Minimum Cut in $\mathcal{O}(m \log^2 n)$ Time

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Slides by Paweł Gawrychowski1

(Global) Minimum Cut

Input: undirected edge-weighted graph $G = (V, E)$

Output: nonempty $S \subset V$ minimizing the total weight of edges between S and $V \setminus S$

Solvable in polynomial time with $n - 1$ maximum flow computations.

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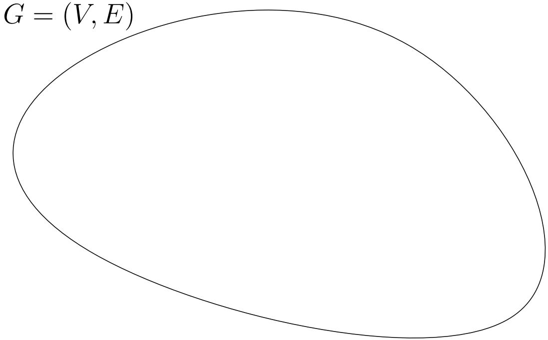

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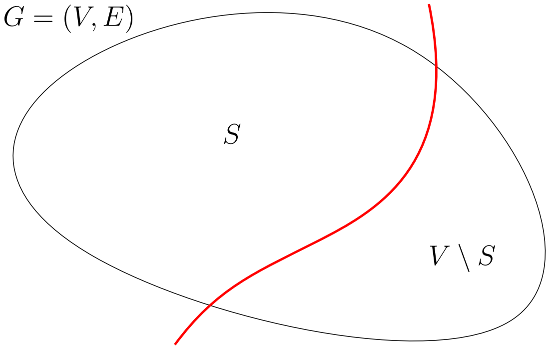

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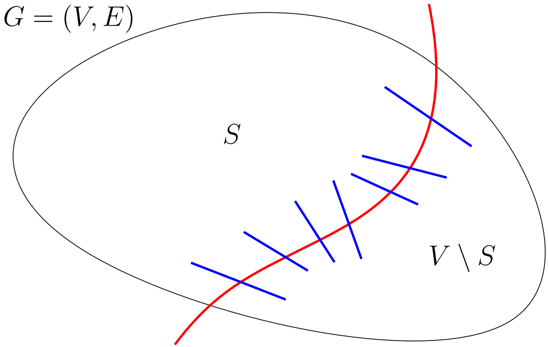

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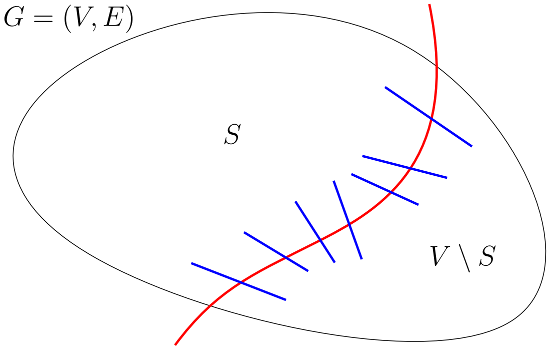

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The fastest maximum flow algorithm works in $\mathcal{O}(nm)$ time, so applying it $n-1$ times results in $\mathcal{O}(n^2 m)$ complexity. Is there a faster algorithm?

Hao and Orlin 1994

Roughly speaking, a single maximum flow computation suffices, resulting in $\mathcal{O}(mn \log(n^2/m))$ complexity.

Nagamochi and Ibaraki 1992

In $\mathcal{O}(m + n \log n)$ time we can either find the global minimum cut or isolate an edge that doesn't cross it. This edge can then be contracted and the procedure repeated, resulting in $\mathcal{O}(mn + n^2 \log n)$ complexity.

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Minimum k-respecting cut

Given a spanning tree T, find the minimum cut crossed by exactly k of its edges.

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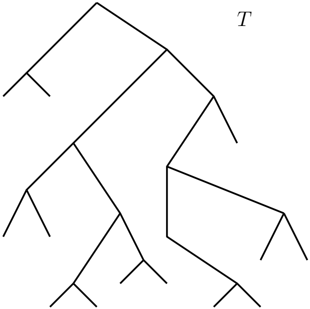

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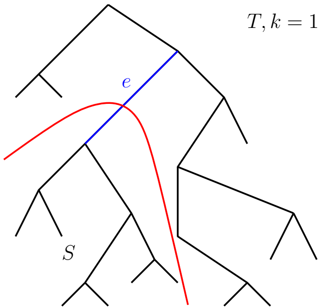

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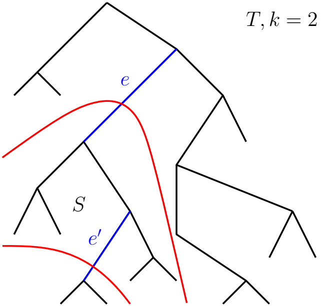

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Nash-Williams 1961

An unweighted graph with minimum cut c contains a set of $c/2$ edge-disjoint spanning trees (and clearly not more than c such trees).

If we had such $c/2$ edge-disjoint trees then the average number of edges from the minimum cut per tree is 2, hence the minimum cut 1- or 2-respects one of these trees.

However, this observation was not straightforward to use:

1. One needs to work with weighted graphs.
2. c might be large, and there might be not enough time to find the $c/2$ trees.
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Weighted tree packing

A set of spanning trees, each with its assigned weight, such that the total weight of all trees containing an edge is at most its weight.

1. Find in $\mathcal{O}(m + n\log n)$ time an unweighted graph H with $\mathcal{O}(n\log n)$ edges and minimum cut $c' = \mathcal{O}(\log n)$, such that the minimum cut in G corresponds to a $7/6$-minimum cut in H.
2. Apply the algorithm of Plotkin-Shmoys-Tardos to find a tree packing of total weight $5/12c'$ in $\mathcal{O}(n\log^3 n)$ time.
3. Fraction of the total weight of all trees in the packing 1- or 2-respected by the minimum cut of G must be at least $1/10$.

Choosing $\mathcal{O}(\log n)$ trees from the packing (with probability equal to the weight) guarantees that w.h.p. we obtain a tree 1- or 2-respected by the minimum cut.

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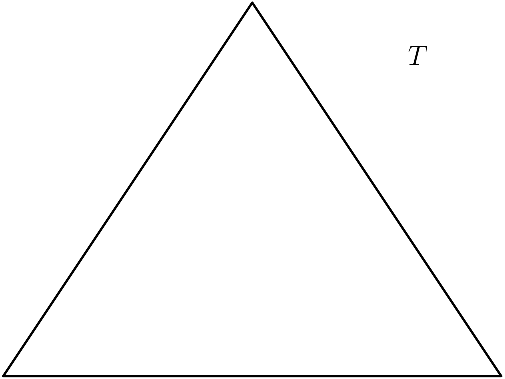
Karger showed that this is actually fairly easy:



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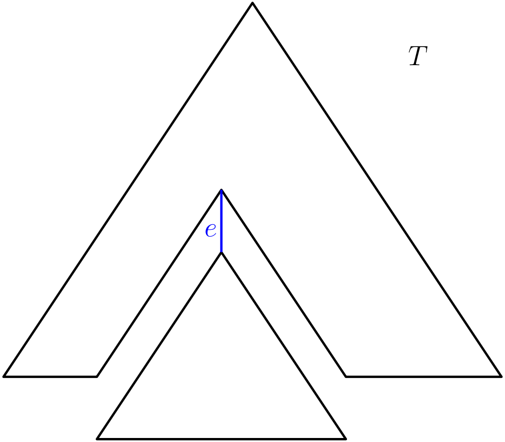

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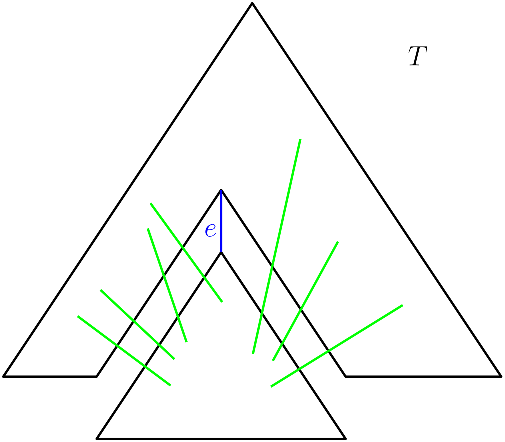

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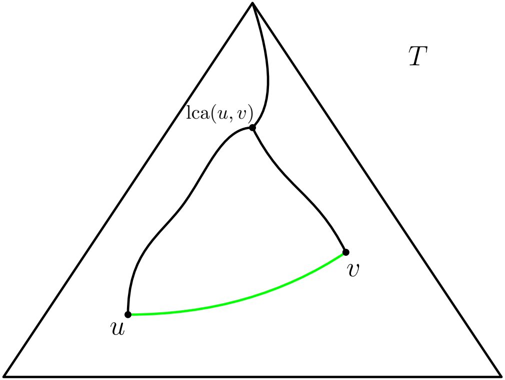

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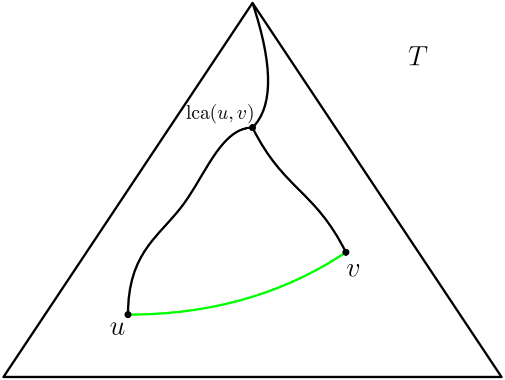

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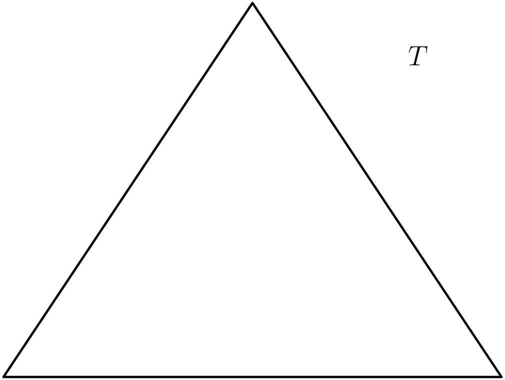

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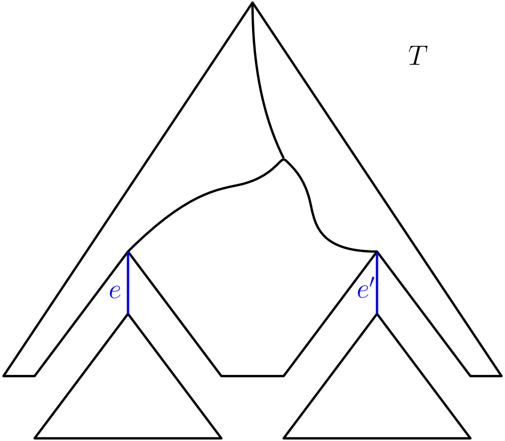

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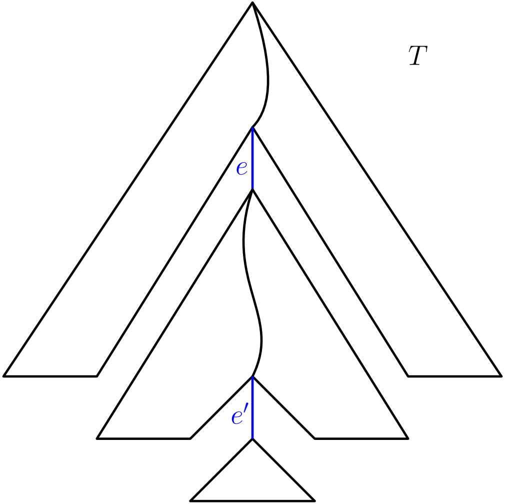

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For both cases, Karger's algorithm operates in $\mathcal{O}(\log n)$ phases that iteratively contracting paths (called *boughs*) consisting of nodes with exactly one child terminating at a leaf. Each phase is implemented with link-cut trees in $\mathcal{O}(m \log n)$ time.

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Minimum cut that 2-respects a spanning tree can be found in $\mathcal{O}(m \log^2 n)$ time.

But... no one was able to succeed in doing so!

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- Extend the approach for boughs directly to trees in order to avoid the $O(\log n)$ different phases needed for pruning boughs. This too would reduce the running time by a $\log n$ factor.

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1. The main technical contribution is a deterministic $\mathcal{O}(m \log n)$ time solution for the 2-respecting problem.
2. To obtain an improvement on the overall $\mathcal{O}(m \log^3 n)$ complexity of Karger's algorithm, we also design an alternative sampling procedure that produces the $\mathcal{O}(\log n)$ spanning trees in $\mathcal{O}(m \log^2 n)$ instead of $\mathcal{O}(m + n \log^3 n)$ time.
3. This gives us the minimum cut in $\mathcal{O}(m \log^2 n)$ time.

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Faster Sampling

1. Find constant approximation of c using Matula's algorithm. We implement it in $\mathcal{O}(m \log^2 n)$ time.
2. Edges with weight larger than c can be now contracted, and we think of an edge with weight w as w parallel unweighted edges.
3. Sample $\lceil pm \rceil$ (unweighted) edges, where $p = \Theta(\log n)/c$, to obtain graph H with minimum cut $c' = \mathcal{O}(\log n)$. This can be implemented in $\mathcal{O}(mc \cdot \log m) = \mathcal{O}(m \log^2 n)$ time w.h.p.
4. Apply the following specialised instantiation of Young's variant of the Lagrangian packing technique:

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$\mathcal{O}(c' \cdot \log n) = \mathcal{O}(\log^2 n)$ iterations, each in $\mathcal{O}(m)$ time.

Faster Sampling

1. Find constant approximation of c using Matula's algorithm. We implement it in $\mathcal{O}(m \log^2 n)$ time.
2. Edges with weight larger than c can be now contracted, and we think of an edge with weight w as w parallel unweighted edges.
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We obtain in $\mathcal{O}(m \log^2 n)$ time a collection of $\mathcal{O}(\log n)$ spanning trees $T_1, T_2, \ldots$ such that the minimum cut 1- or 2-respects some T_i.

Now we iterate over every tree T_i and:

1. Find the minimum 1-respecting cut in $\mathcal{O}(m)$ time.
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Dependent Edges

Link-cut trees implement the following operations in $\mathcal{O}(\log n)$ time:

1. add Δ to the score of every edge on a path,
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We binarise T and sweep over its edges maintaining the scores.

For every $e' \in T(e)$, the weight of a cut defined by $\{e, e'\}$ is the score of e' + the total weight of all edges with exactly 1 endpoint in $T(e)$.

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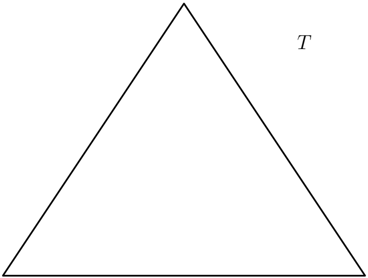

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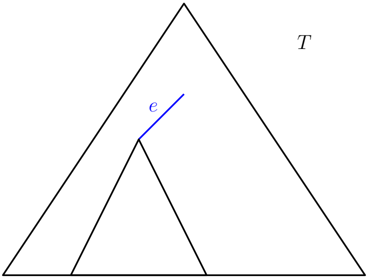

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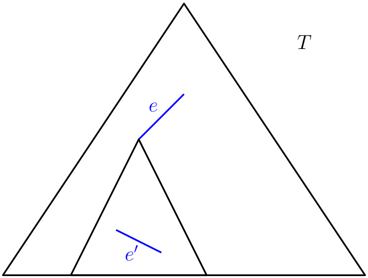

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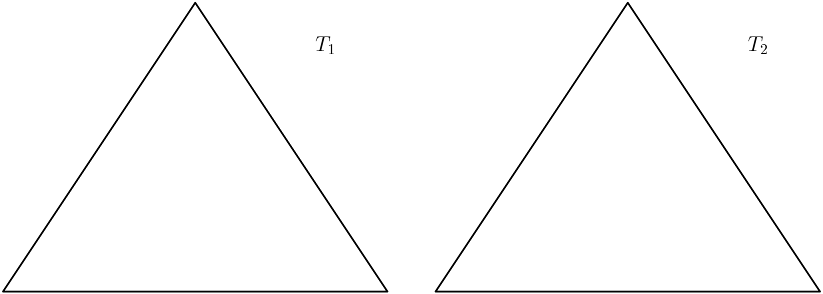
Bipartite Problem

We are given two trees and a number of cross-edges:



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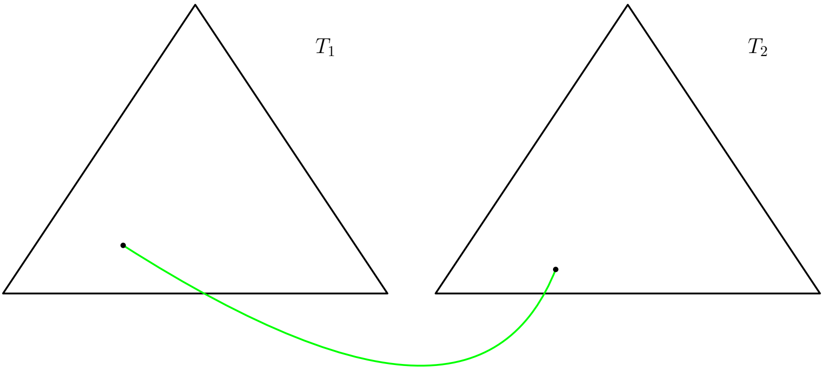

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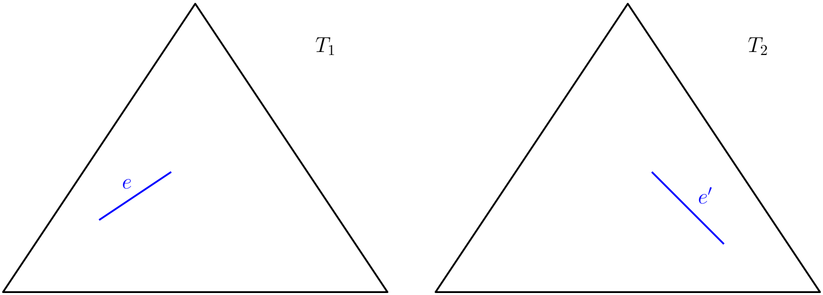

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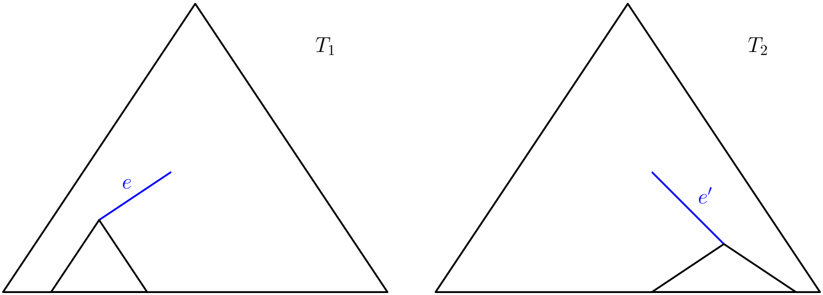

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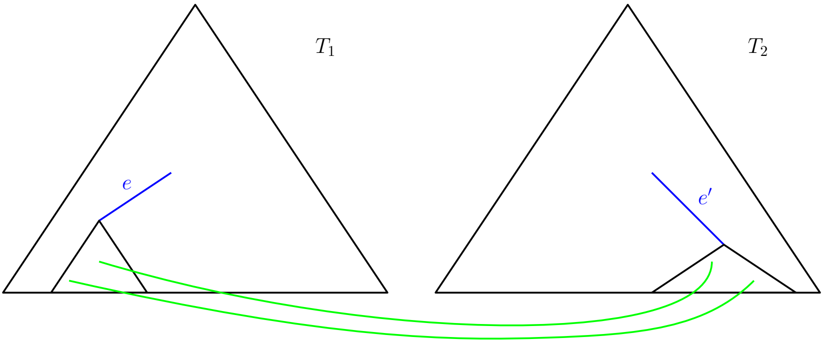

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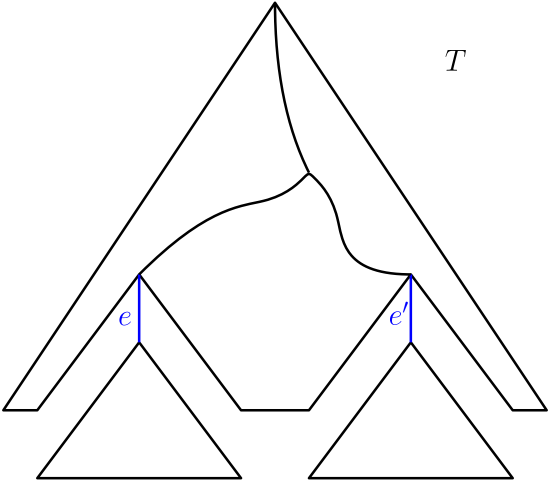
Reduction to Bipartite Problems

We create a separate instance for every node x with two children:



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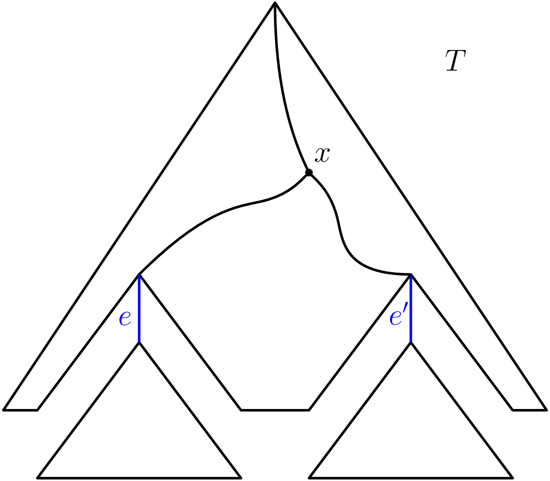

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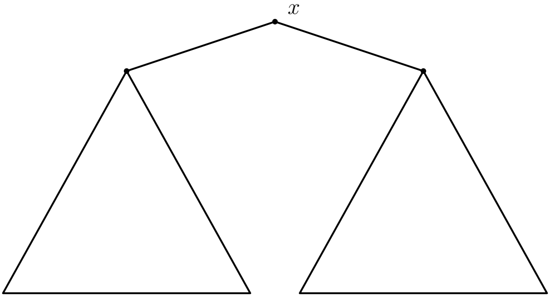

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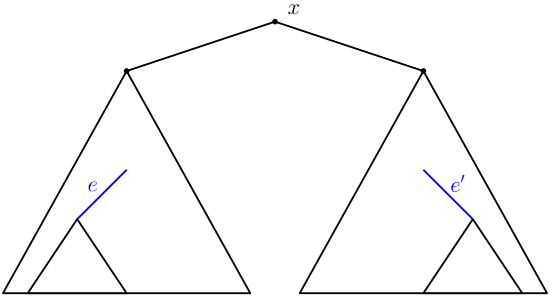

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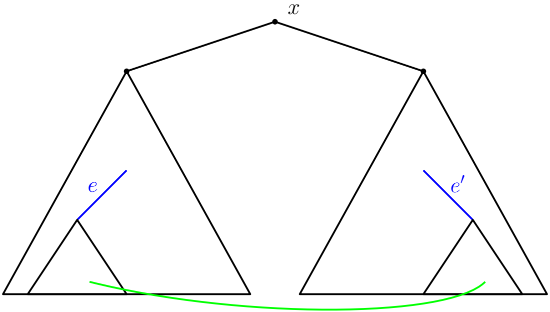

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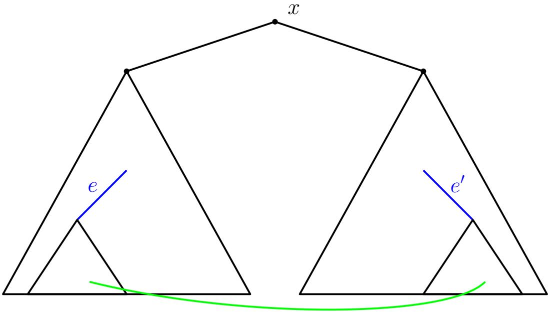

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$s =$ number of edges (u, v) with $\mathrm{lca}(u, v) = x$.

Even though both trees might be large, only $\mathcal{O}(s)$ edges really matter.

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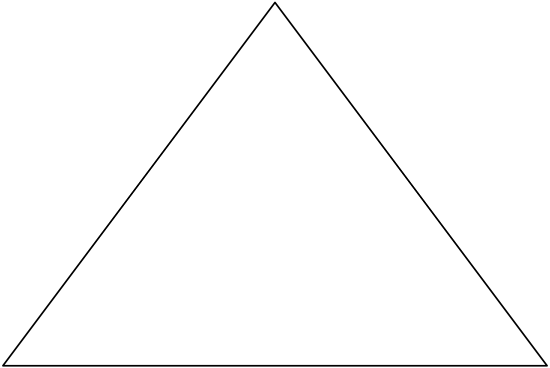

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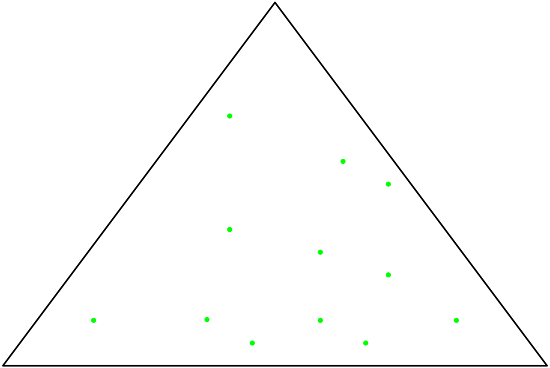

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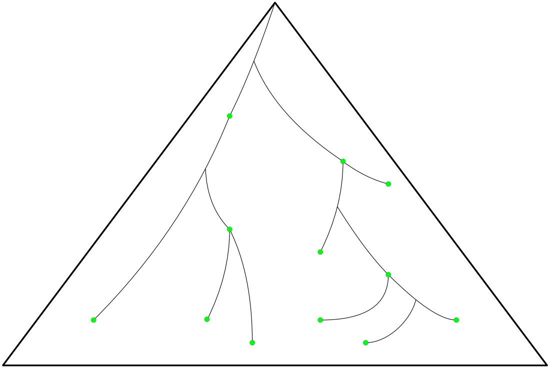

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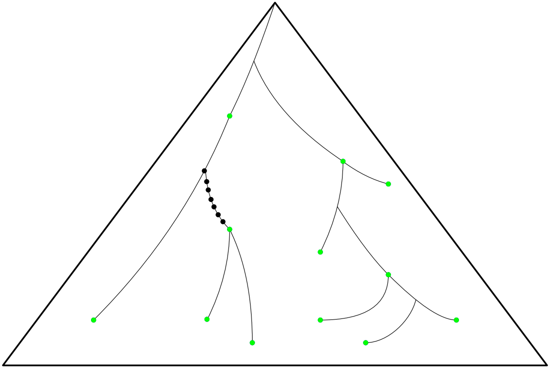

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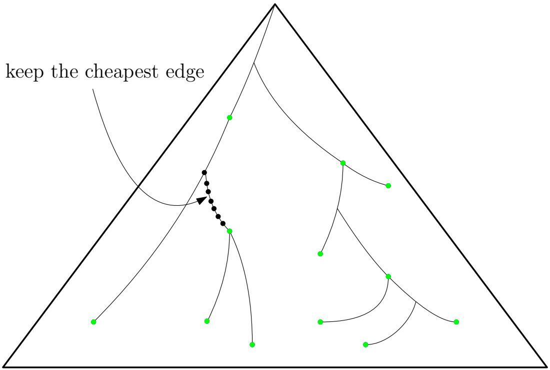

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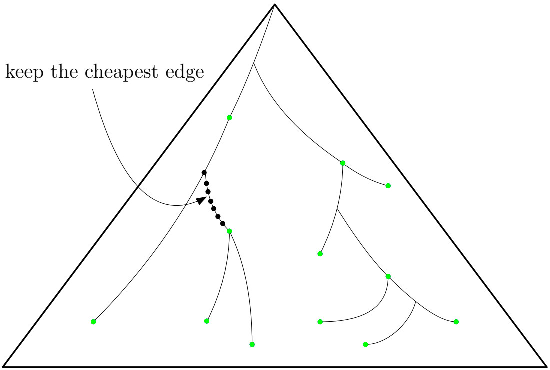

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In every recursive call we operate on a fragment of T_1: the subtree rooted at u_1 (possibly) without the subtree rooted at u_k.

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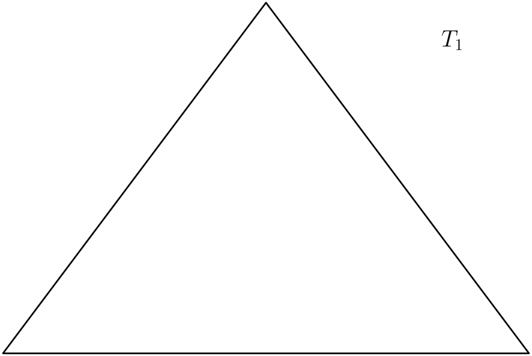

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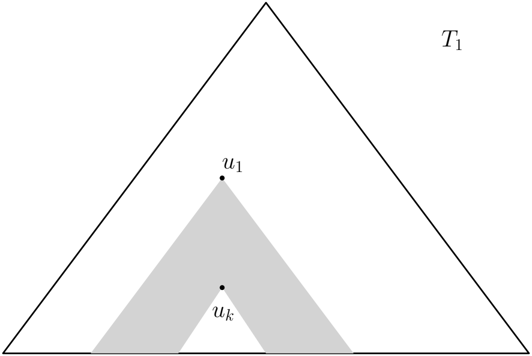

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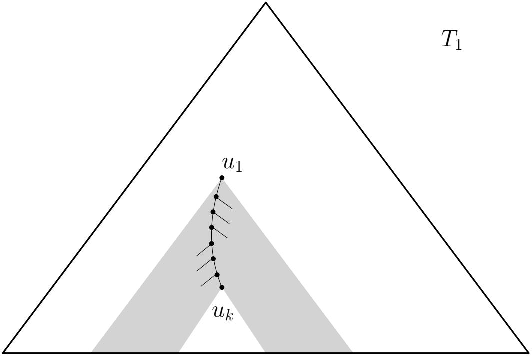

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By appropriately splitting the current fragment into smaller fragments, we ensure that the depth of the recursion is $\mathcal{O}(\log n)$, and fragments on every level of the recursion are disjoint. But what about T_2?

We maintain a compressed representation of the relevant part of T_2.

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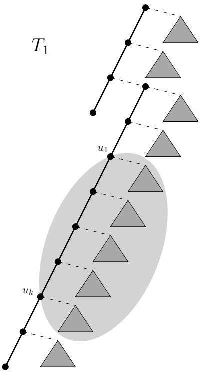

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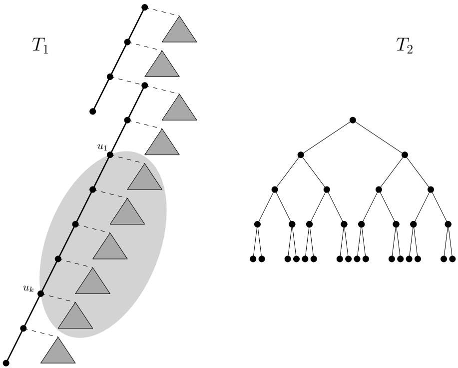

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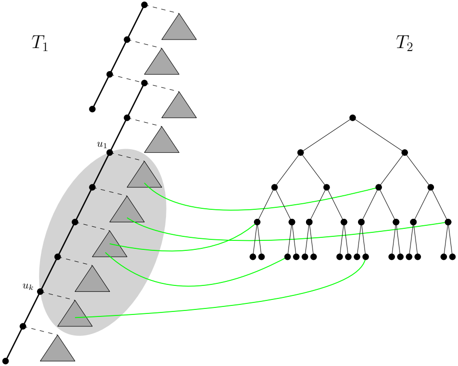

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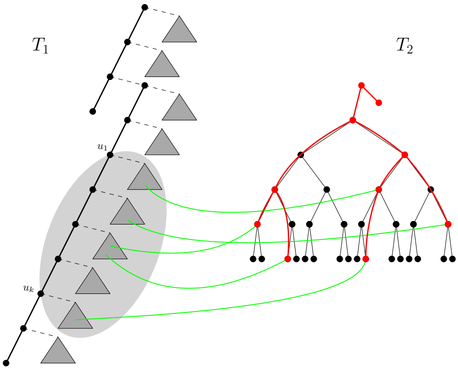

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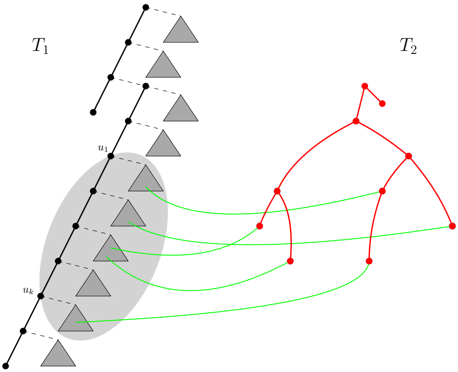

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Recursion

In every recursive call we have a fragment f of T_1 and a compressed representation of the relevant part of T_2 of size $\mathcal{O}(s)$, where s is the number of cross-edges with one endpoint in f. Then:

1. We find the cheapest $e' \in T_2$ for $\mathcal{O}(1)$ edges $e \in T_1$ by considering all $\mathcal{O}(s)$ edges in the compressed representation.
2. We partition the remaining into $\mathcal{O}(1)$ fragments $f_1, f_2, \ldots$.
3. For every f_i, we extract a compressed representation of its relevant part of T_2 from the current compressed representation in $\mathcal{O}(s)$ time, and recurse.

We make sure that the depth of the recursion is $\mathcal{O}(\log n)$, and then the whole running time becomes $\mathcal{O}(m \log n)$.

There are some technicalities, in particular we actually maintain two separate compressed representations, please see the paper.

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1. Is there a faster algorithm?
2. Is there a near-linear time deterministic algorithm?

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