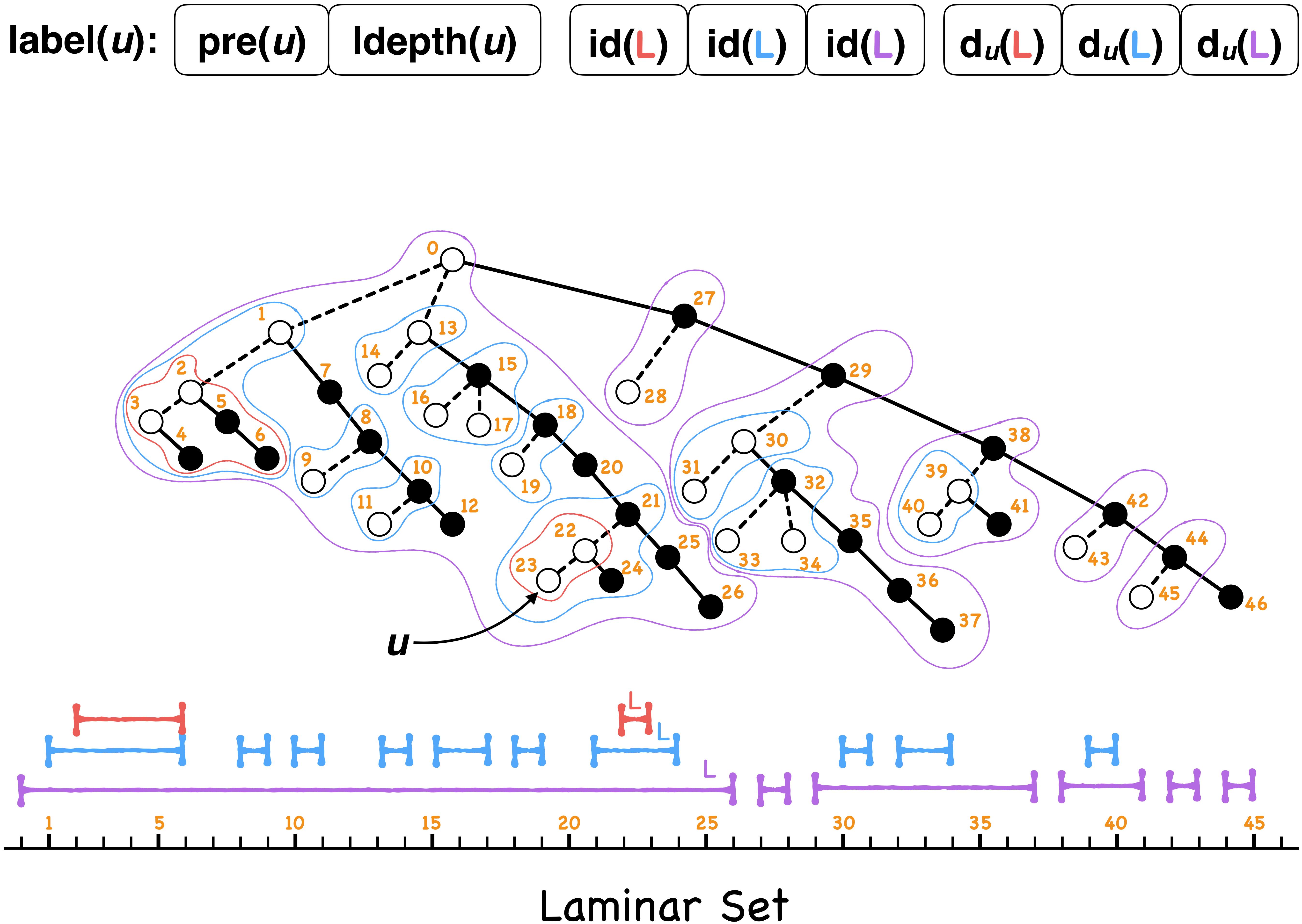
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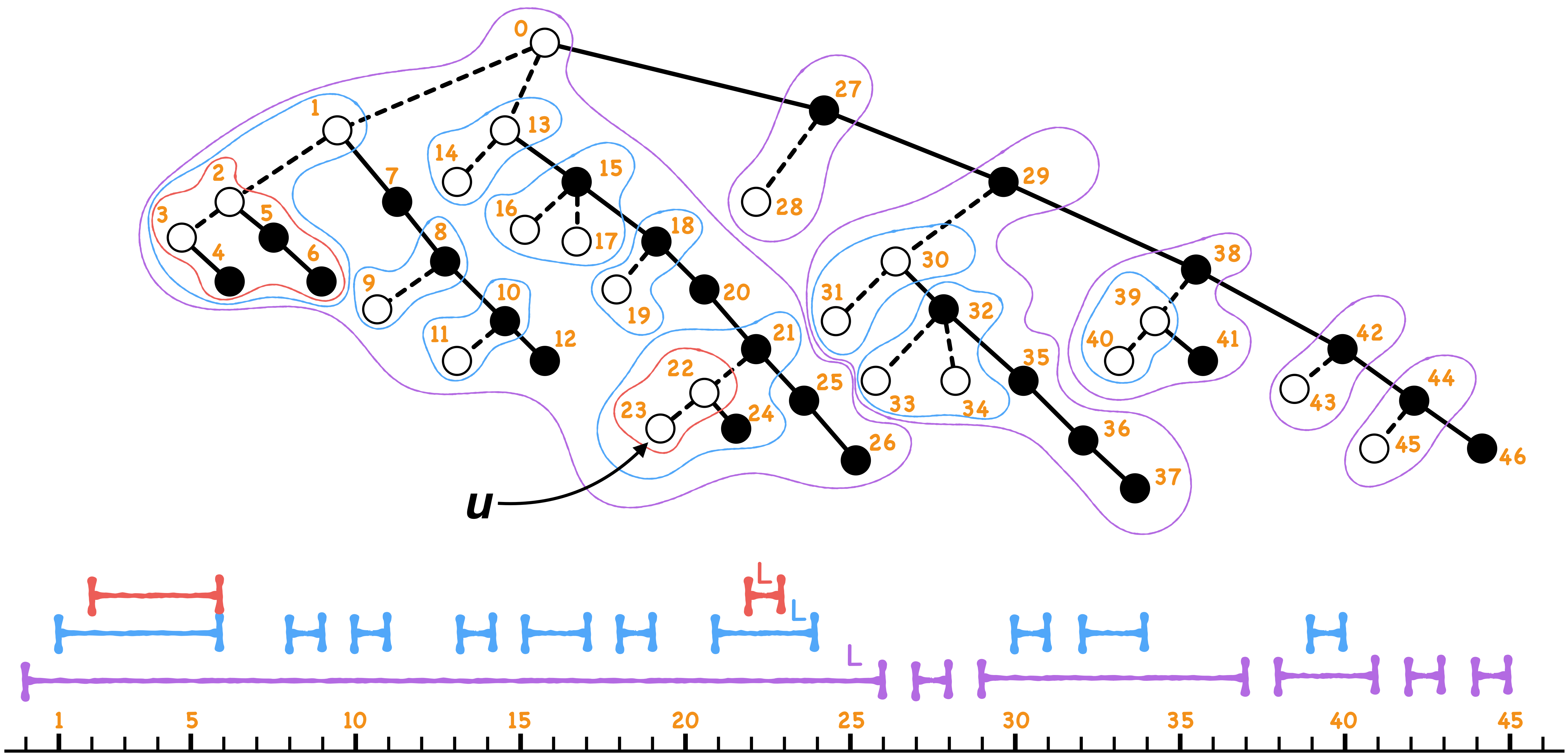

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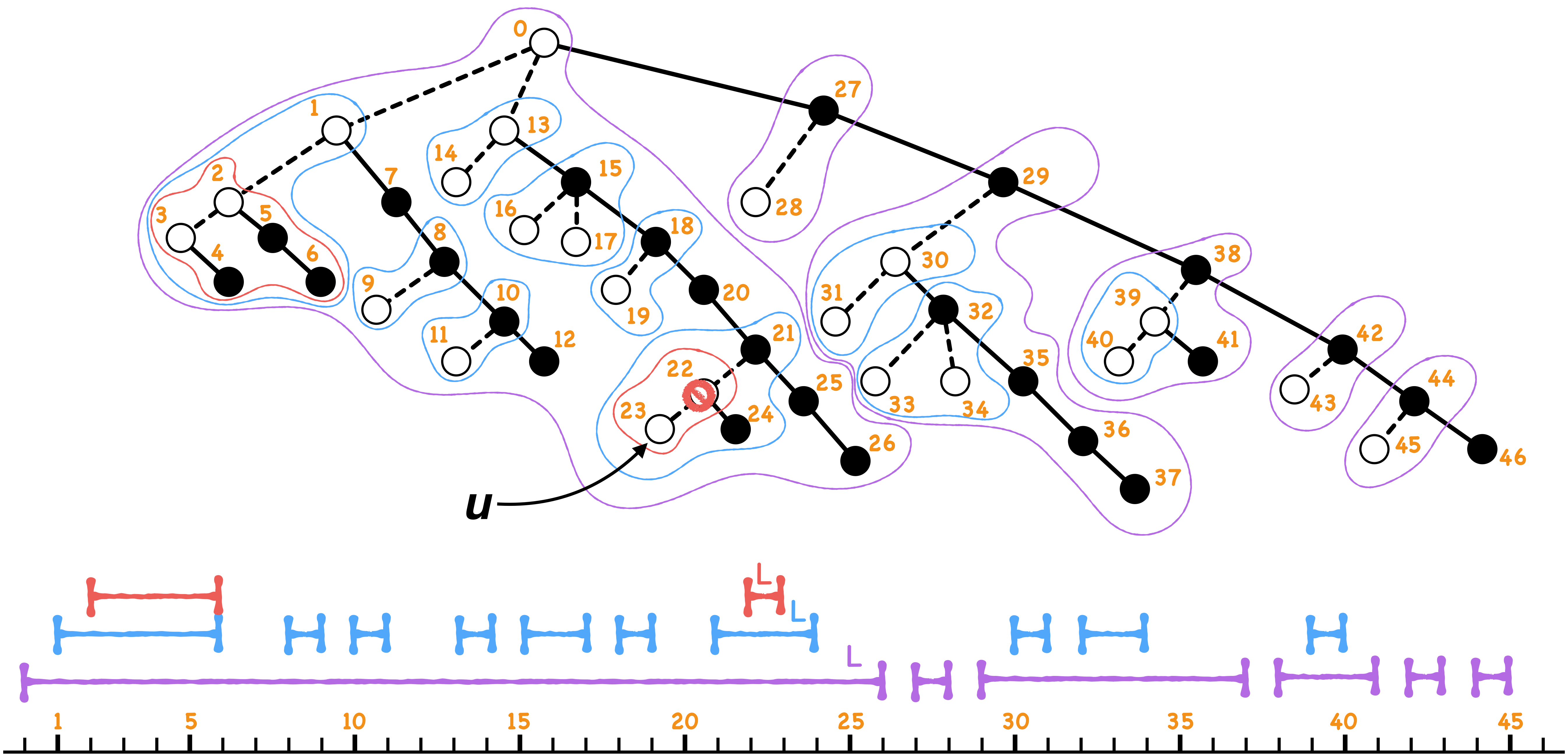

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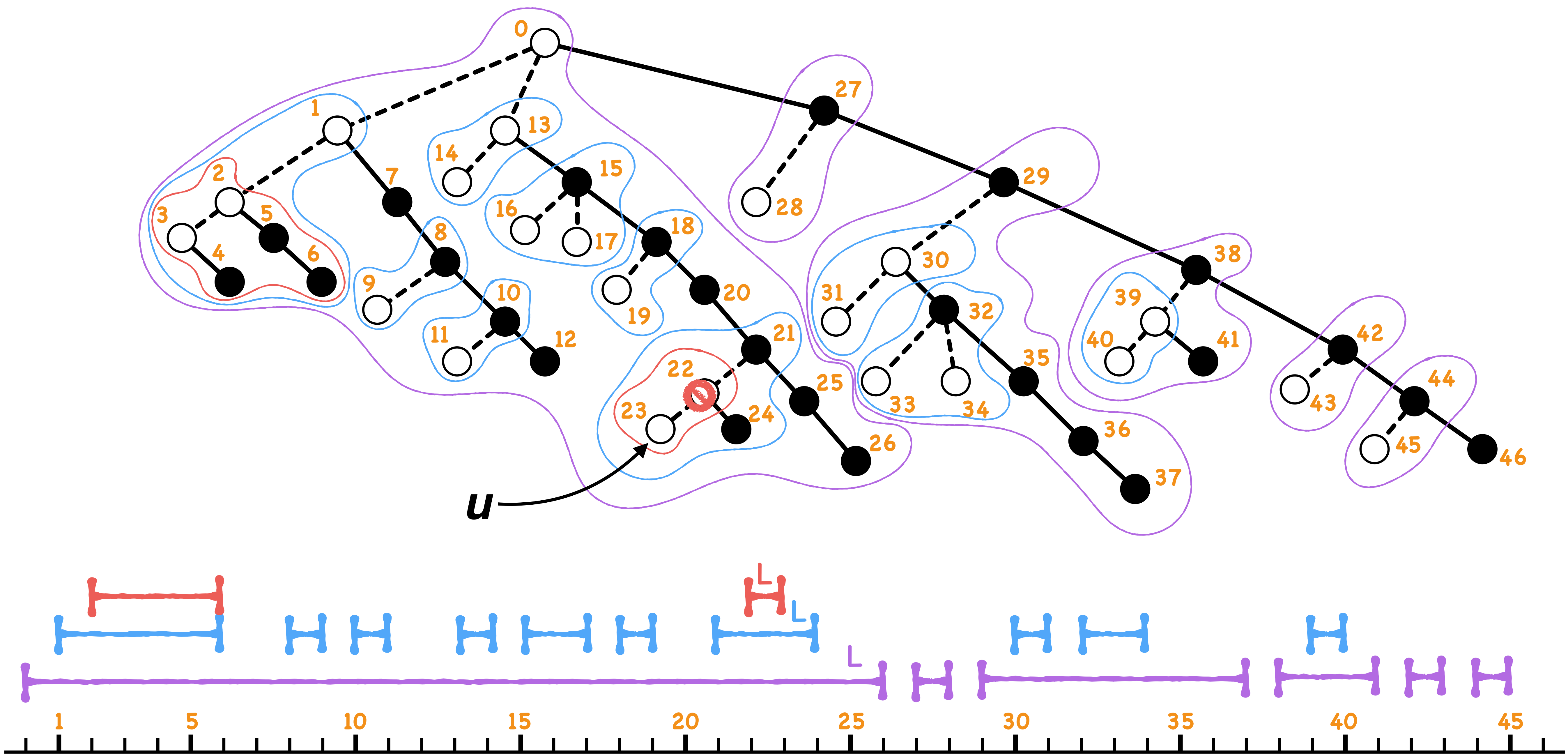

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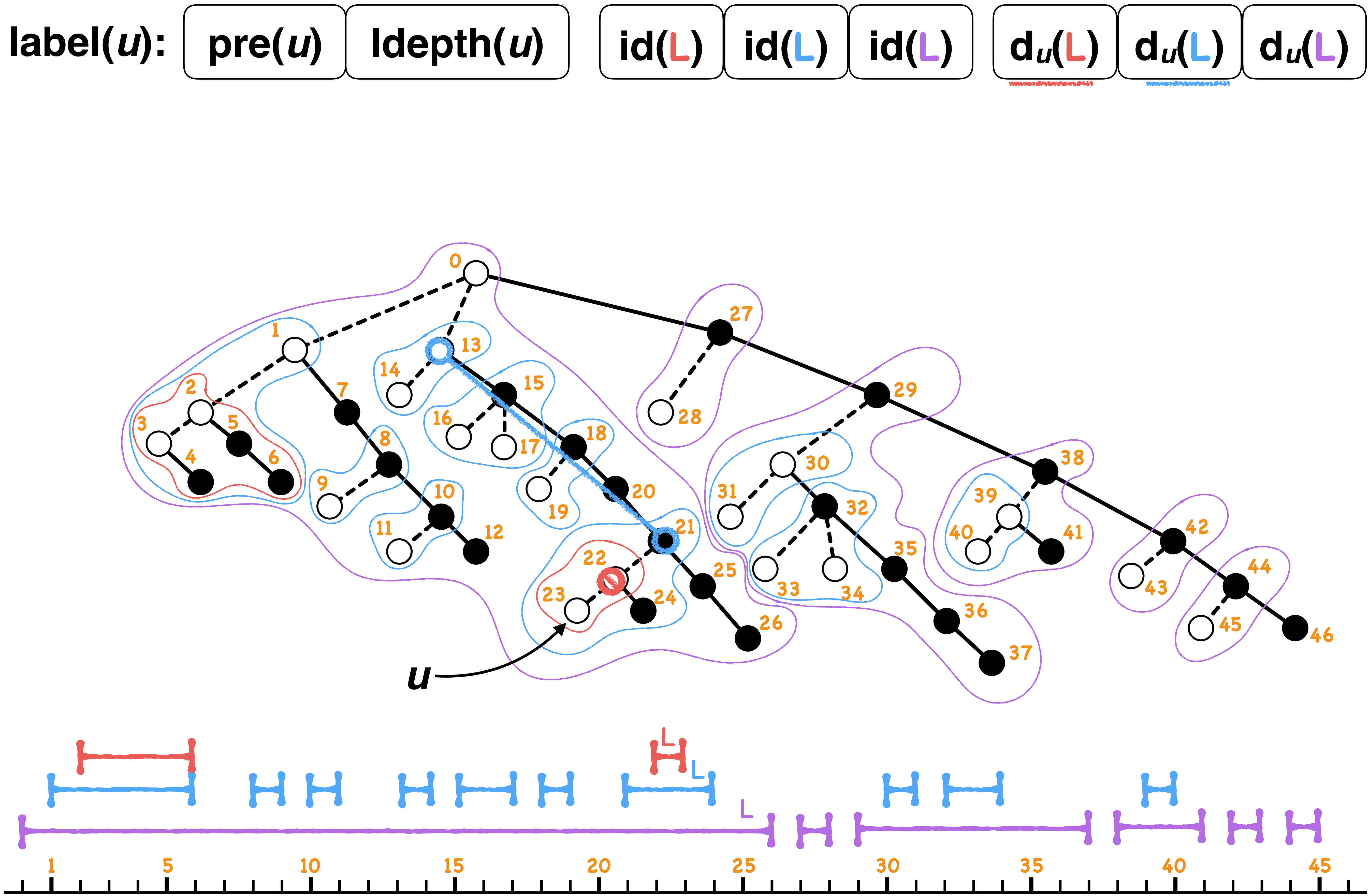
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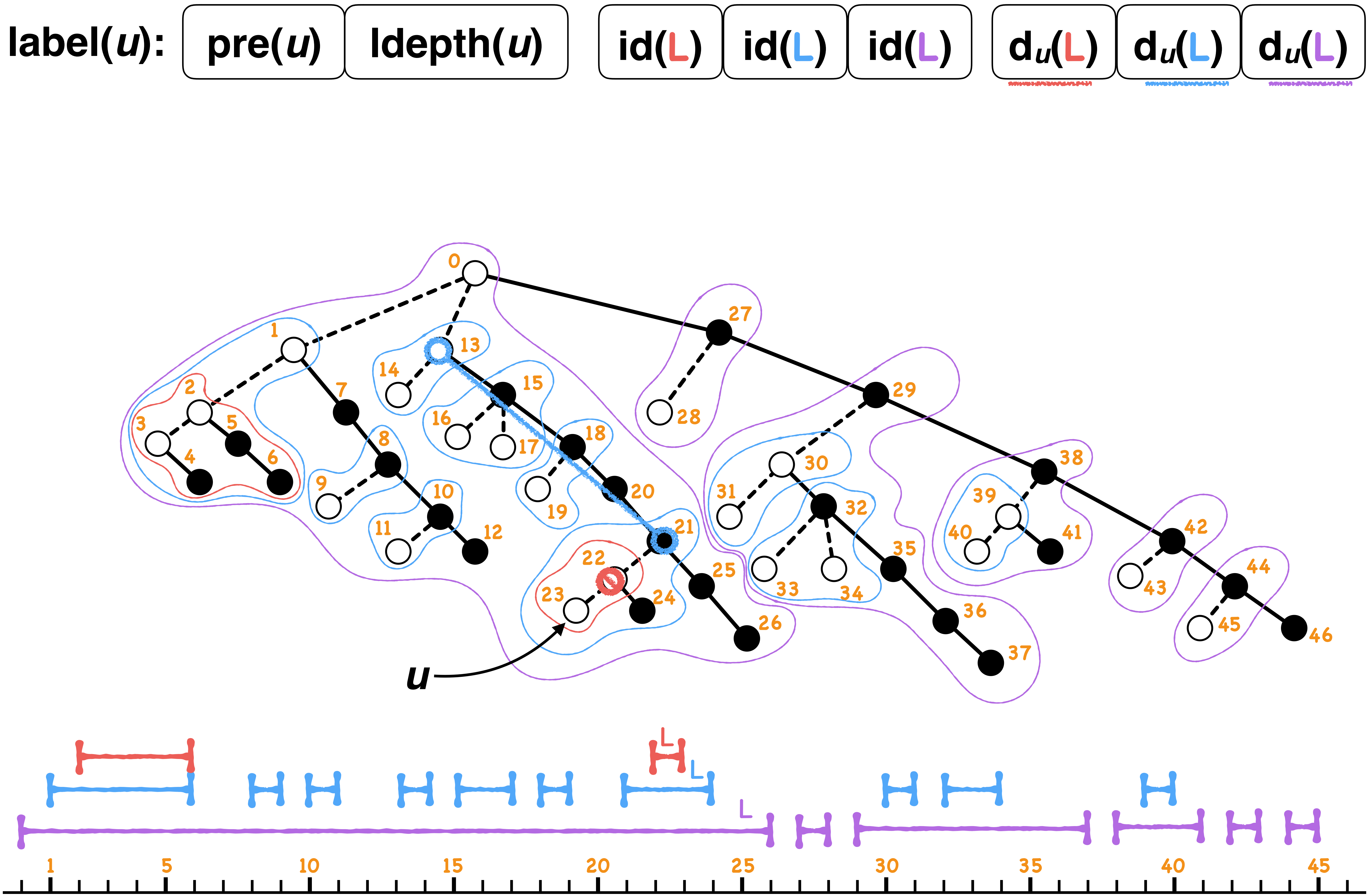

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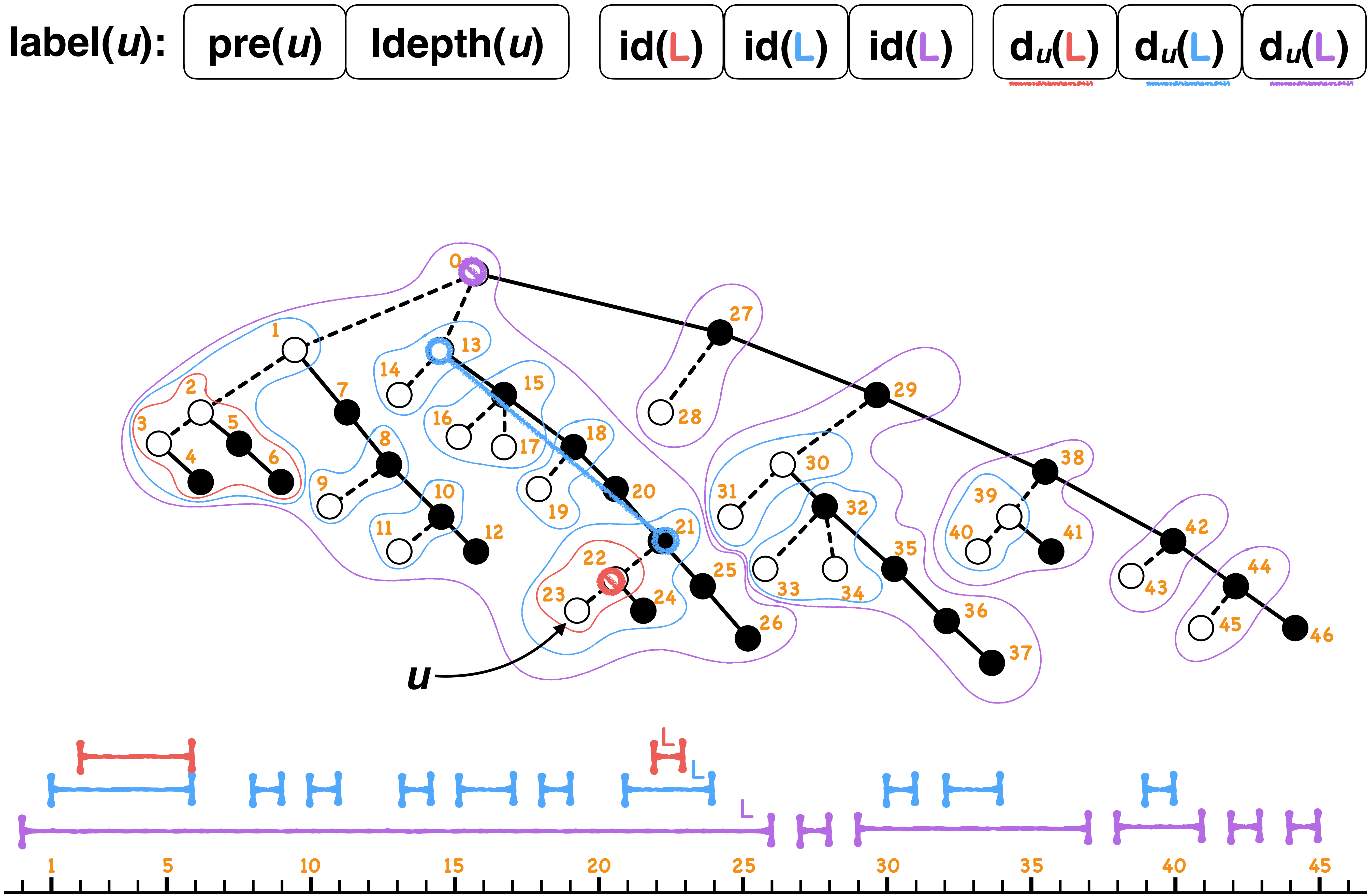
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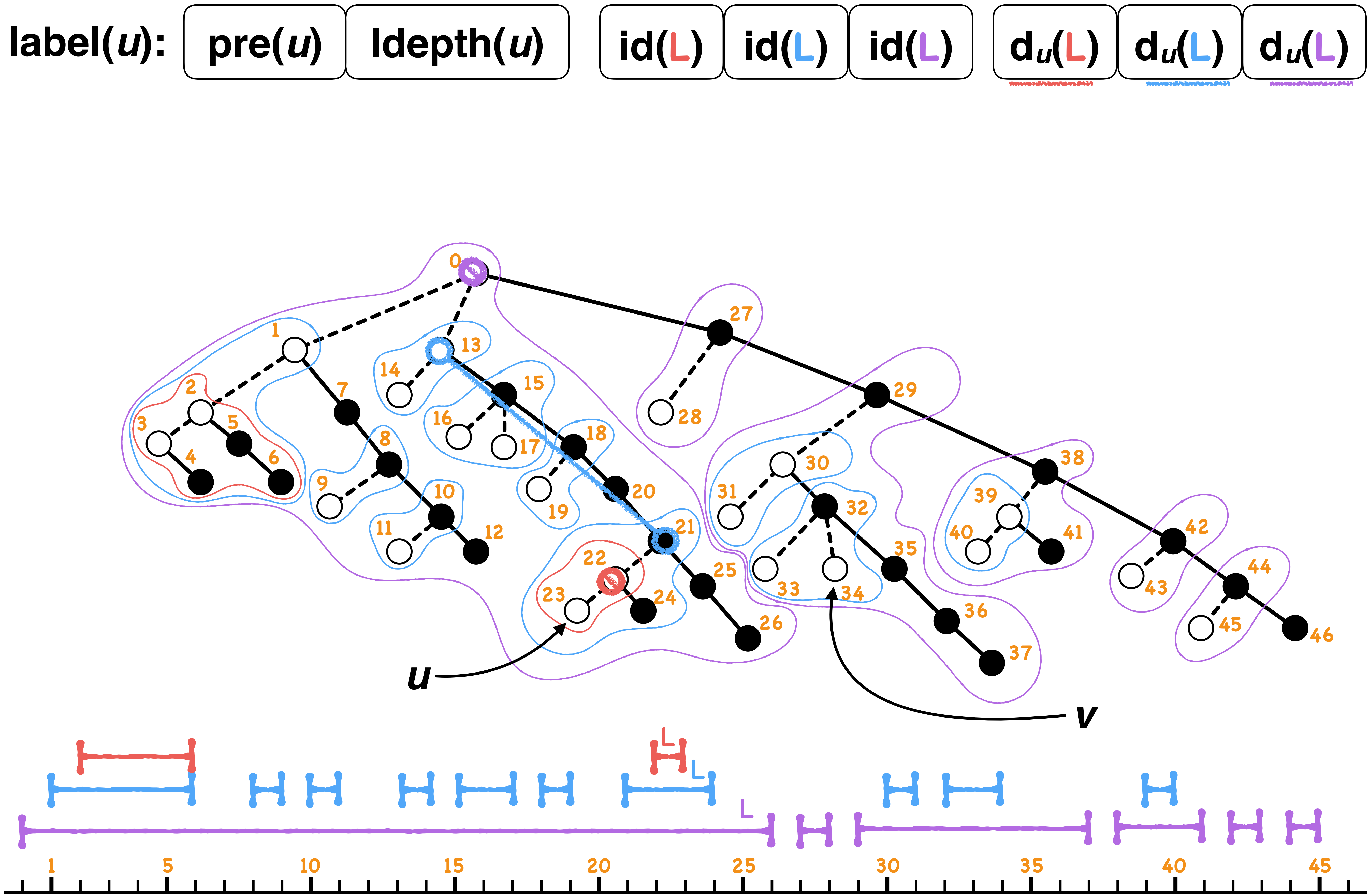
Laminar Set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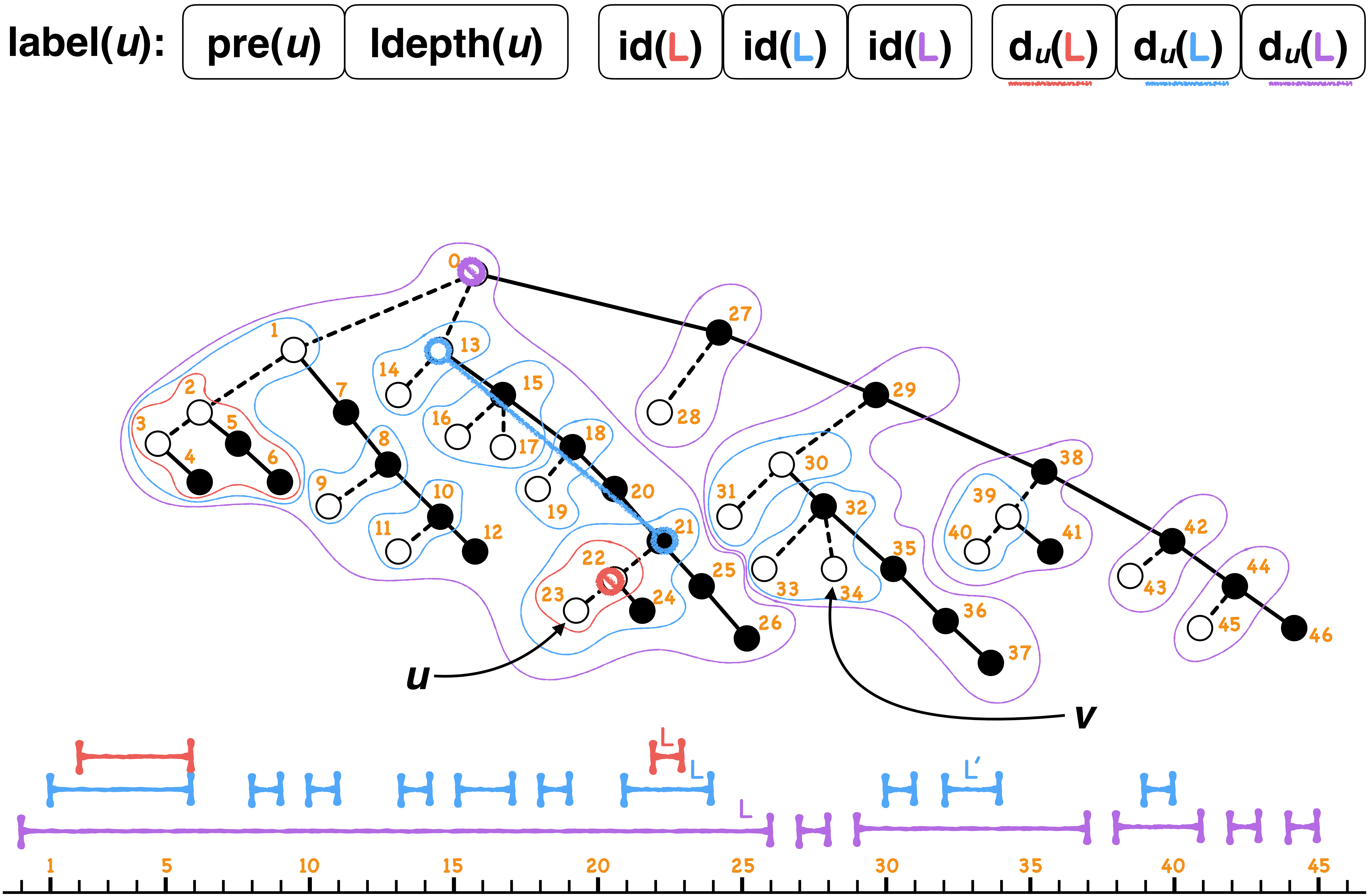

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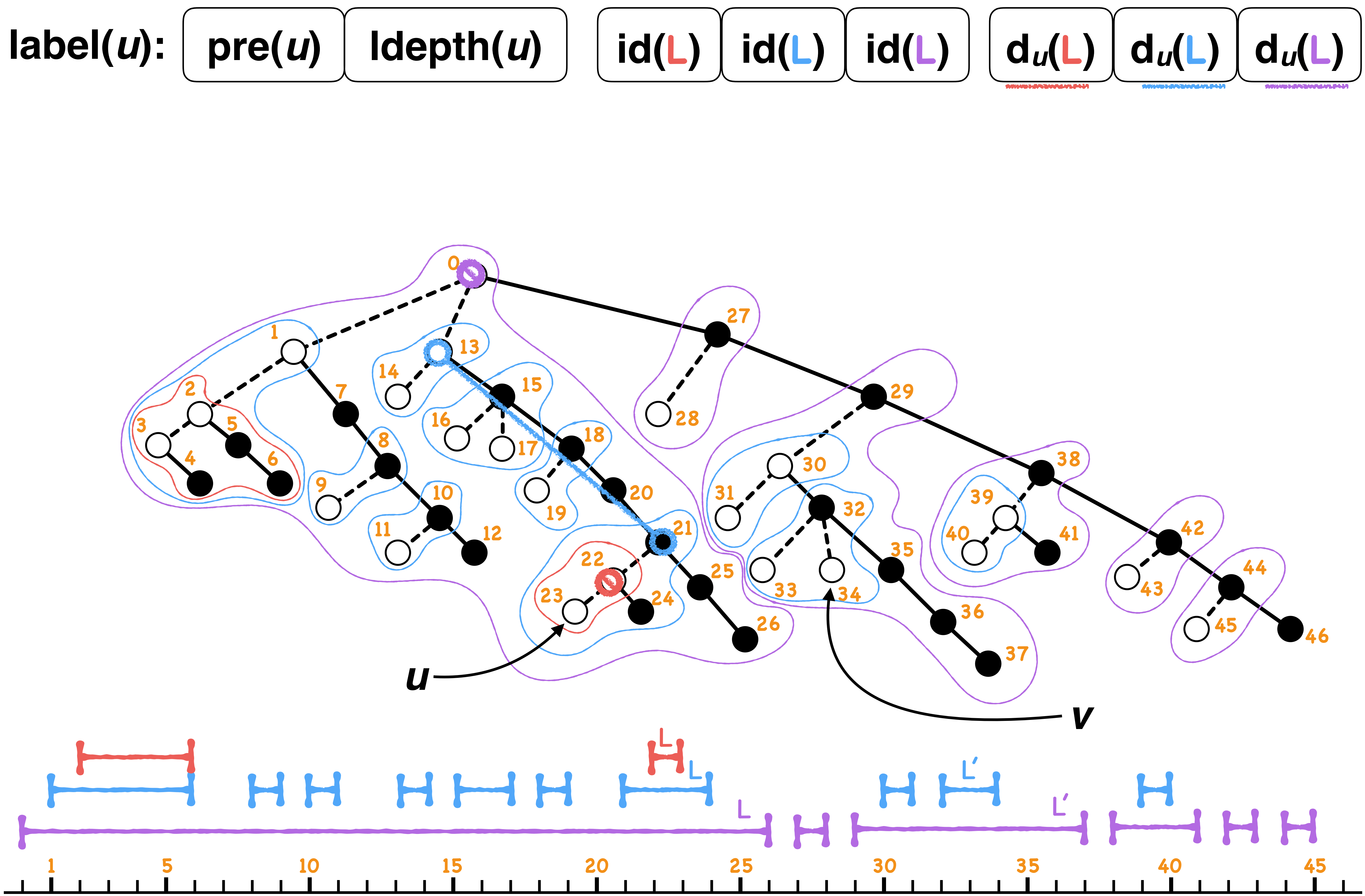
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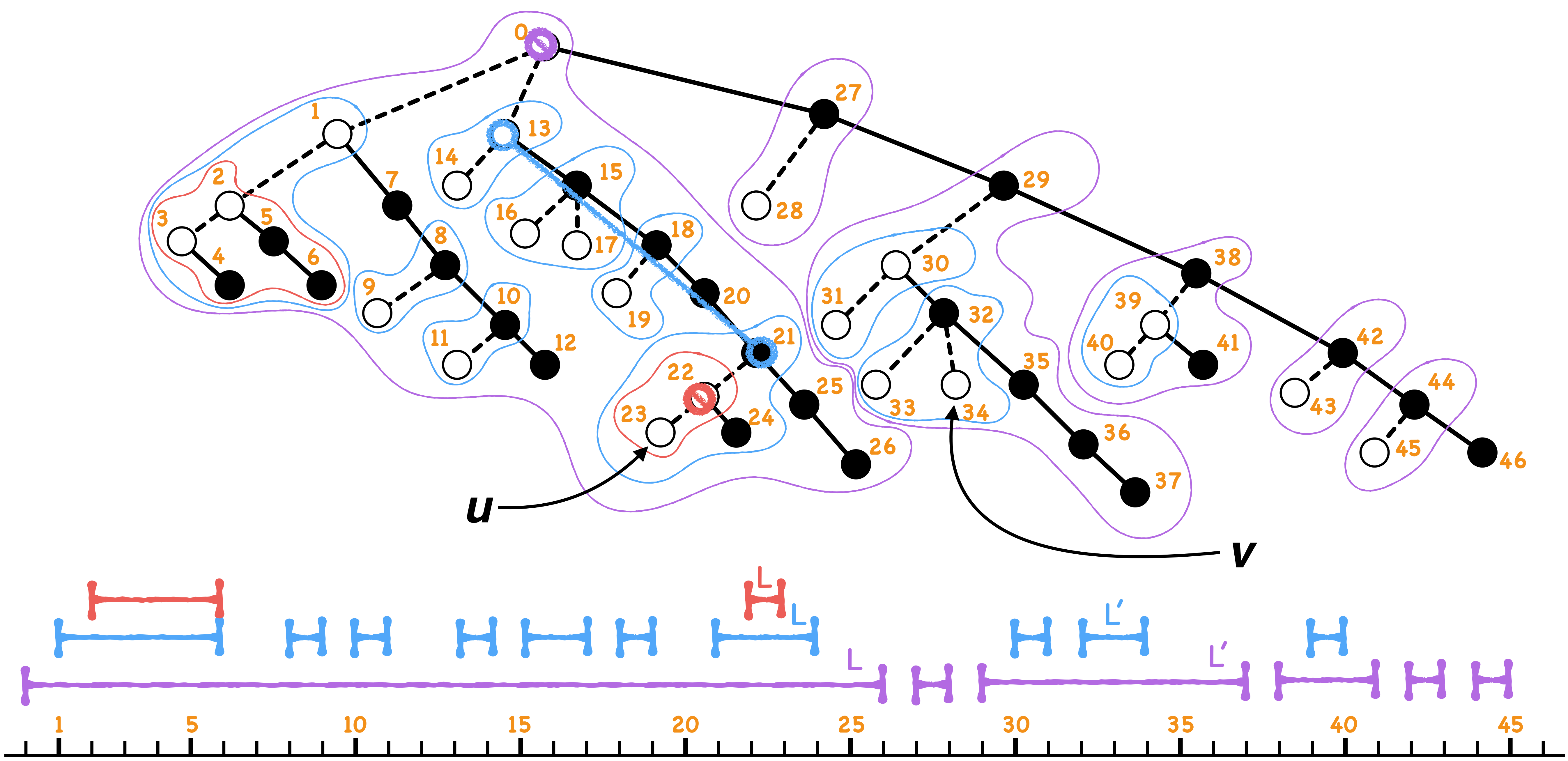

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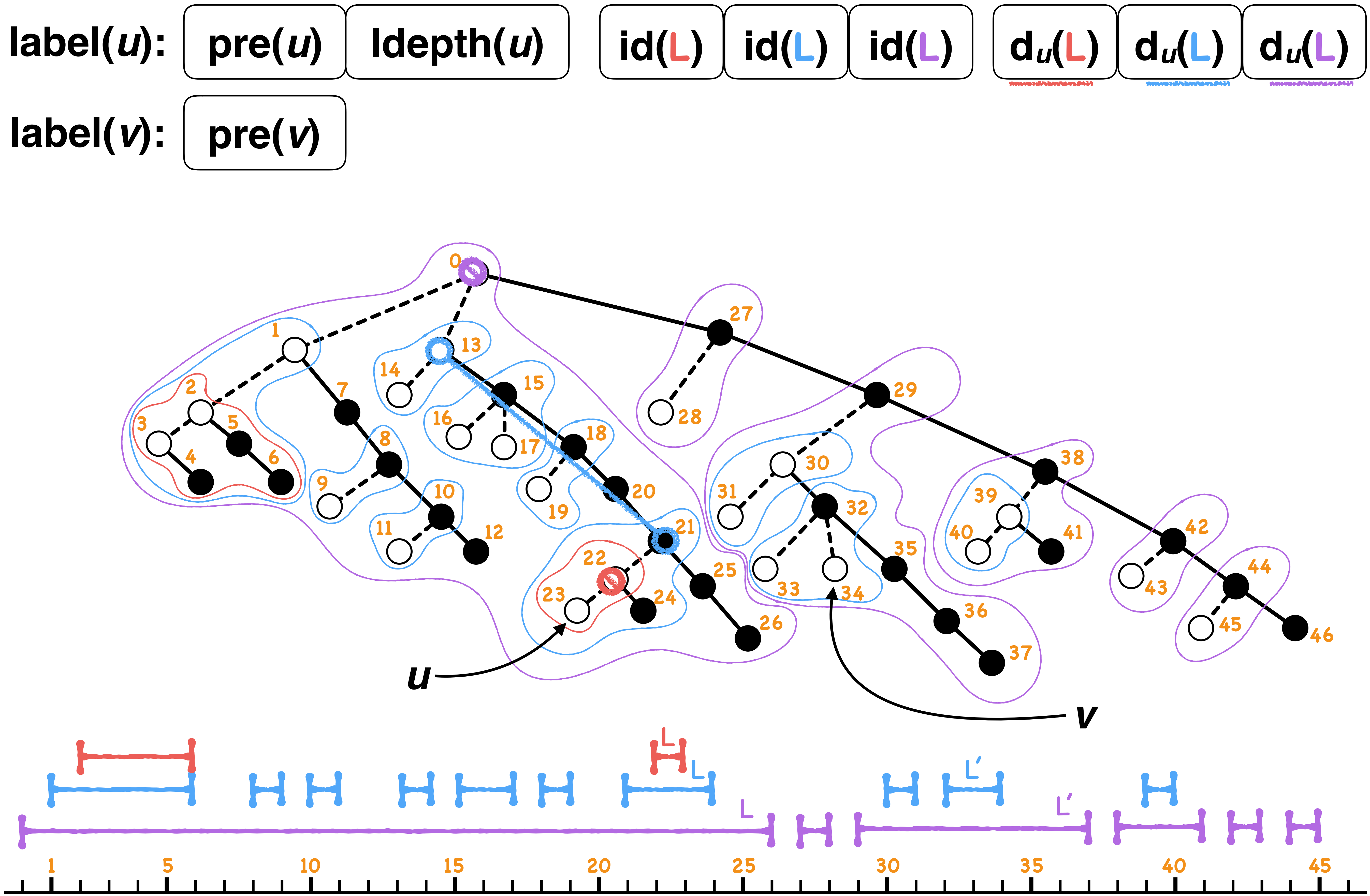

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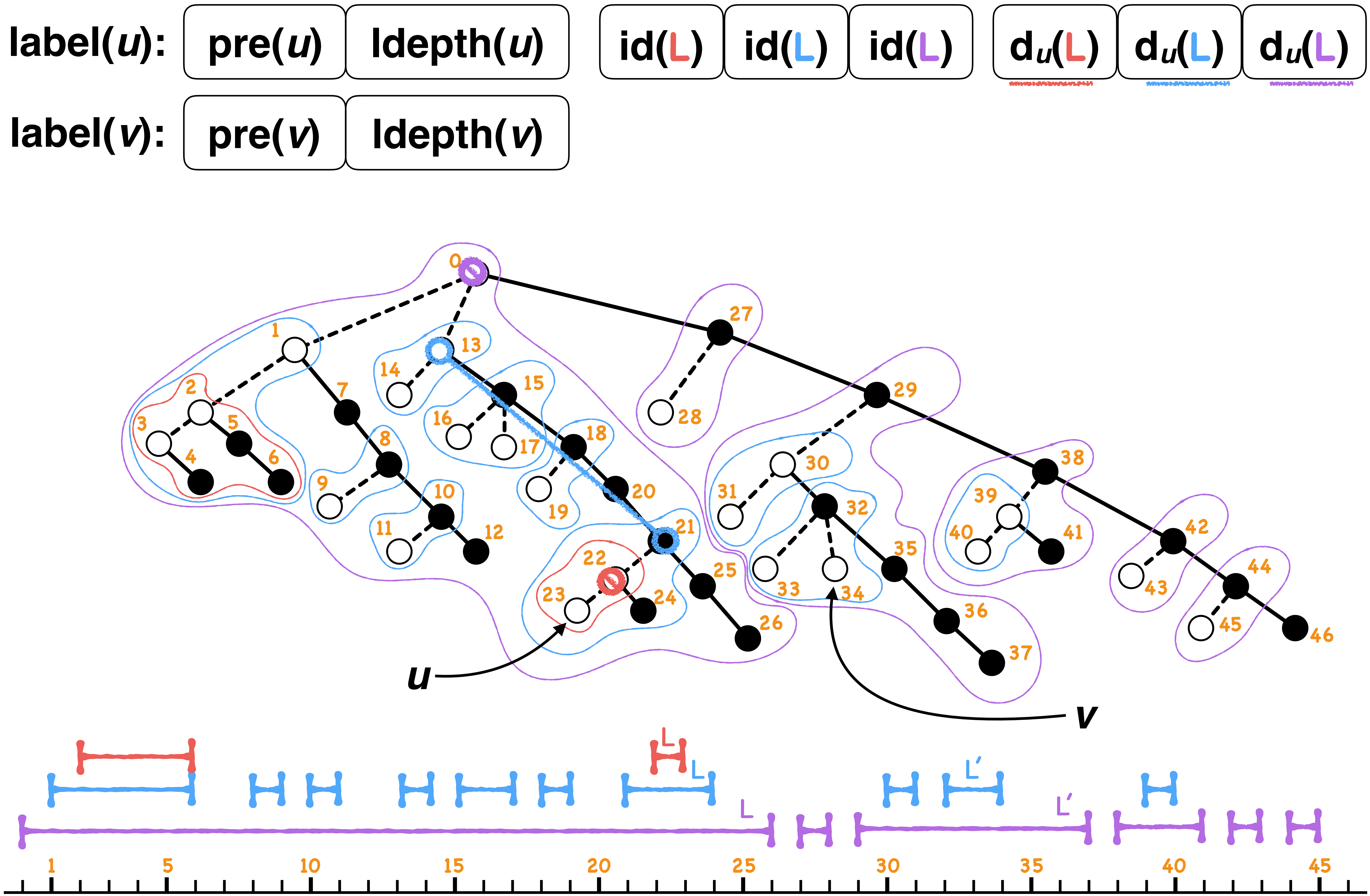

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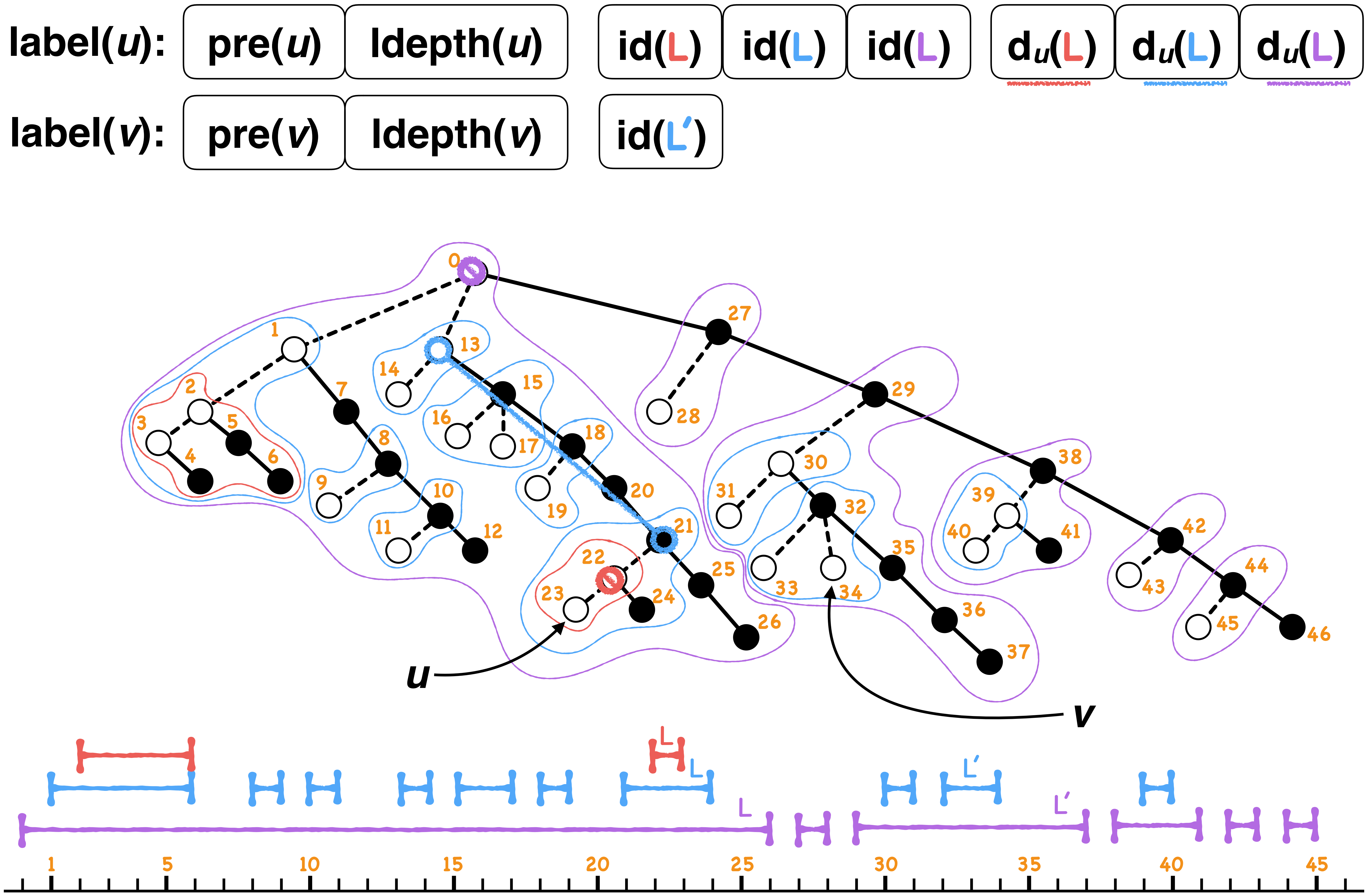

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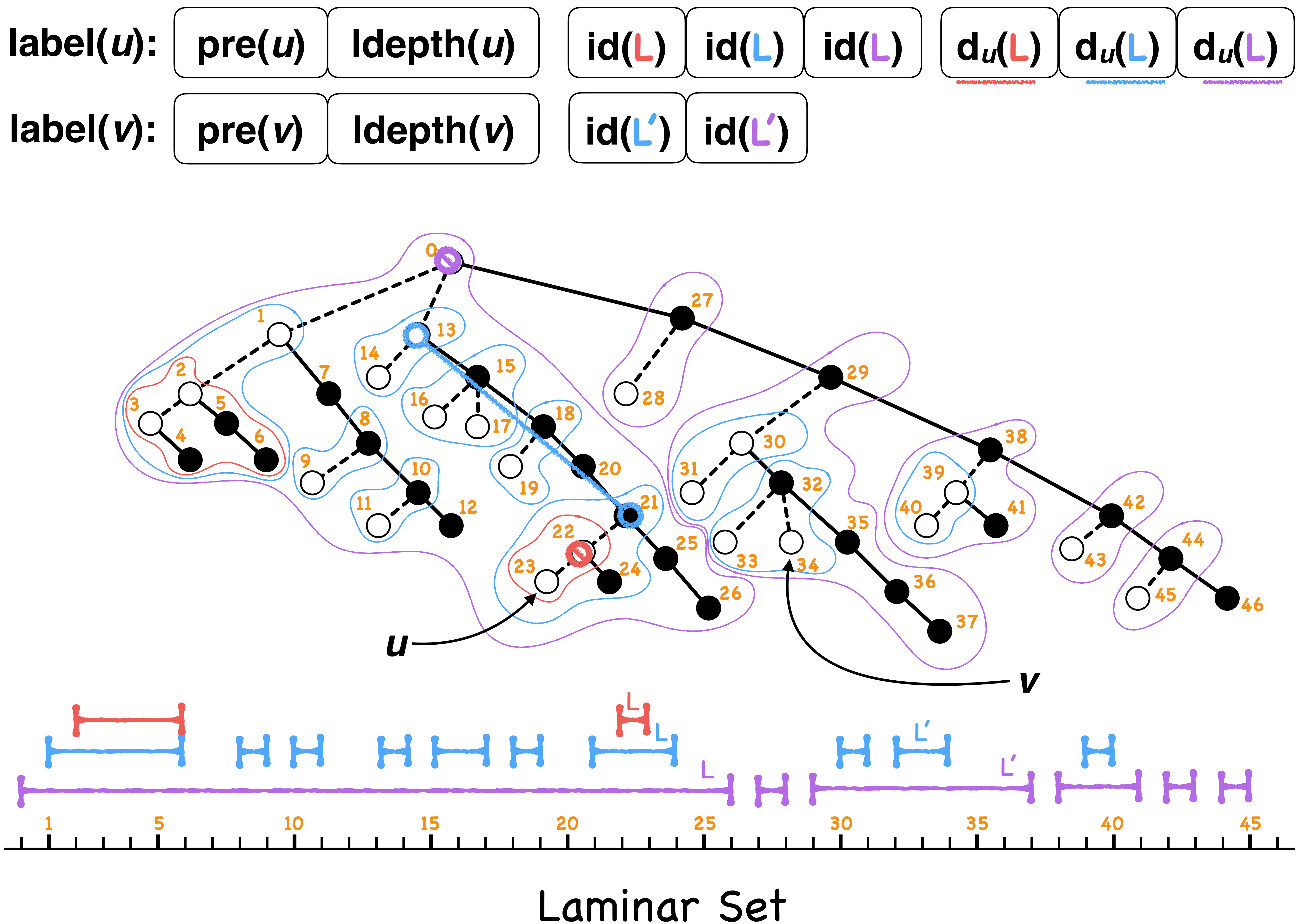

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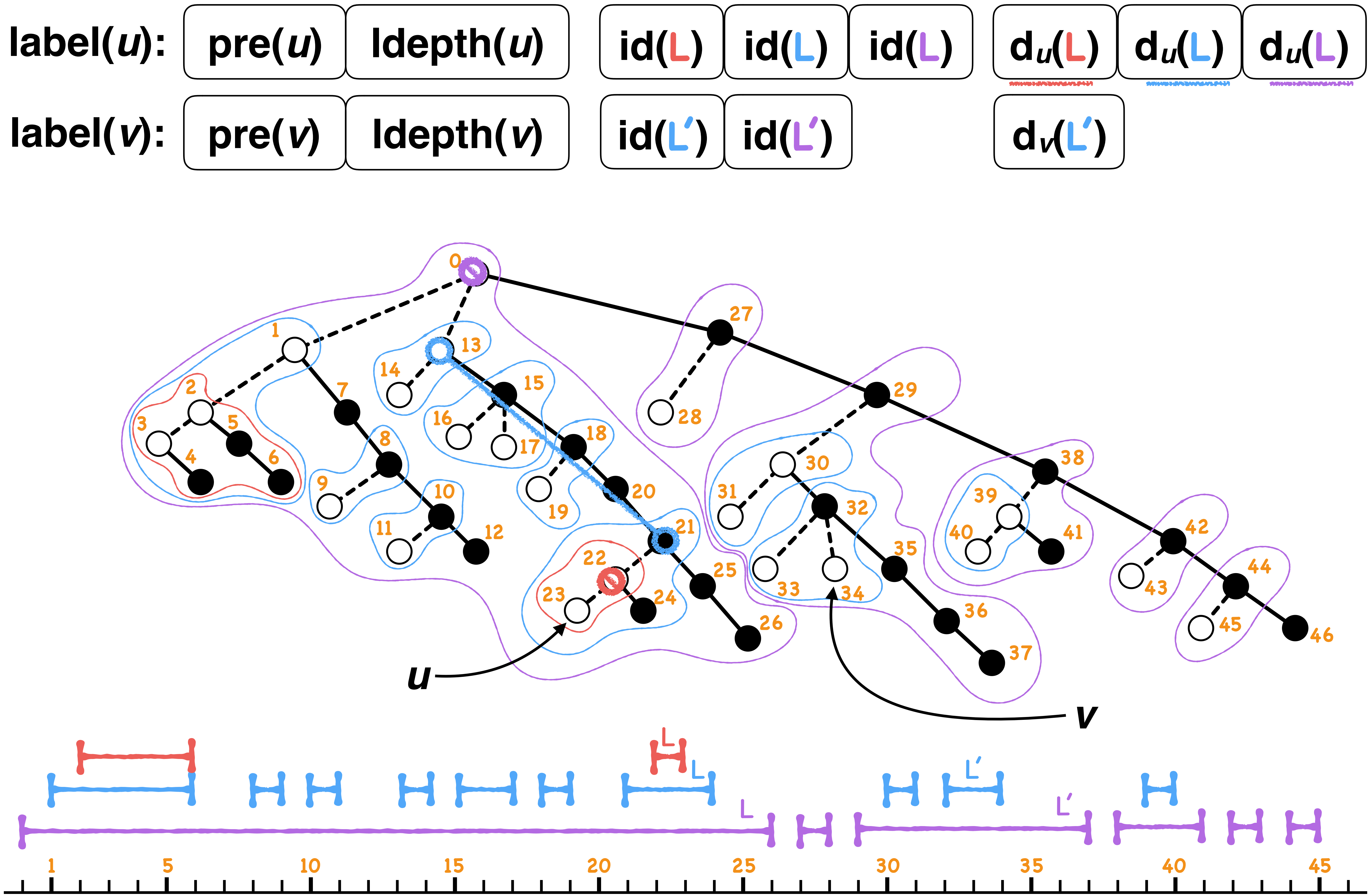
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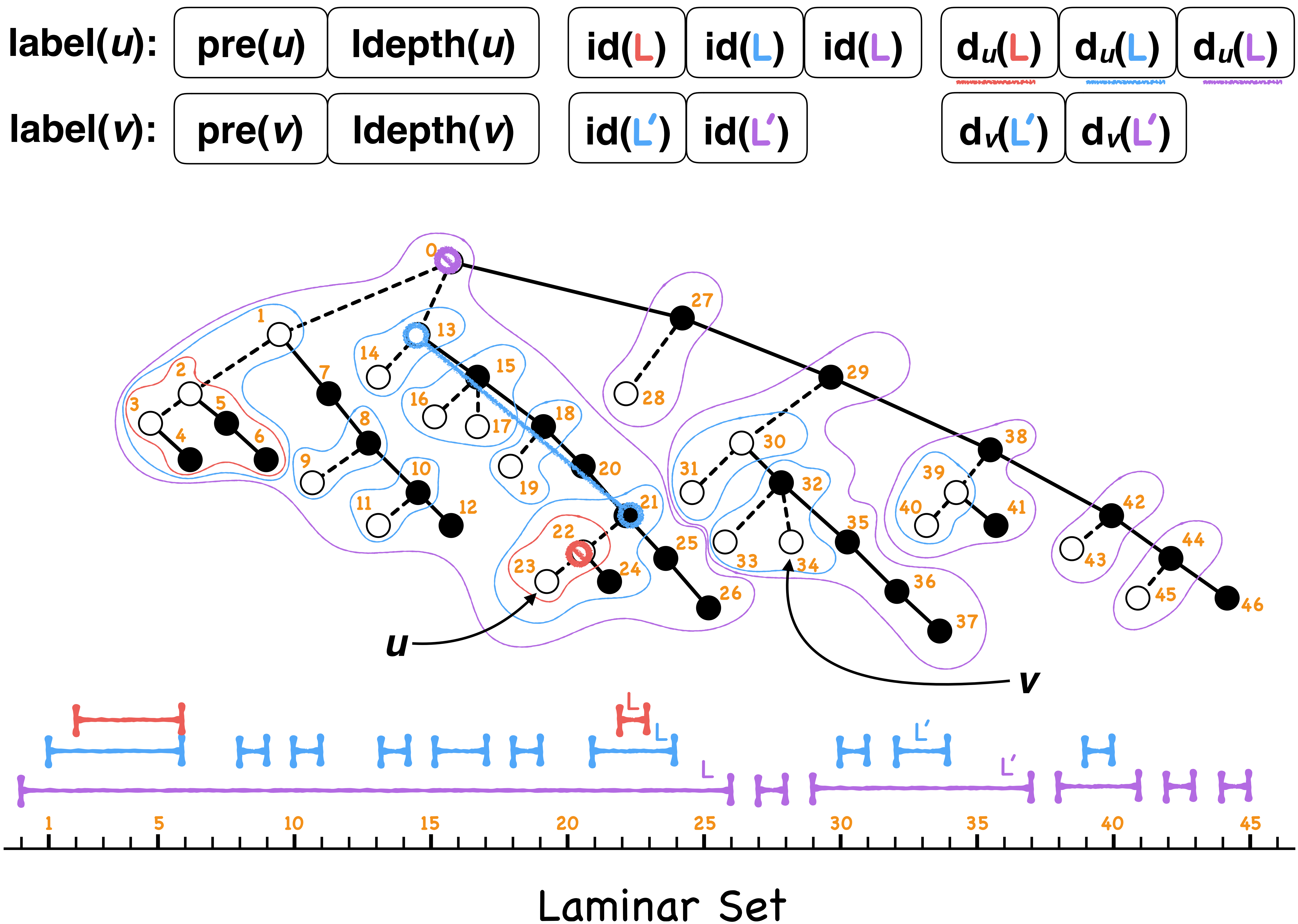
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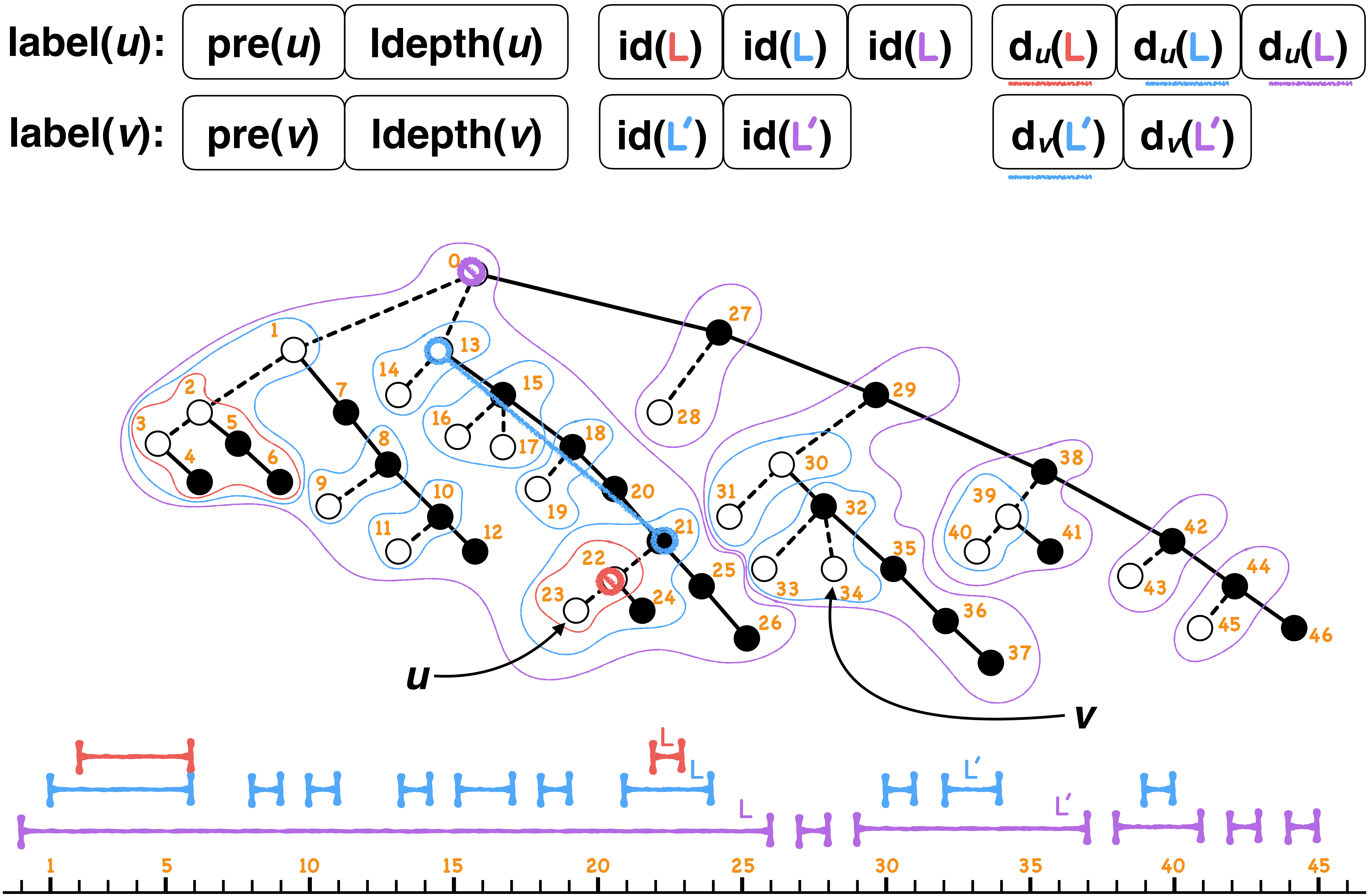

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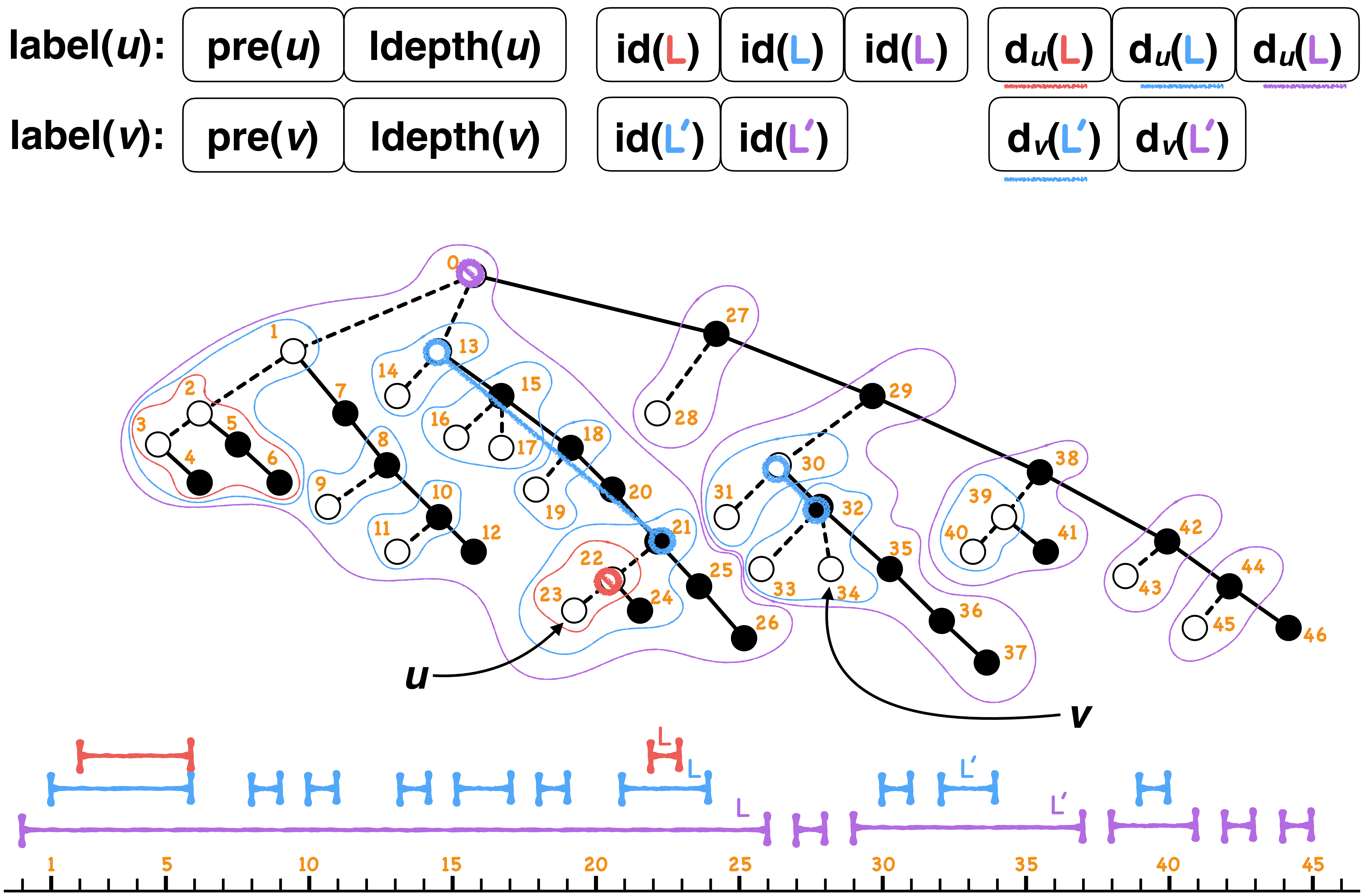

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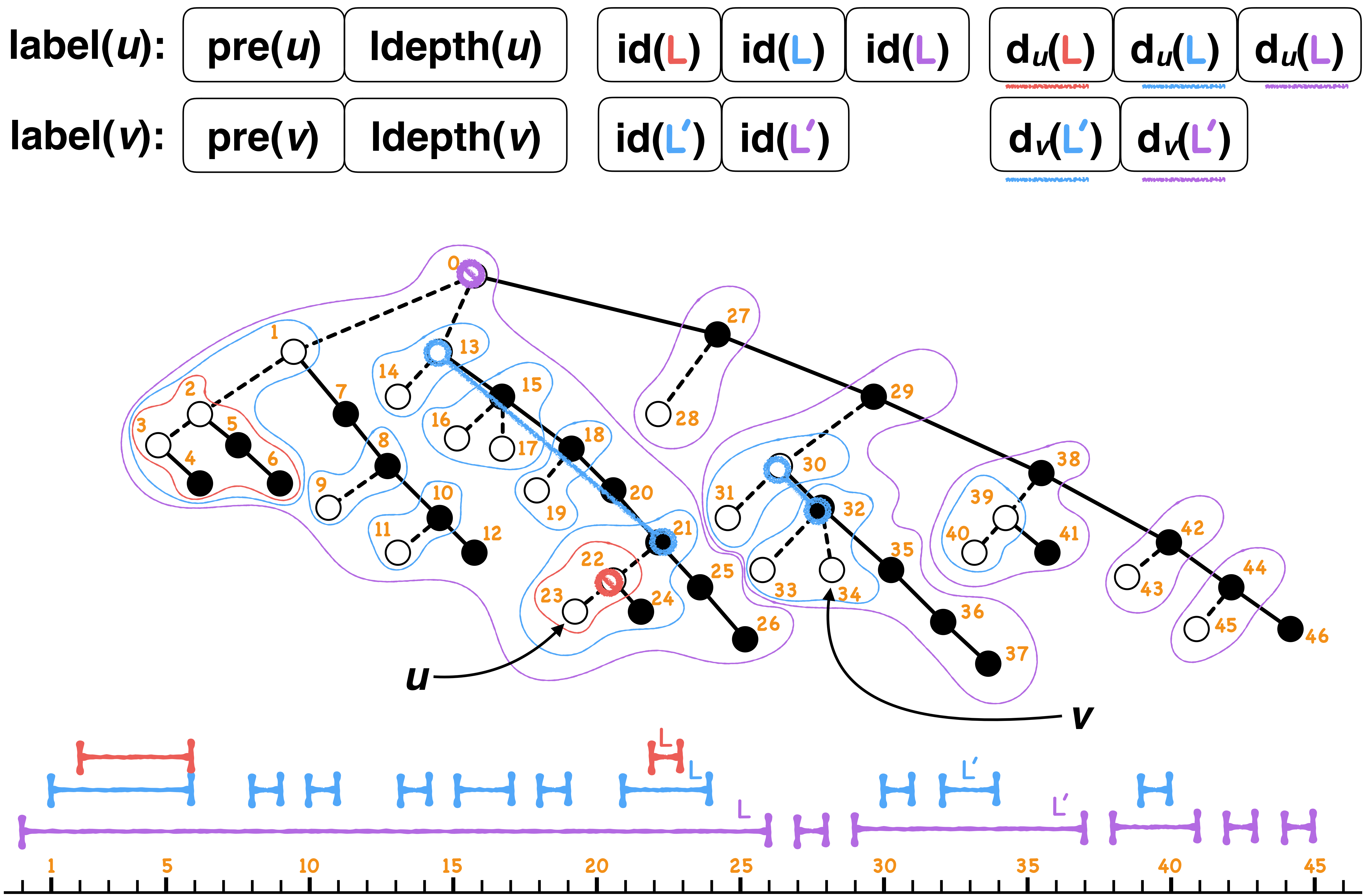
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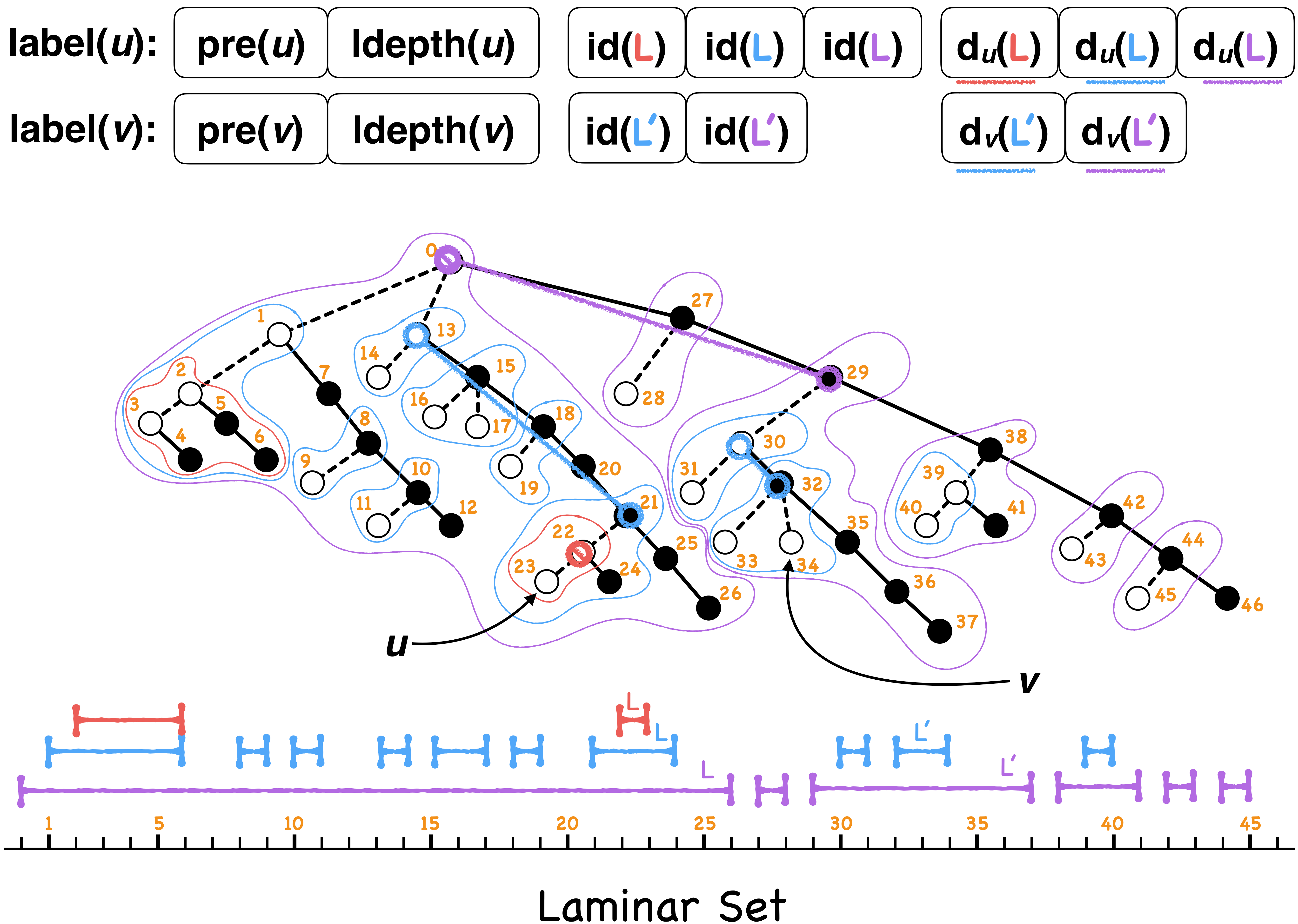

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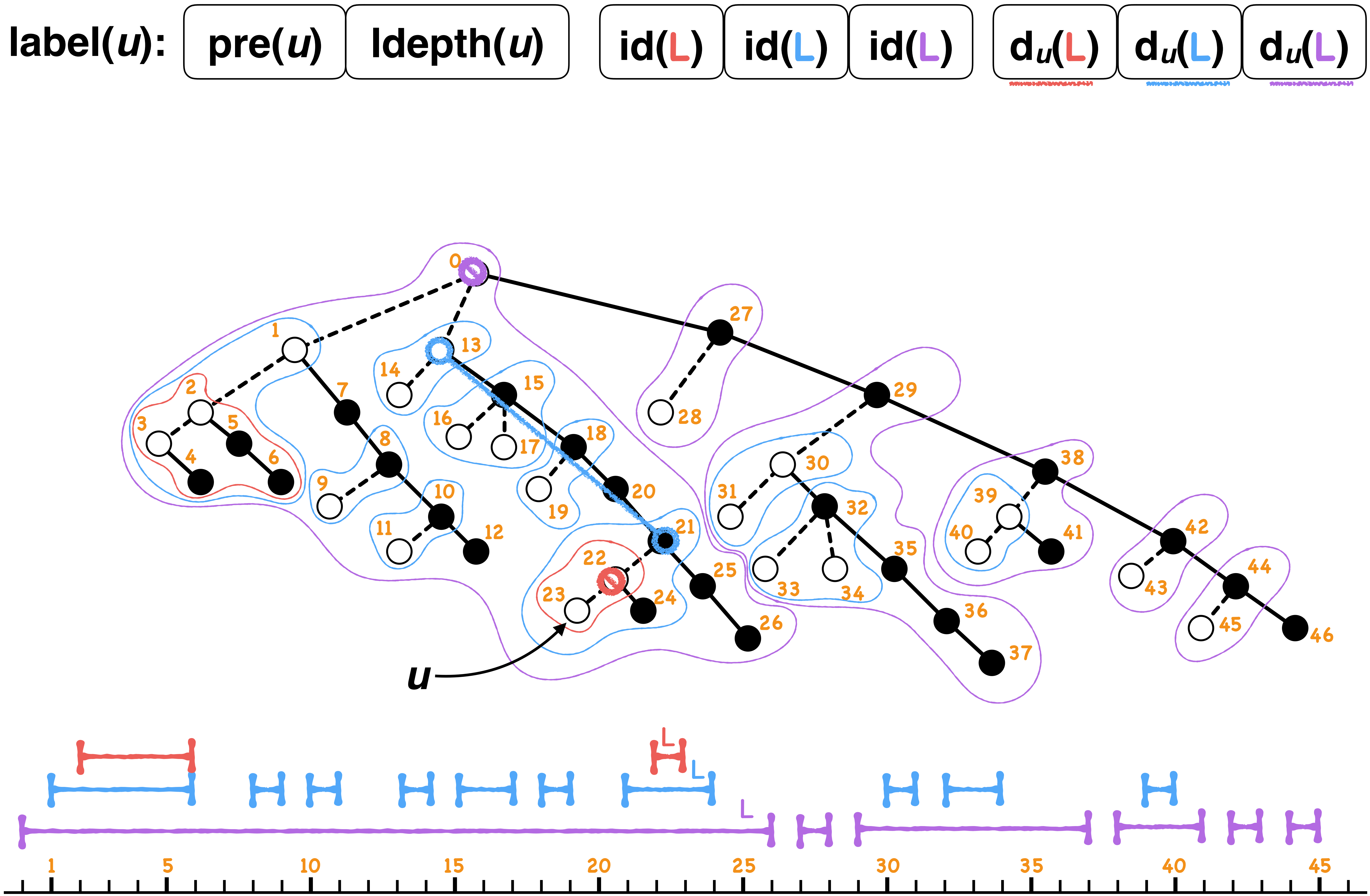
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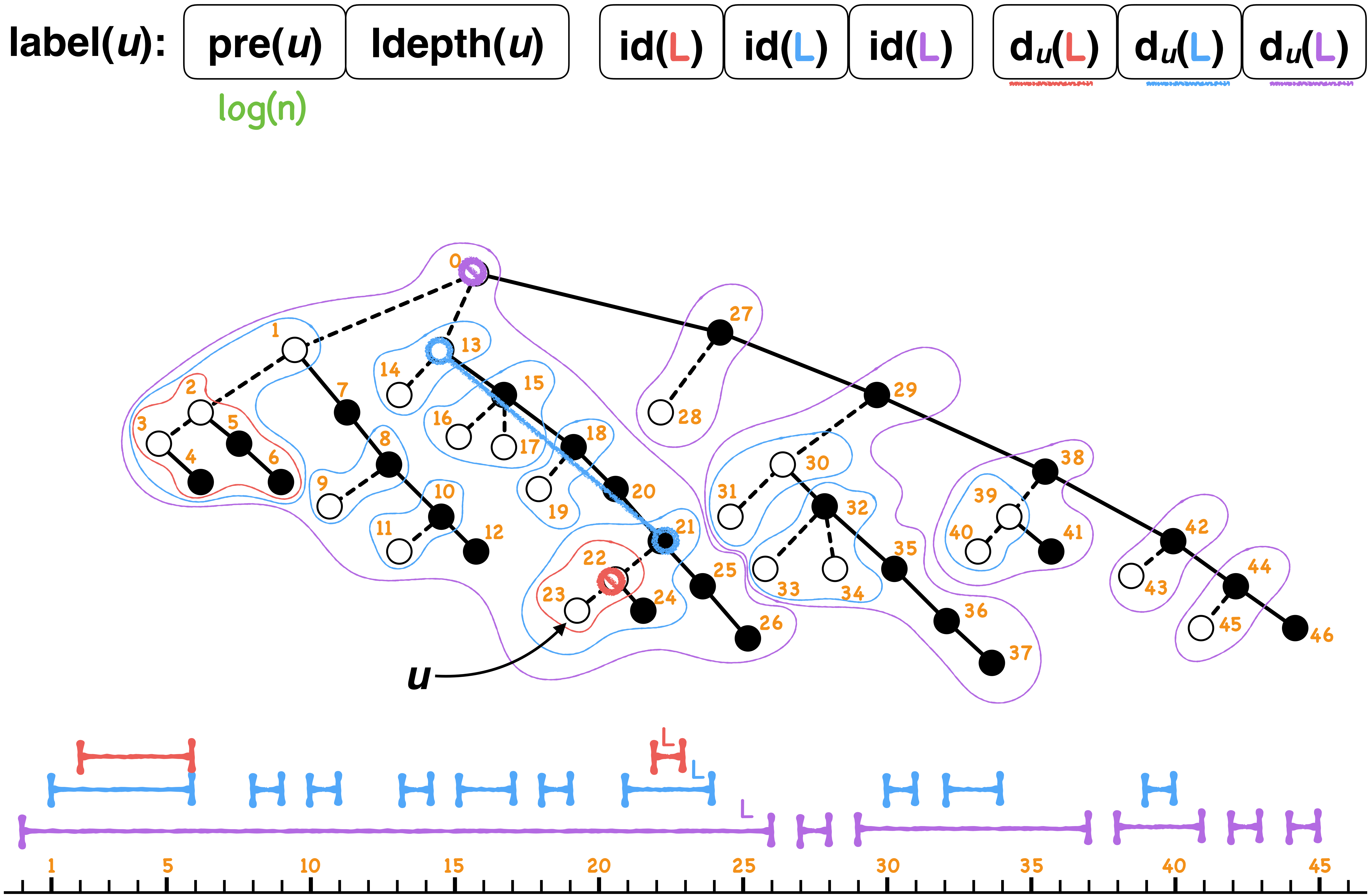

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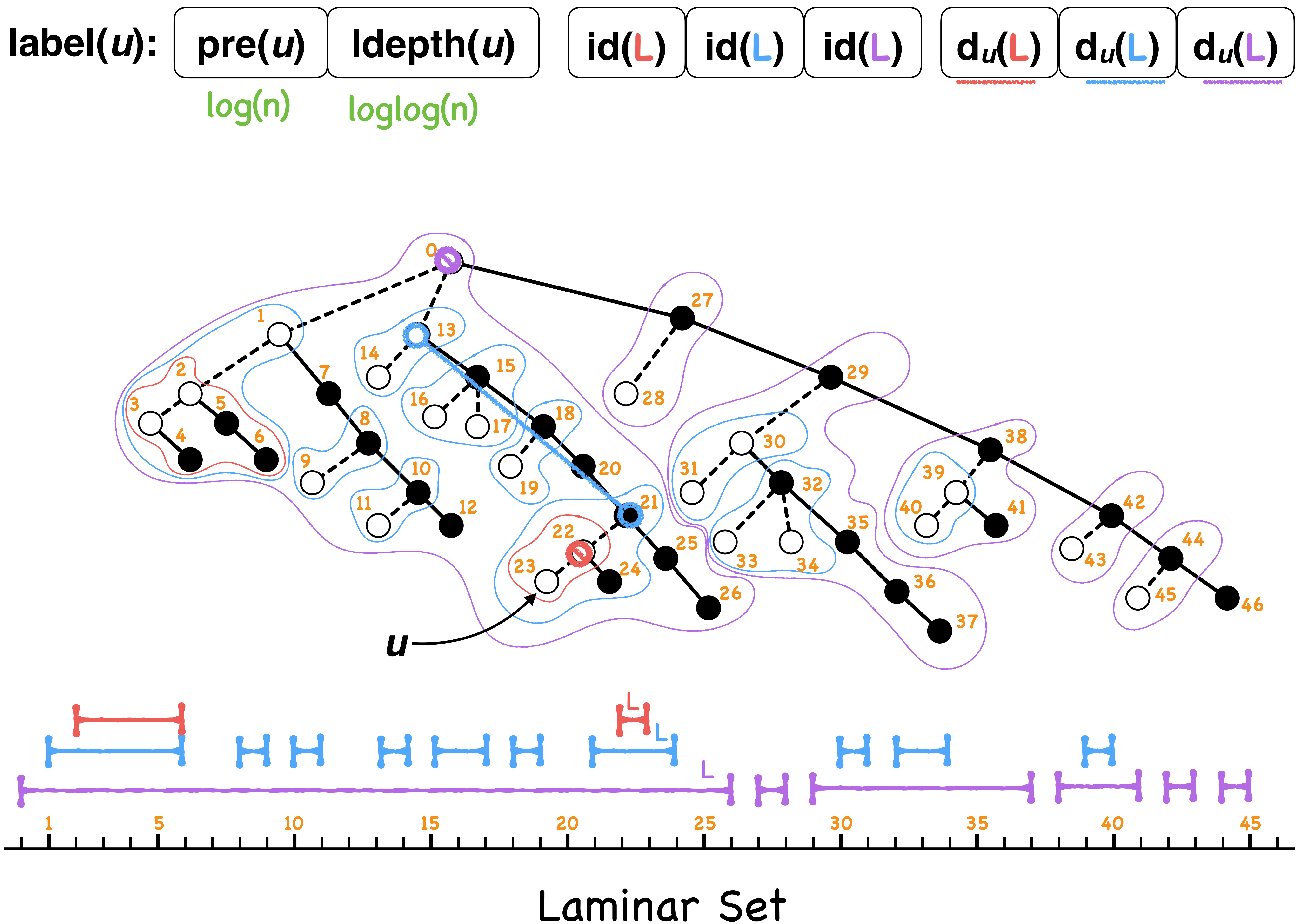

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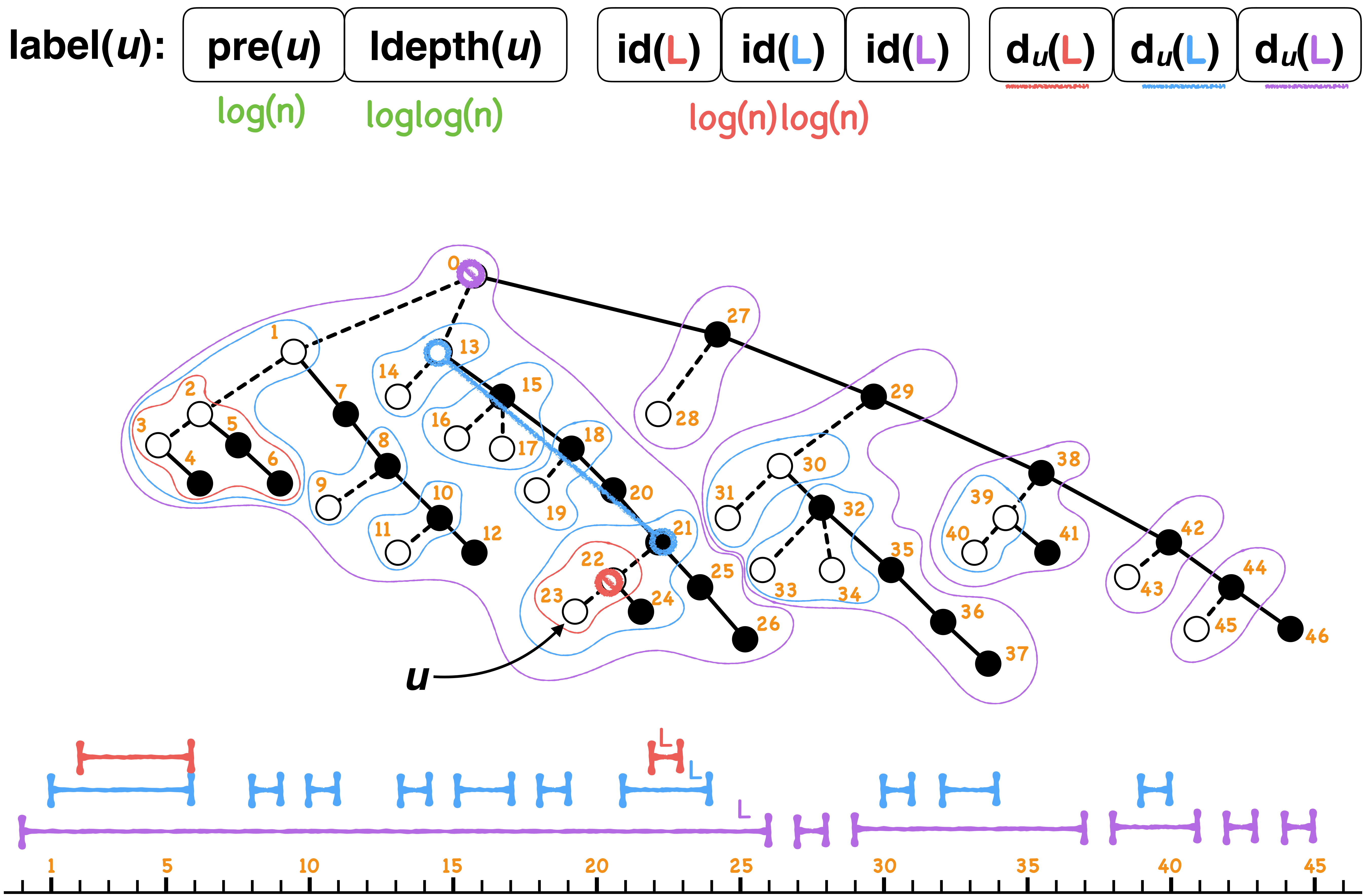

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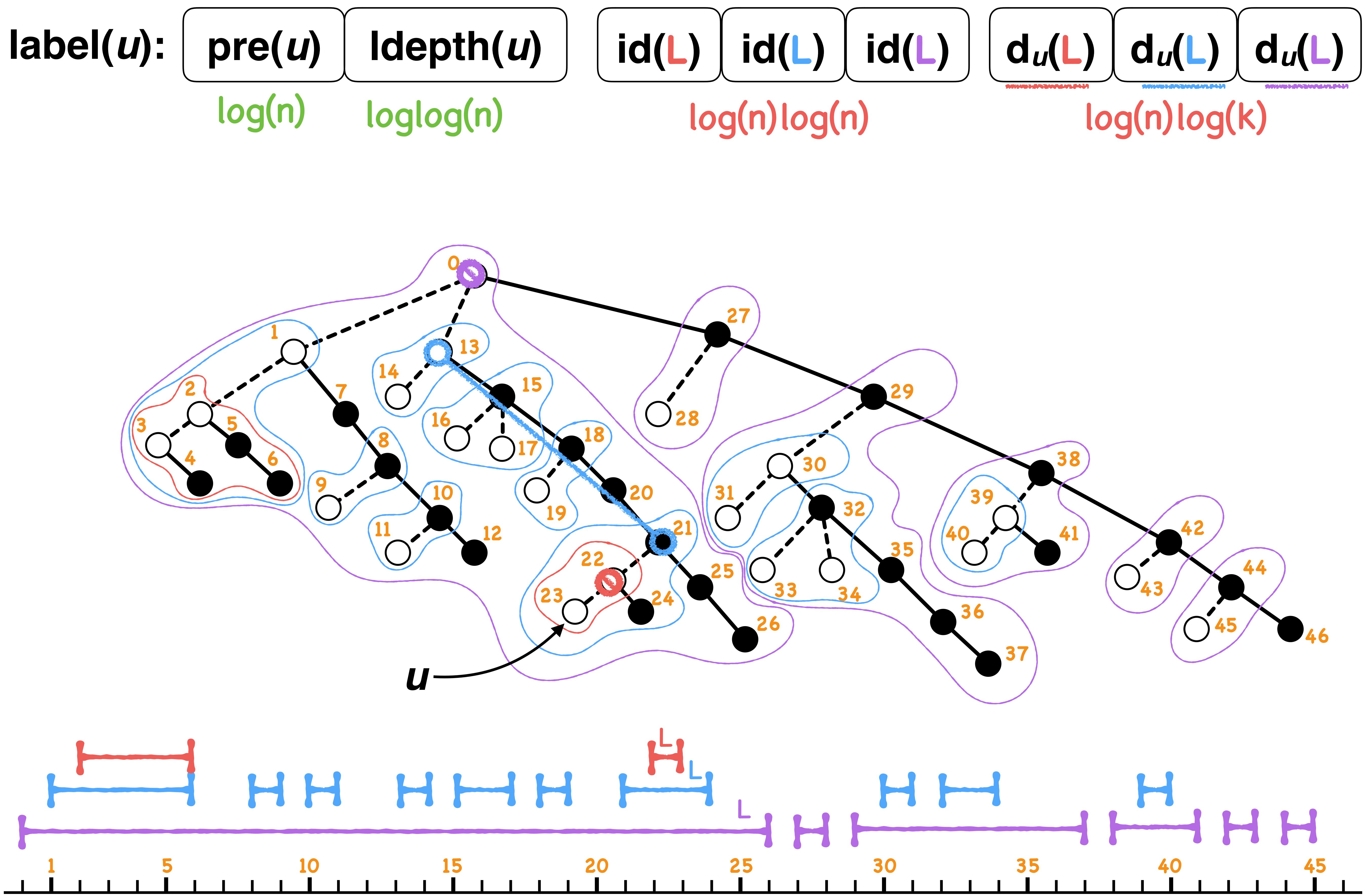

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Laminar Set

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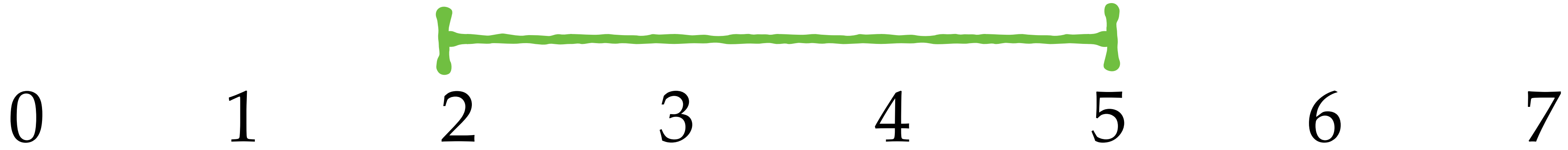

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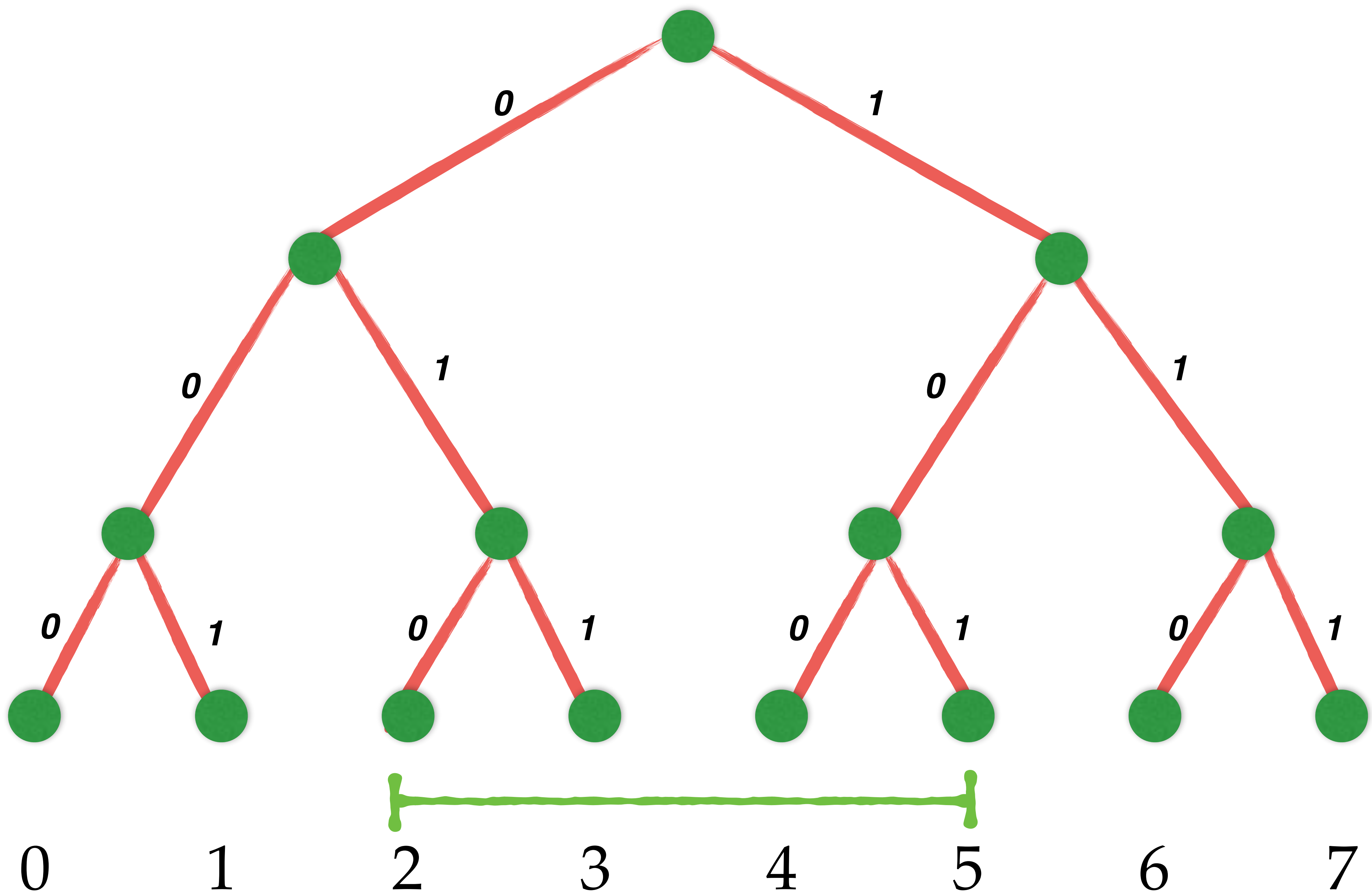

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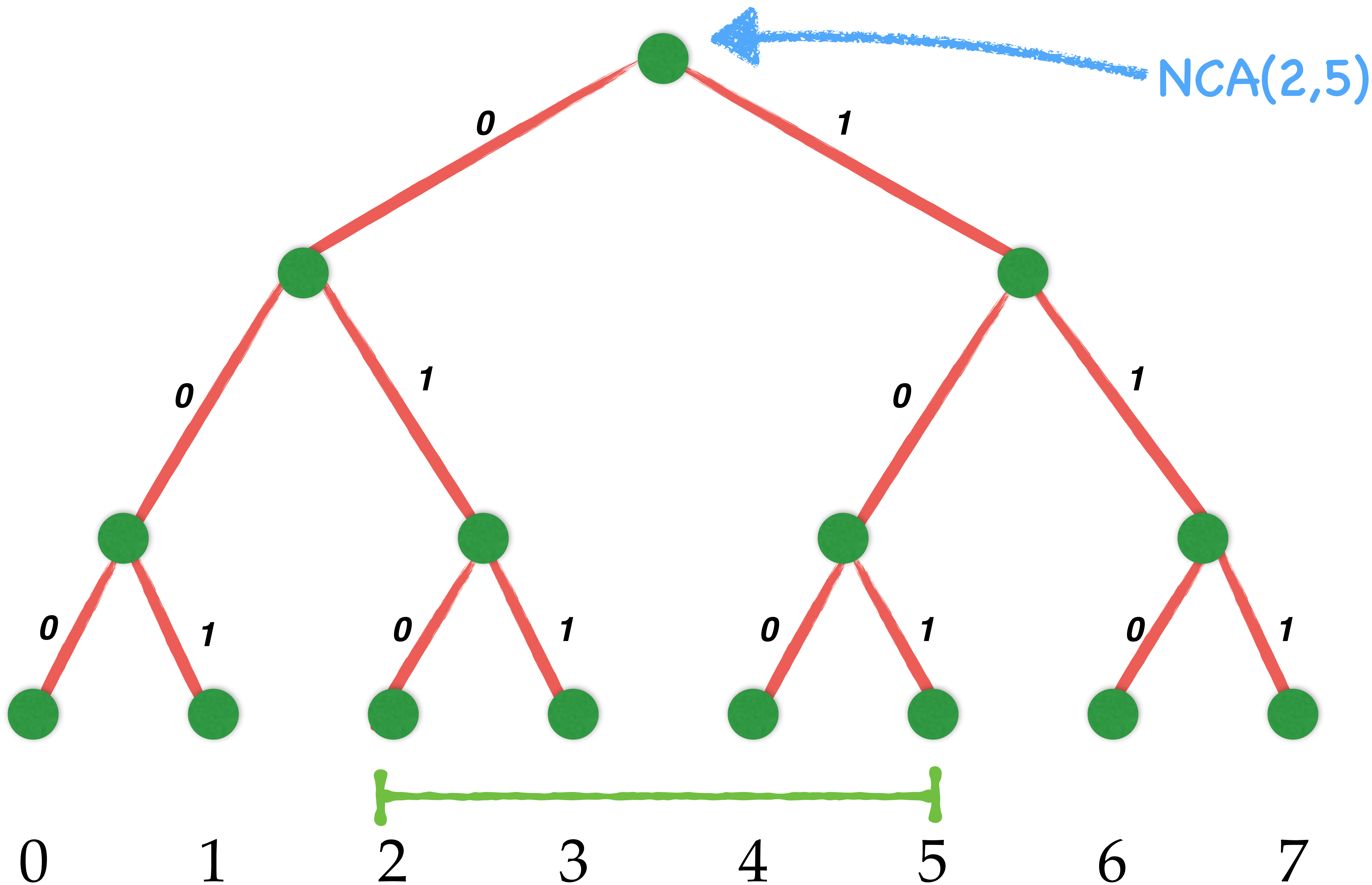

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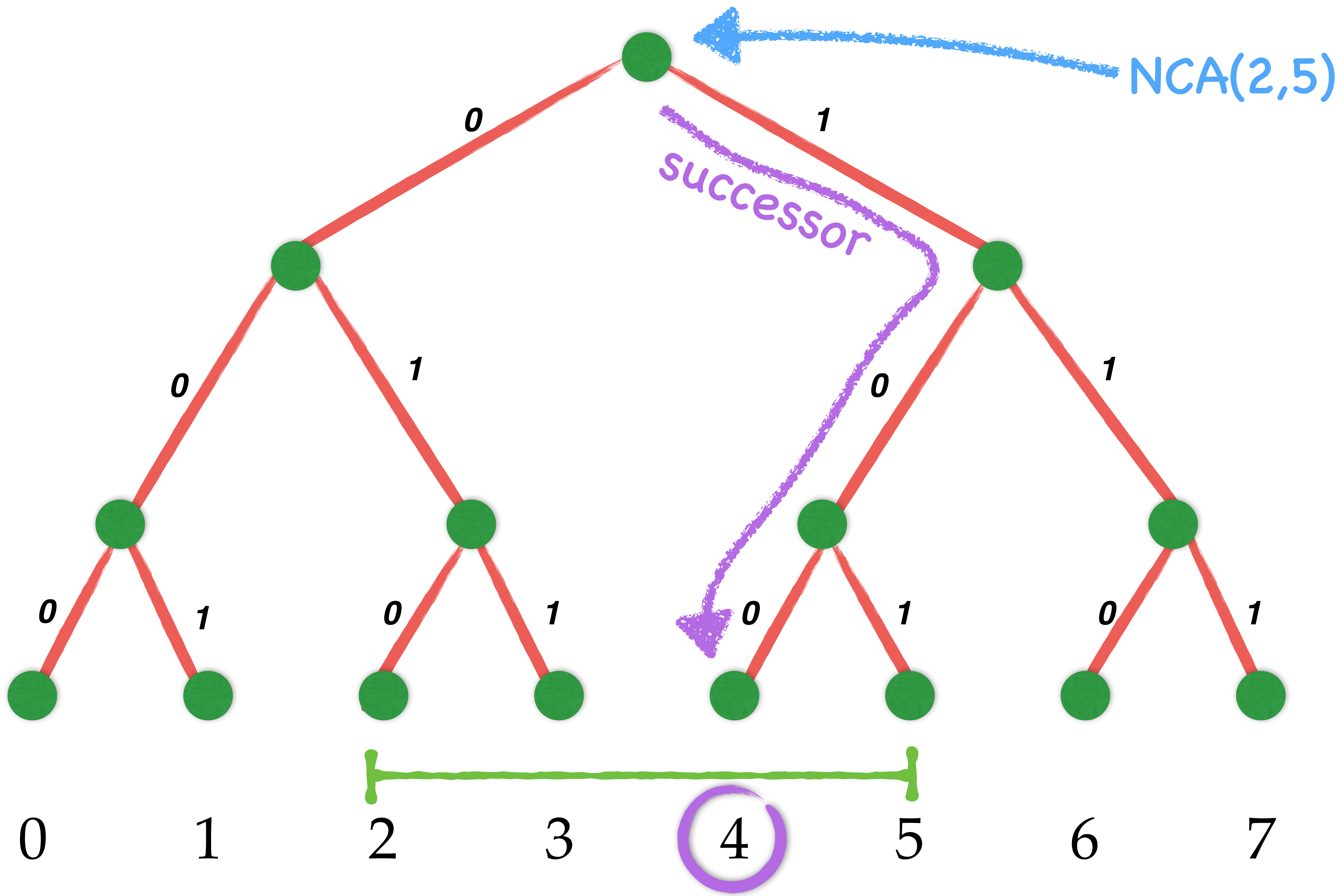

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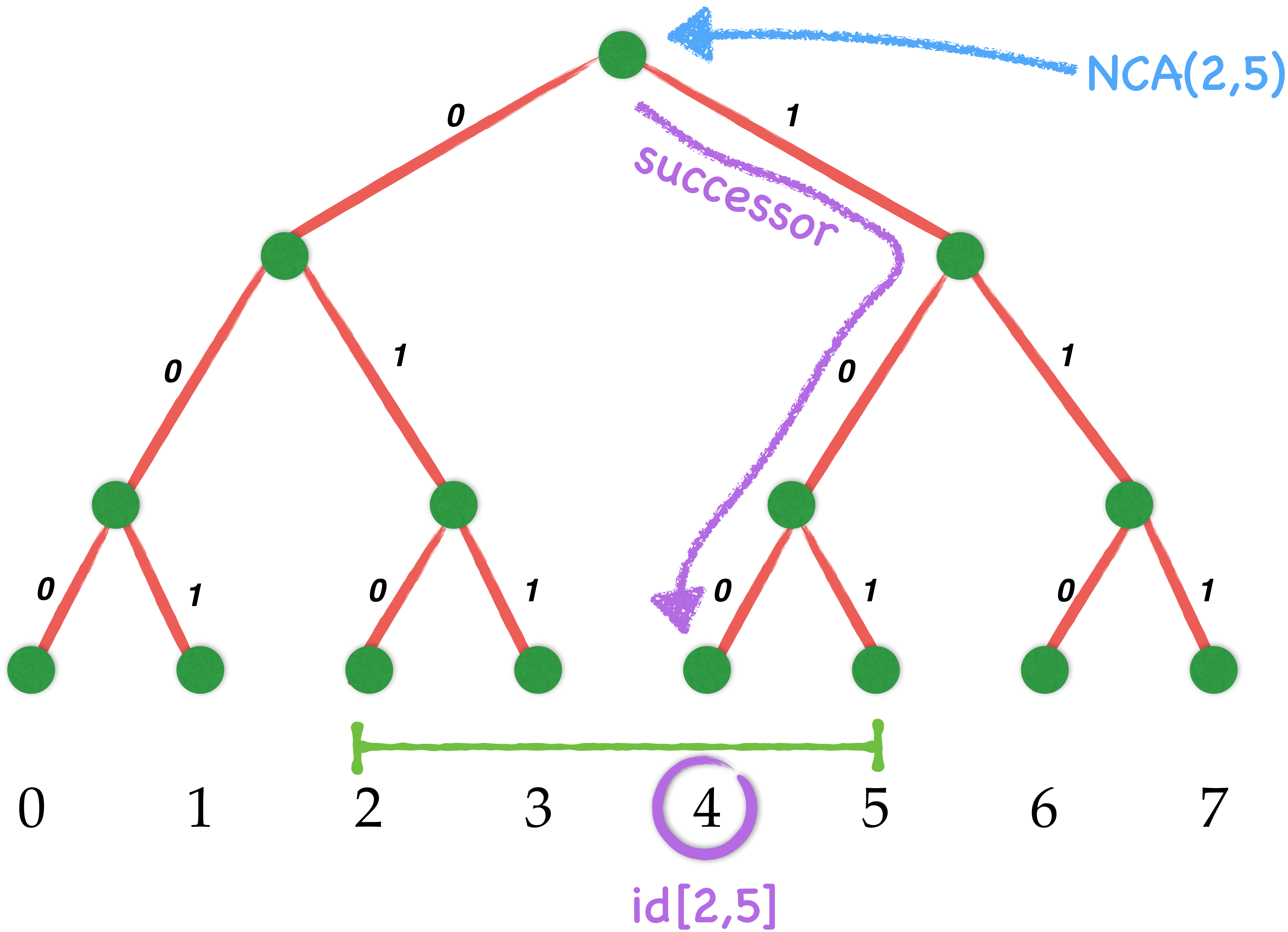

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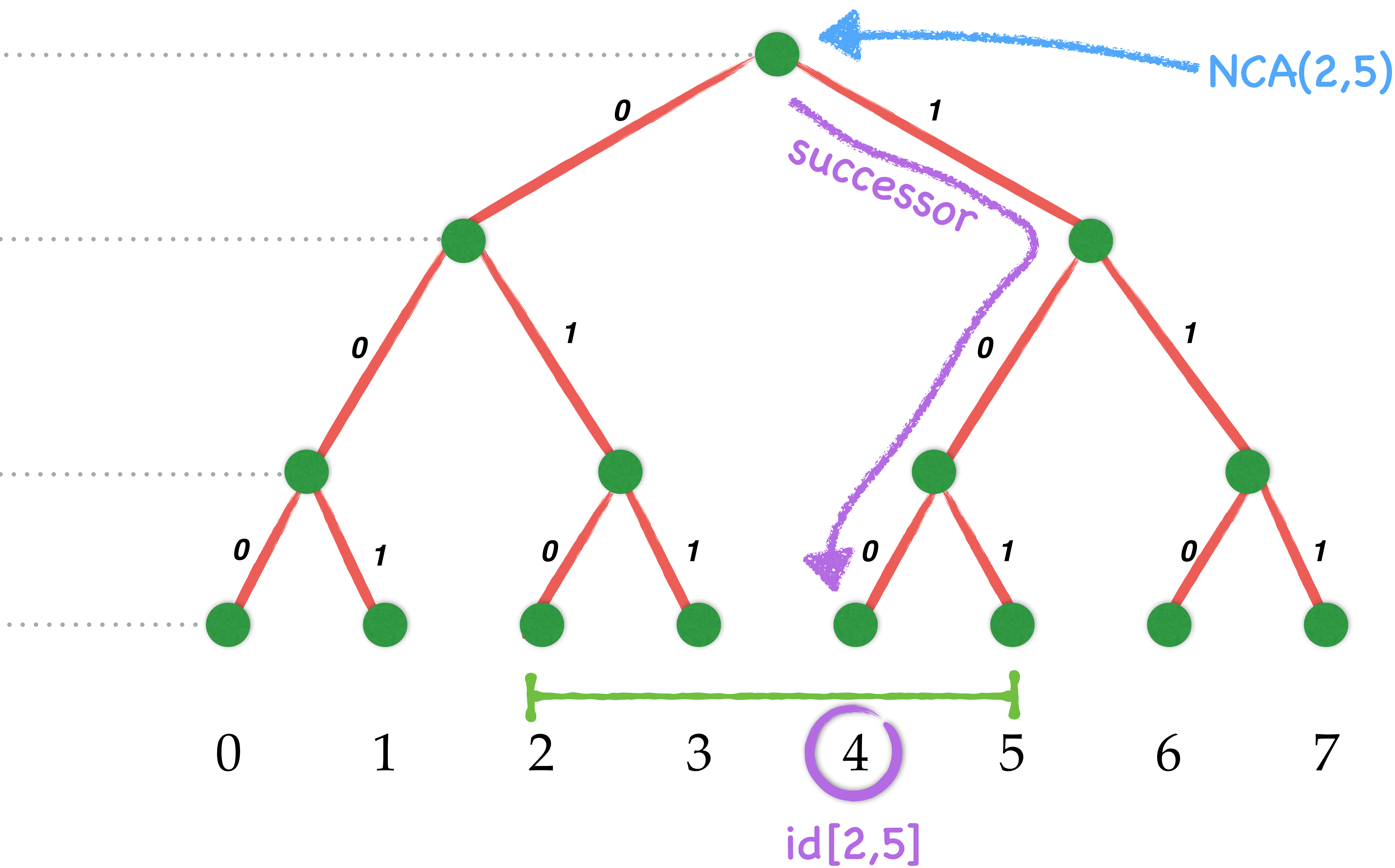

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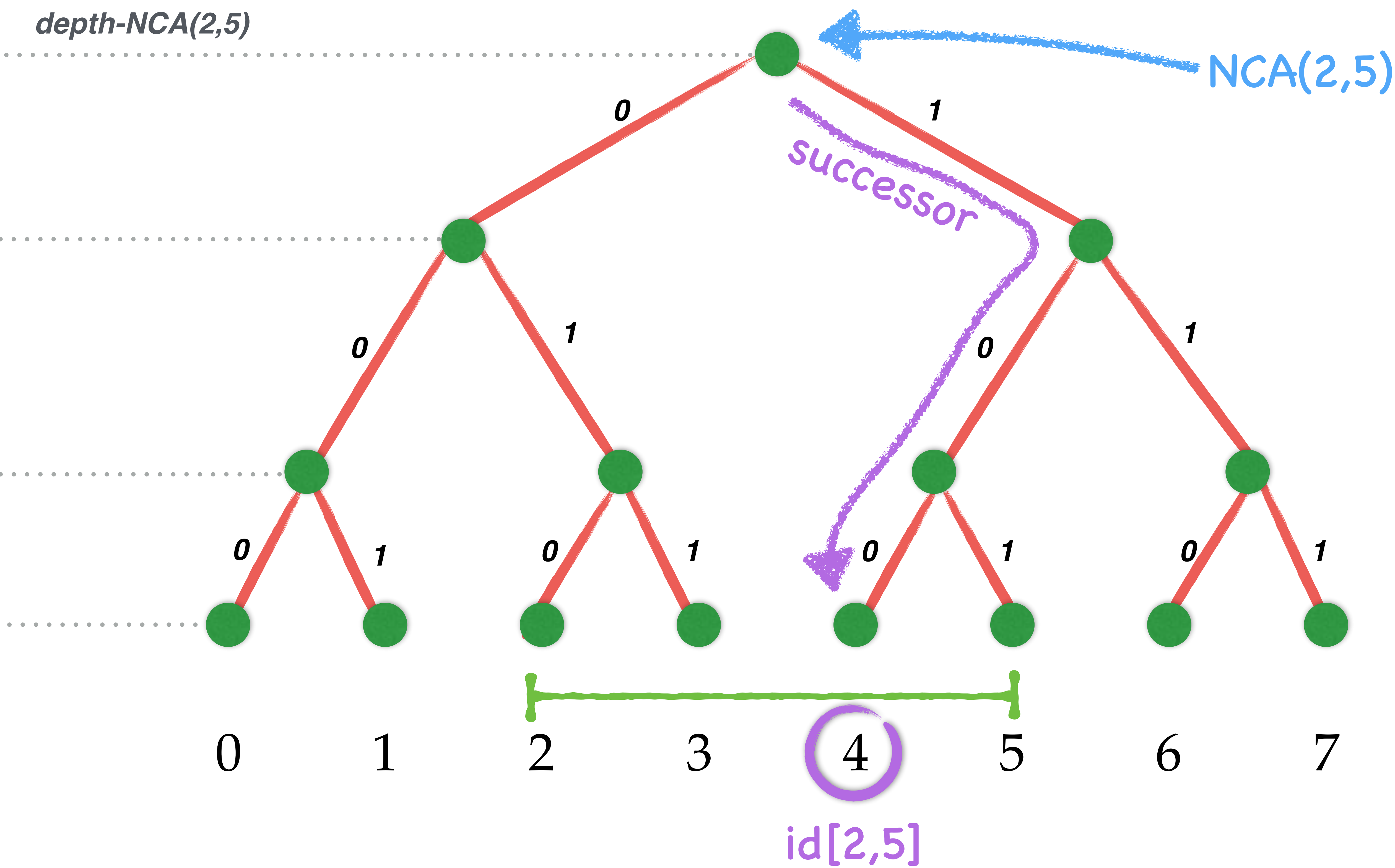
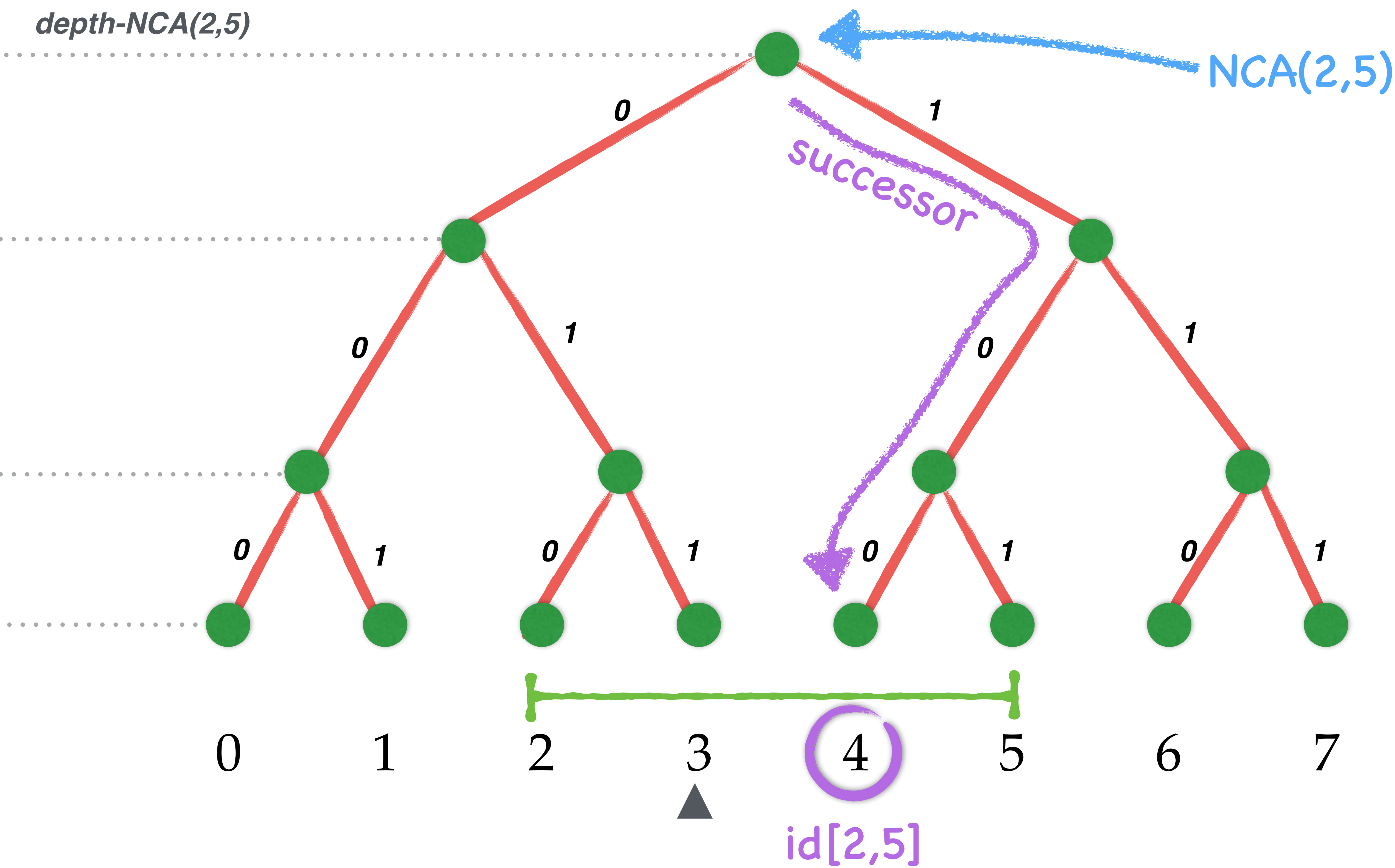


Range Representative



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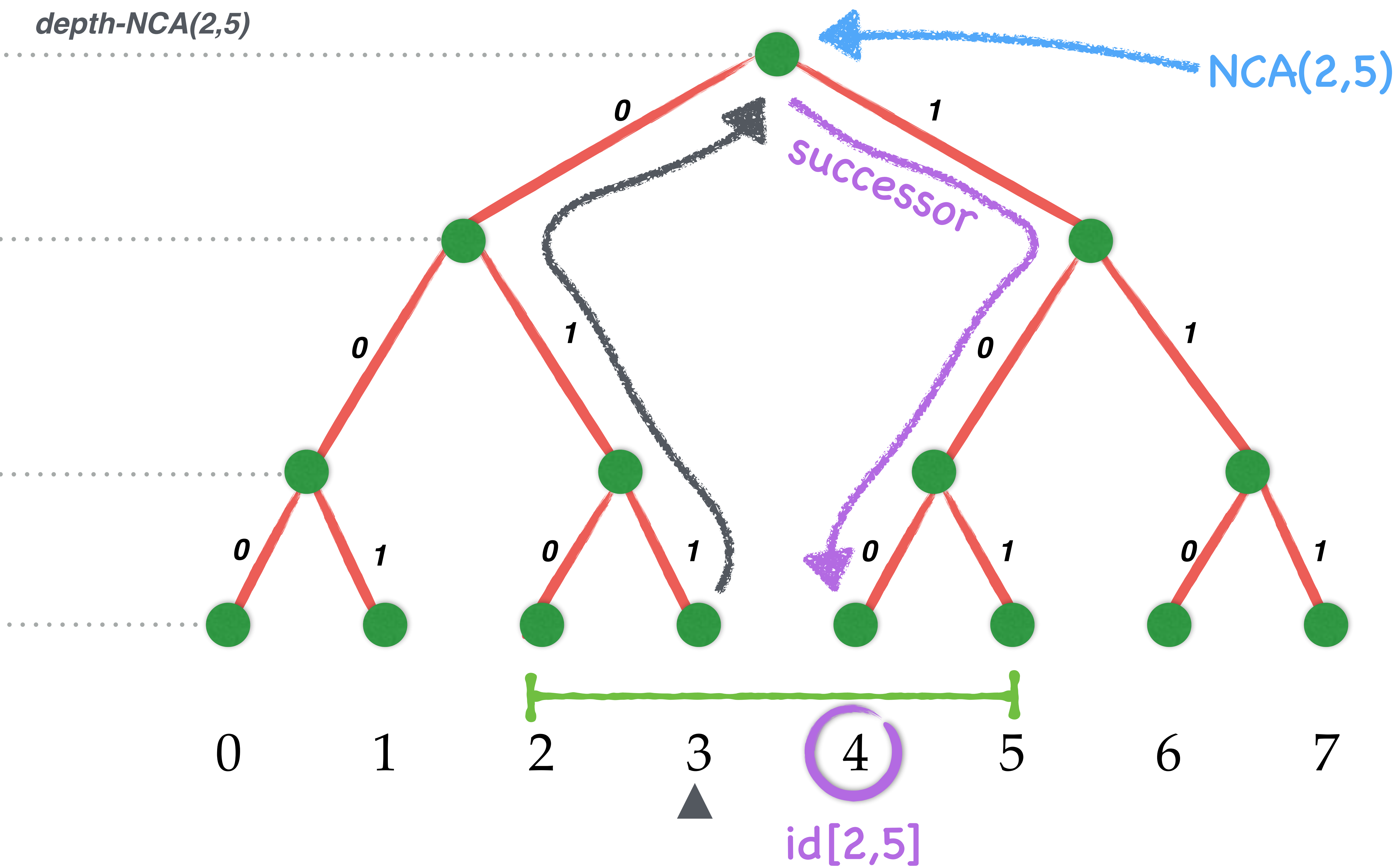

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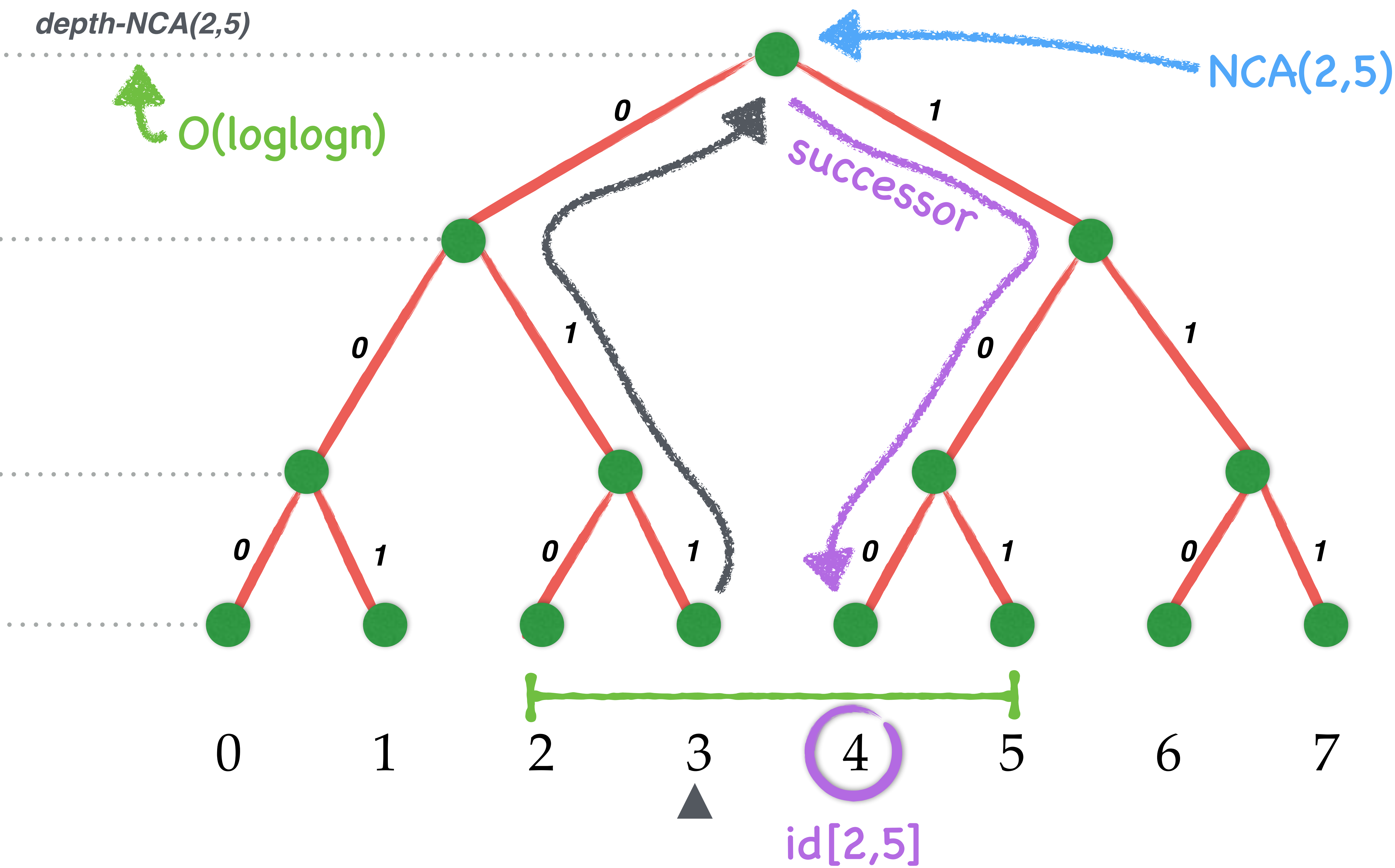

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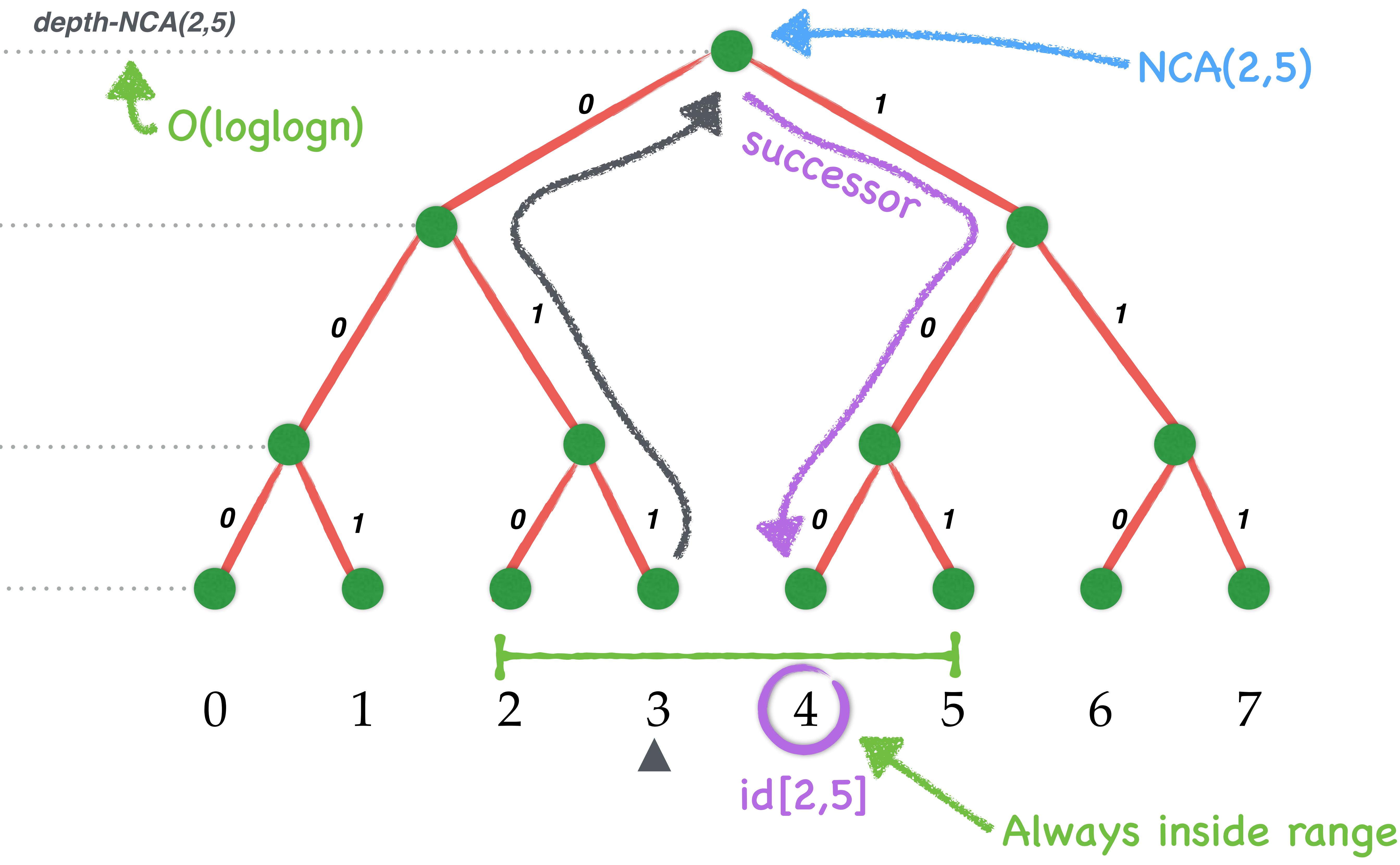

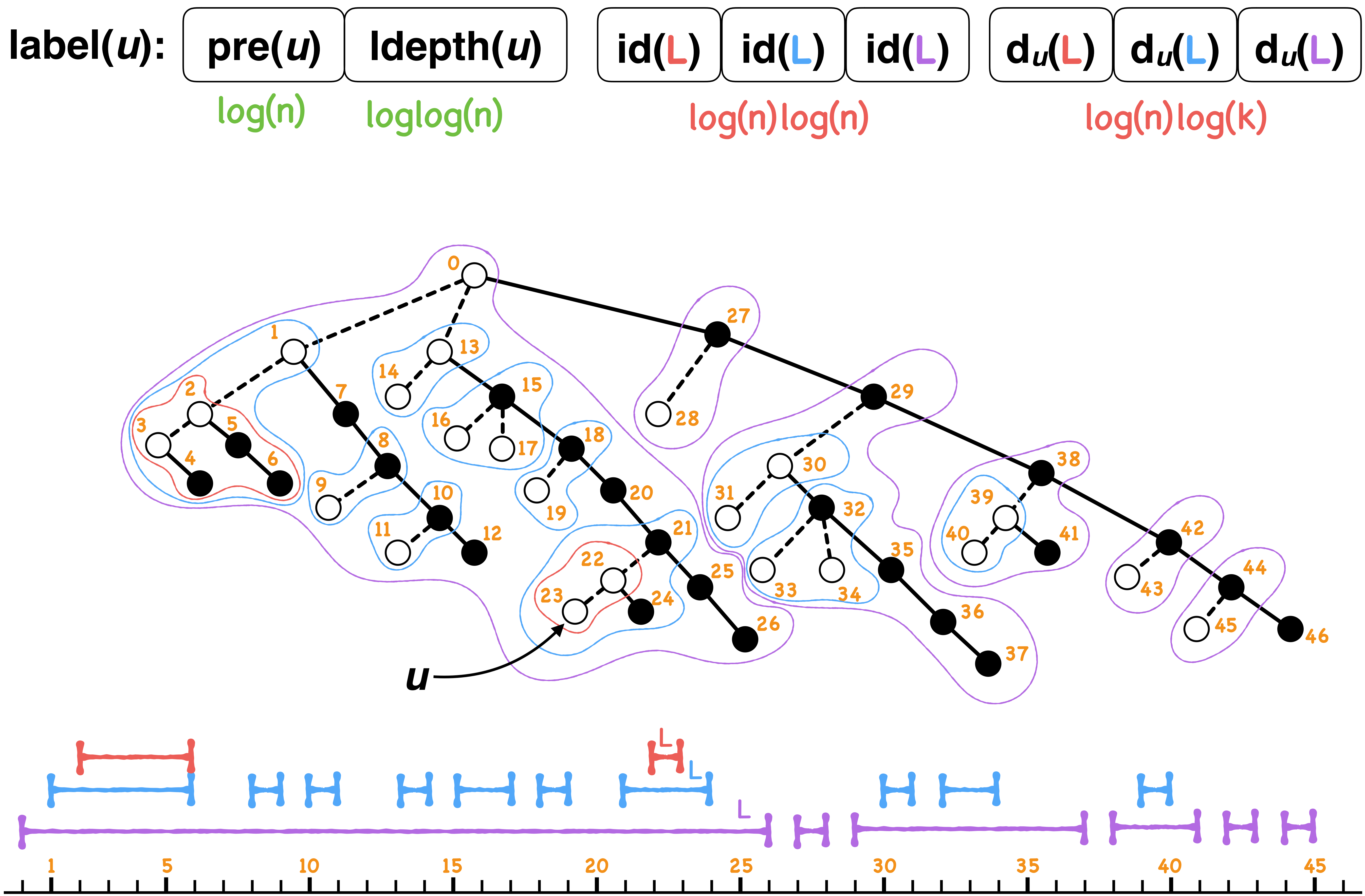

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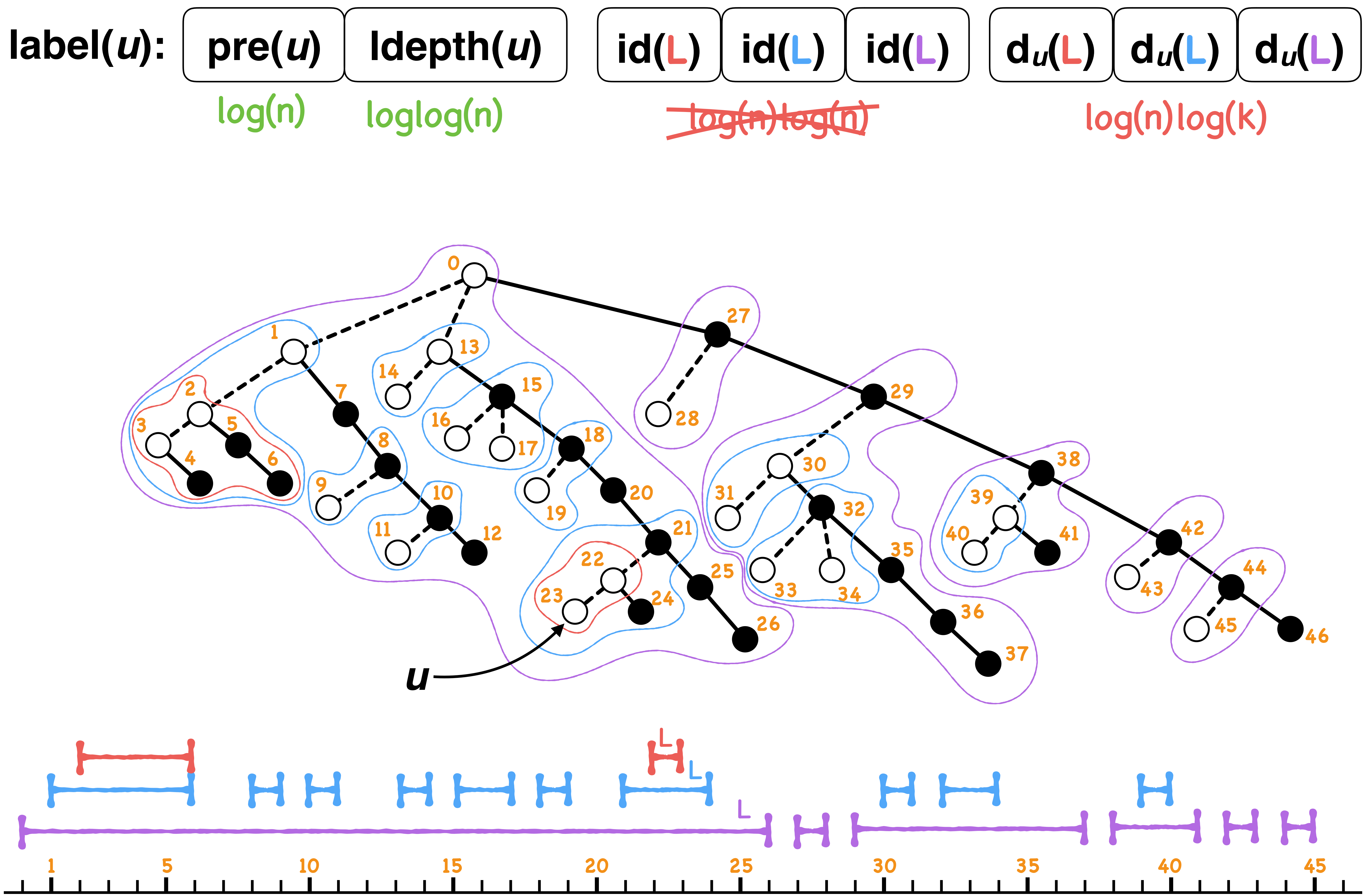

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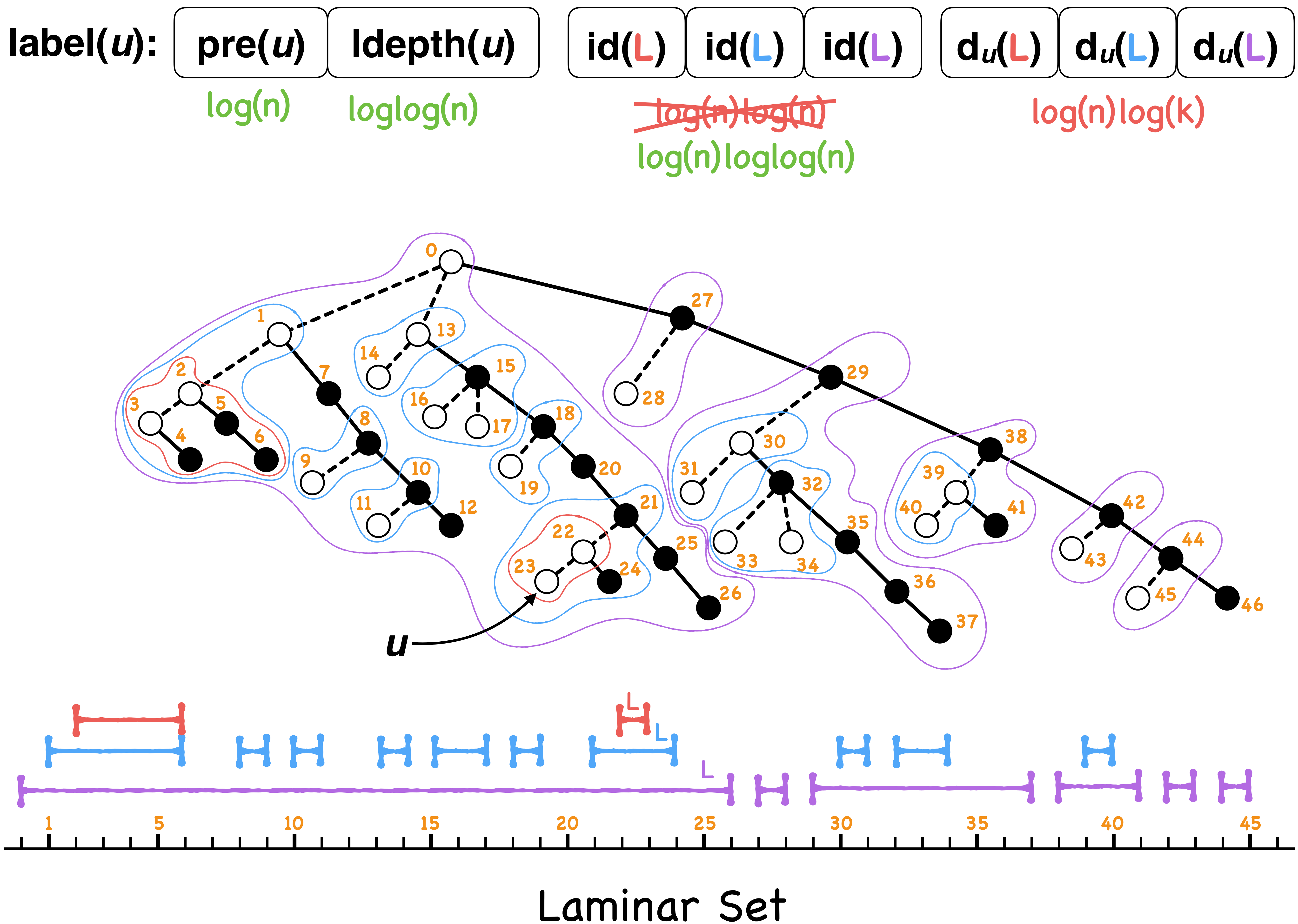
Laminar Set



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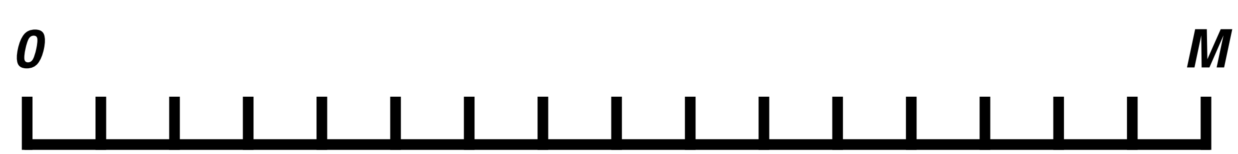
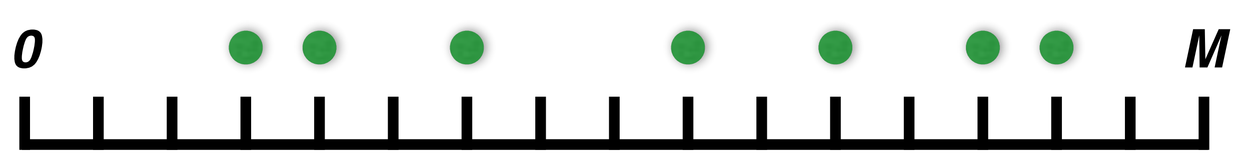
Laminar Set

Bounded monotone sequence

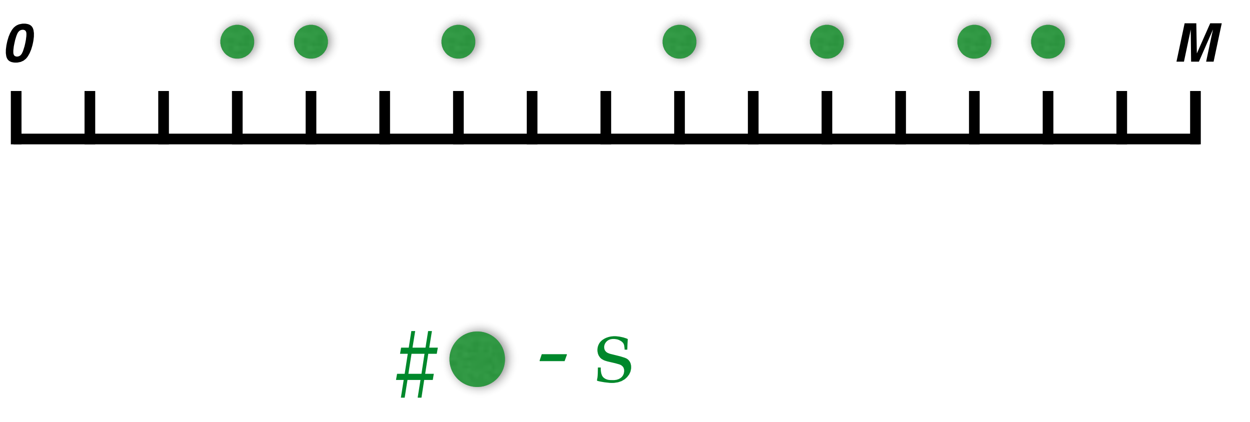
Bounded monotone sequence



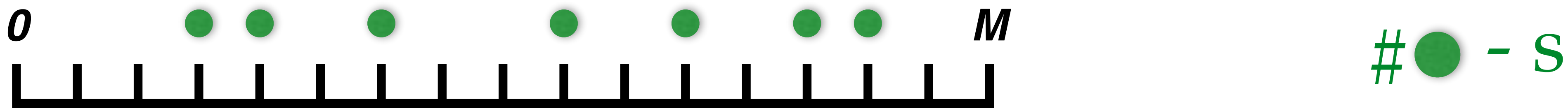
Bounded monotone sequence



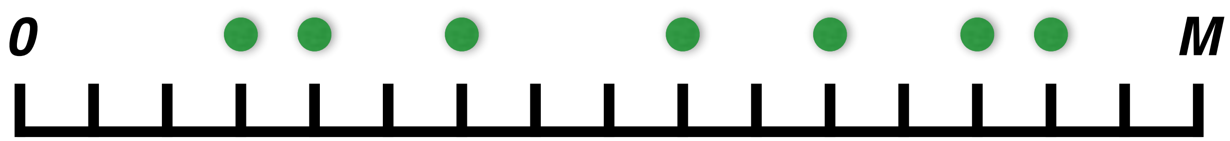
Bounded monotone sequence



Bounded monotone sequence



Bounded monotone sequence



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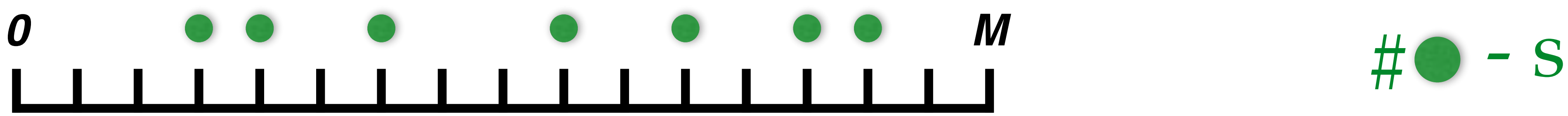
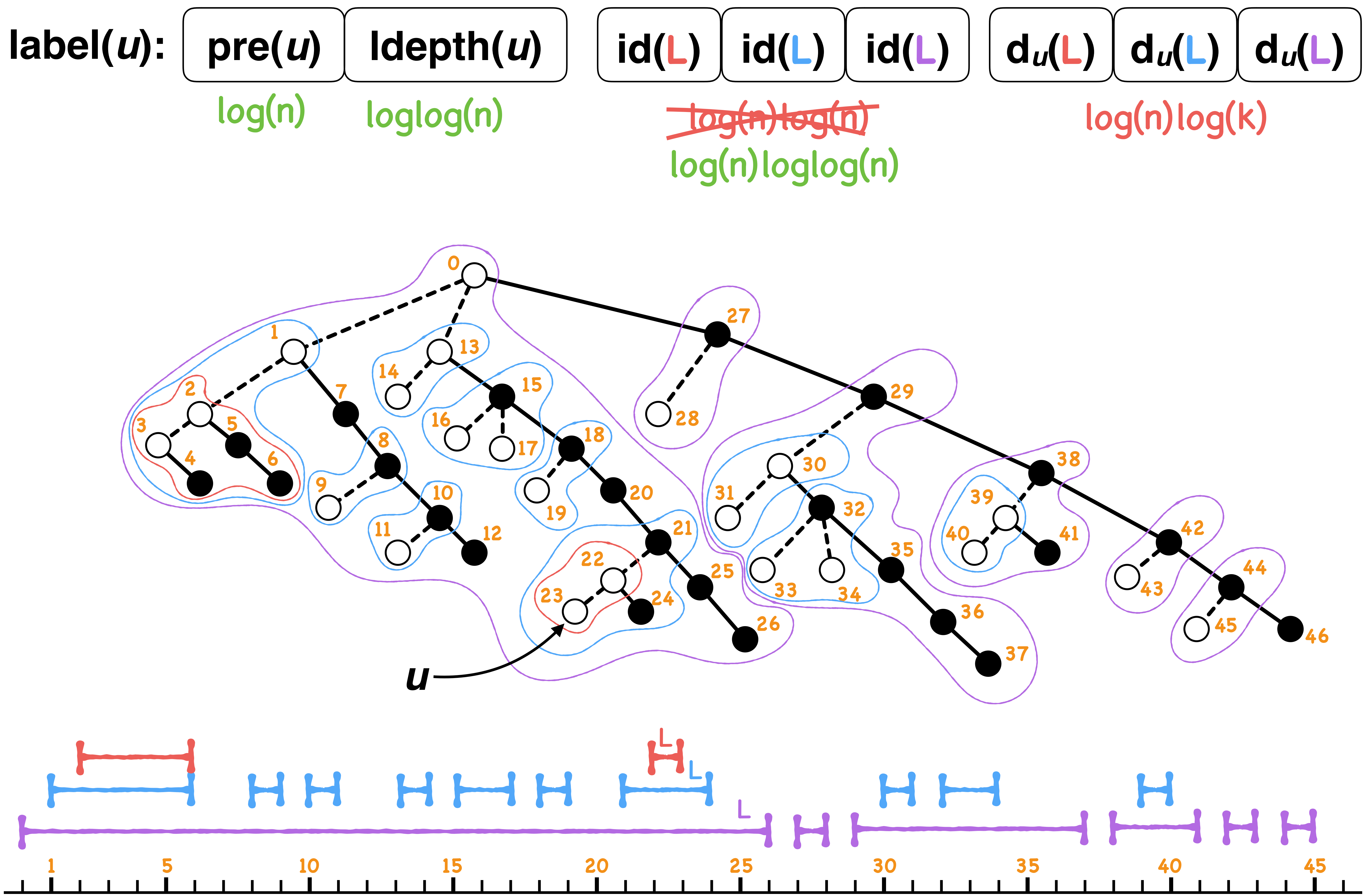
$$\log \binom{M}{s}$$

Bounded monotone sequence



$$\log \binom{M}{s} \cong s \cdot \log \frac{M}{s}$$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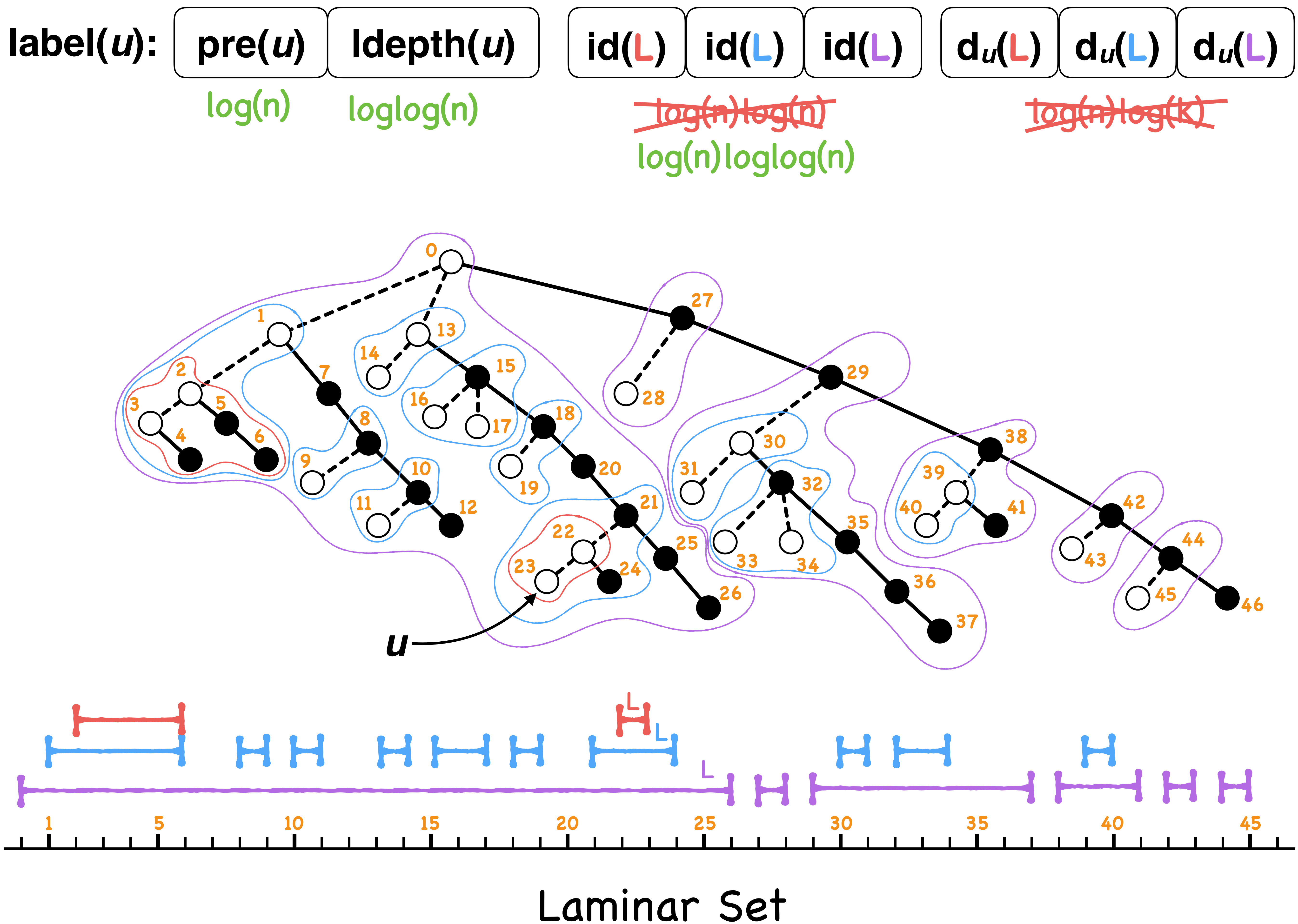
Laminar Set



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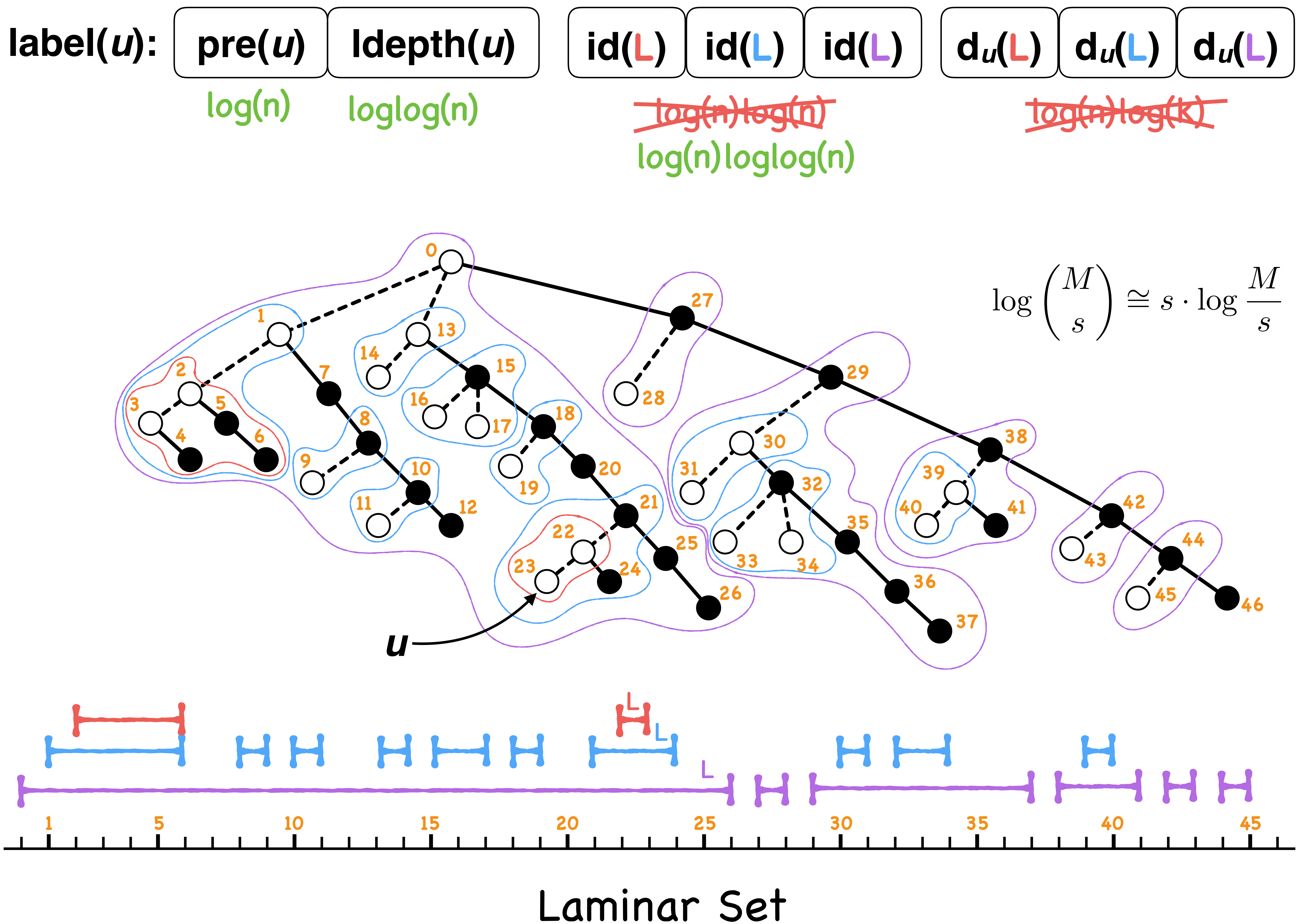

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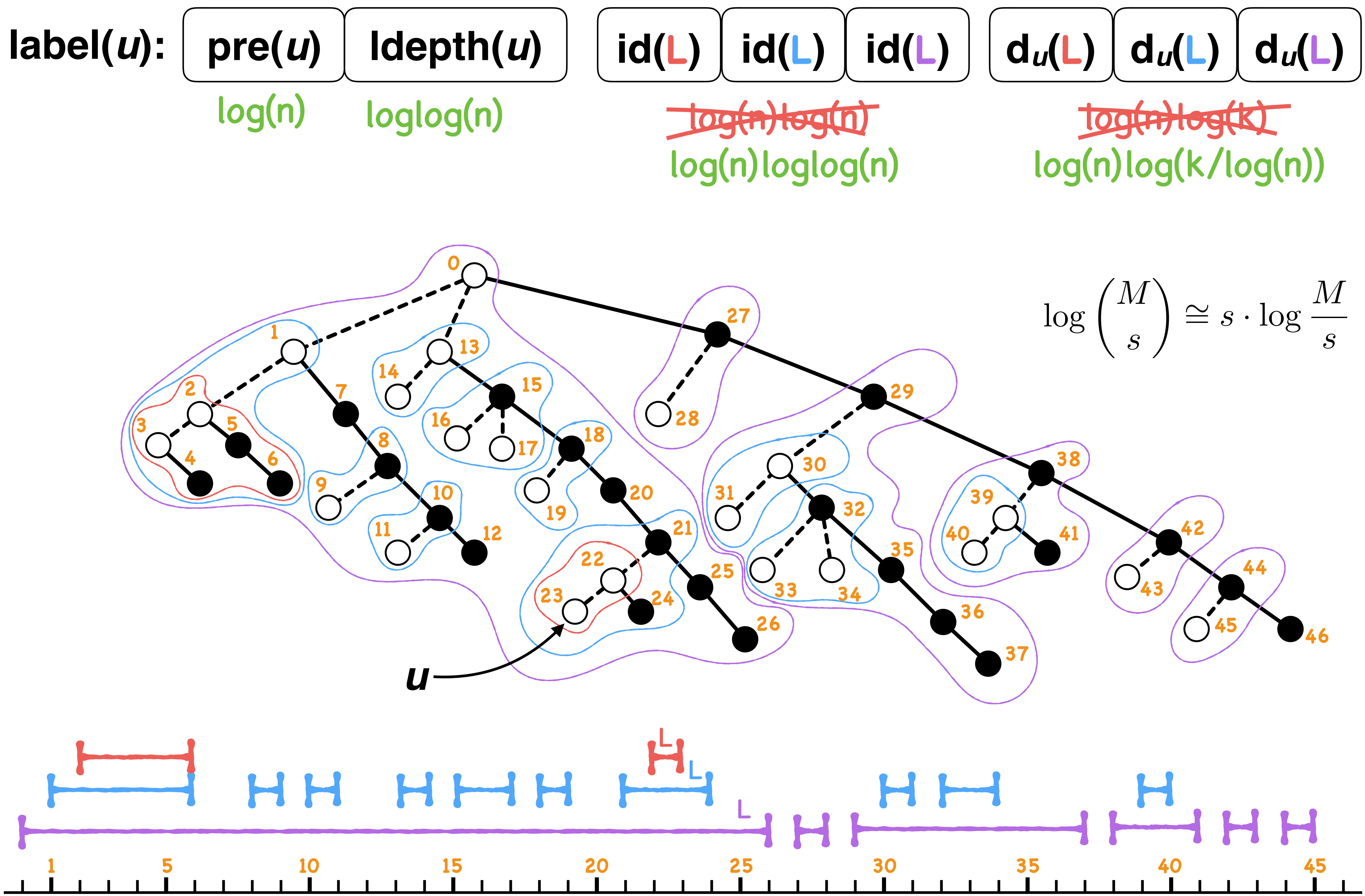

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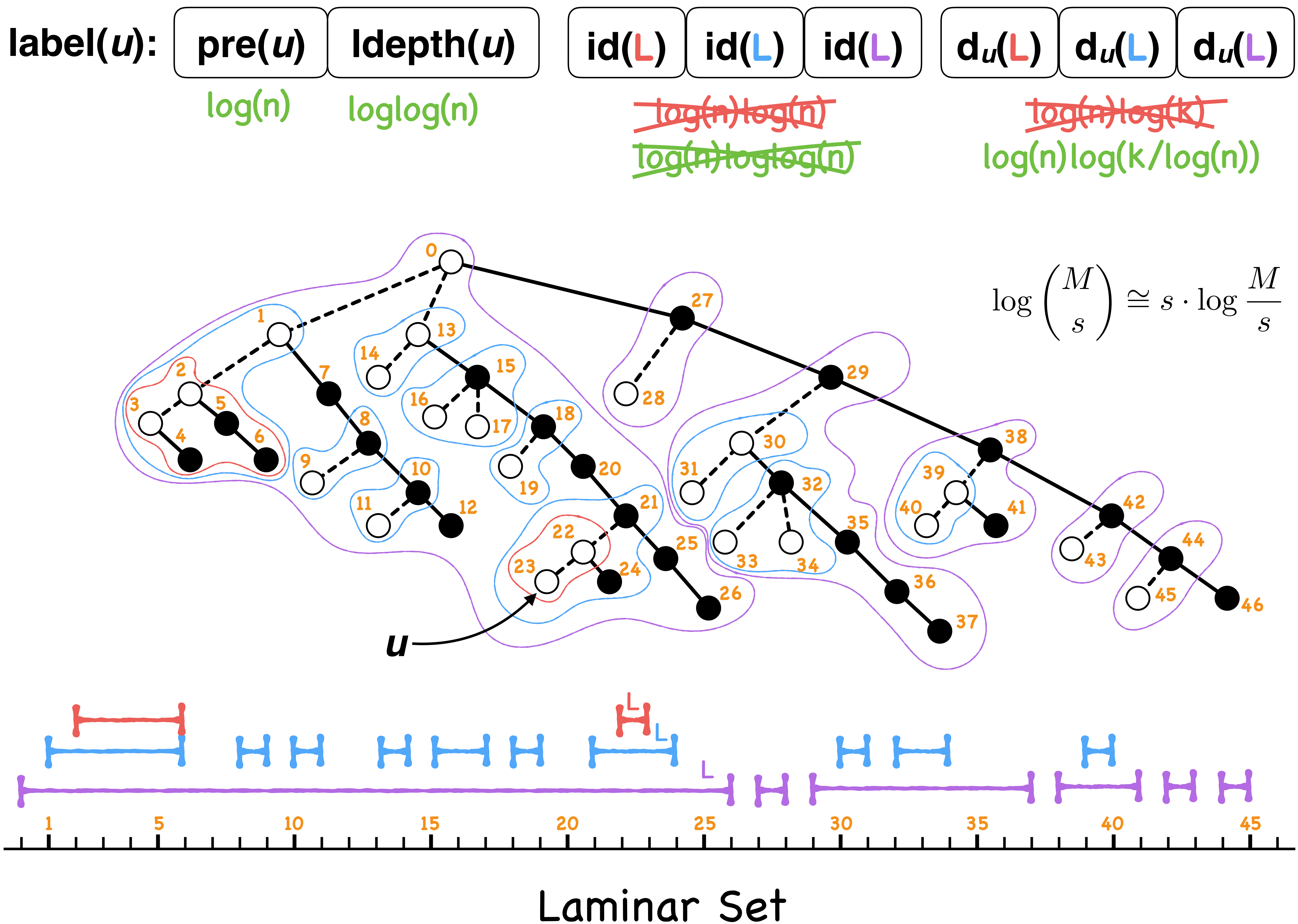
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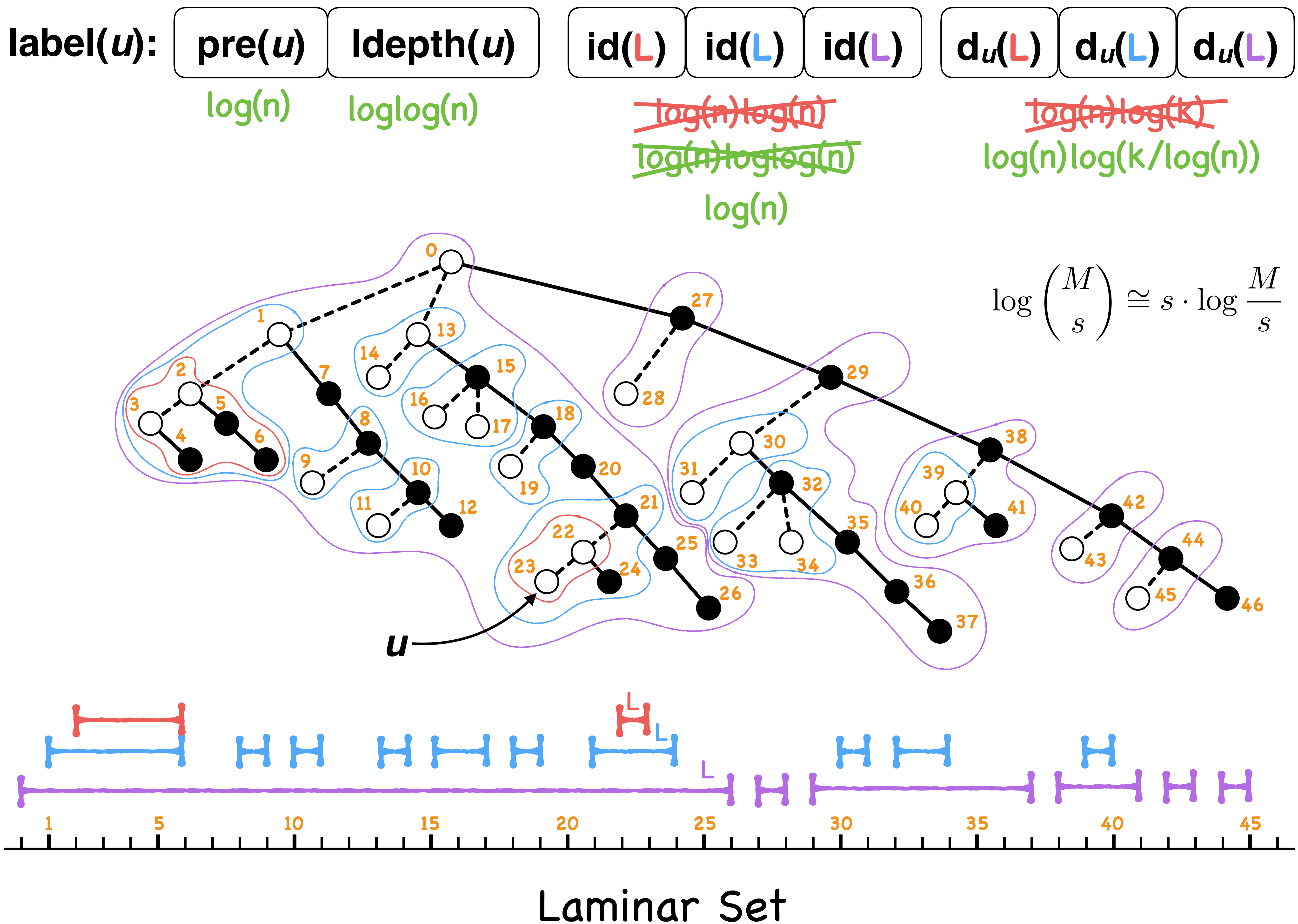

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Laminar Set

Results

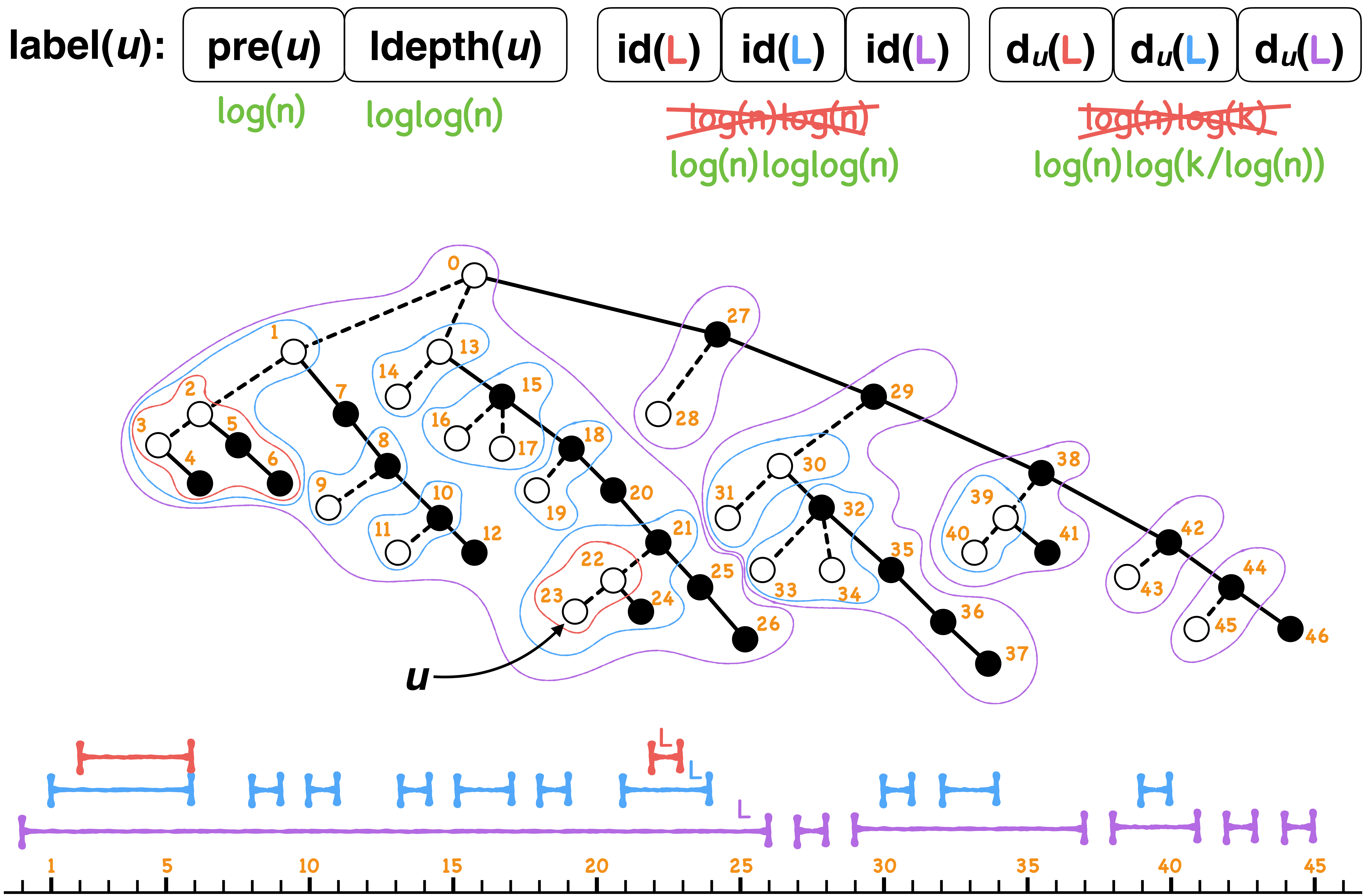
	Upper			Lower		
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	=>	$\frac{1}{4}\log^2 n$	$\frac{1}{8}\log^2 n$ [Gavoille et al. 2001]	=>	$\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	=>	$\log\frac{1}{\varepsilon}\log n$			$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ + $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	=>	$\log n$ + $O(k\log(\log n/k))$			$\log n$ + $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$						

Results

	Upper			Lower	
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	=>	$\frac{1}{4}\log^2 n$	$\frac{1}{8}\log^2 n$ => [Gavoille et al. 2001]	$\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	=>	$\log\frac{1}{\varepsilon}\log n$		$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	=>	$\log n$ $+$ $O(k\log(\log n/k))$		$\log n$ $+$ $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$	$O(\log n \cdot \log(k/\log n))$				

Results

	Upper			Lower	
Exact lower order terms excluded	$\frac{1}{2}\log^2 n$ [Alstrup et al. 2016]	=>	$\frac{1}{4}\log^2 n$	$\frac{1}{8}\log^2 n$ [Gavoille et al. 2001]	=> $\frac{1}{4}\log^2 n$ [Alstrup et al. 2016]
Approximate	$\frac{1}{\varepsilon}\log n$ [Alstrup et al. 2016]	=>	$\log\frac{1}{\varepsilon}\log n$		$\log\frac{1}{\varepsilon}\log n$
k-Distance $k \leq \log n$	$\log n$ $+$ $O(k\log(k\log n/k))$ [Gavoille et al. 2007]	=>	$\log n$ $+$ $O(k\log(\log n/k))$		$\log n$ $+$ $\Omega(k\log(\log n/k\log k))$
k-Distance $k > \log n$	$O(\log n \cdot \log(k/\log n))$			$\Omega(\log n \cdot \log(k/\log n))$	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Laminar Set

Conclusion

Conclusion

1. Distance labeling and Universal Tree **separated**

Conclusion

1. Distance labeling and Universal Tree **separated**

2. **Optimal** distance labeling for trees
   (lower order terms excluded)

Conclusion

1. Distance labeling and Universal Tree **separated**

2. **Optimal** distance labeling for trees
   (lower order terms excluded)

3. k-Distance and Approximate labels for trees **settled**

End