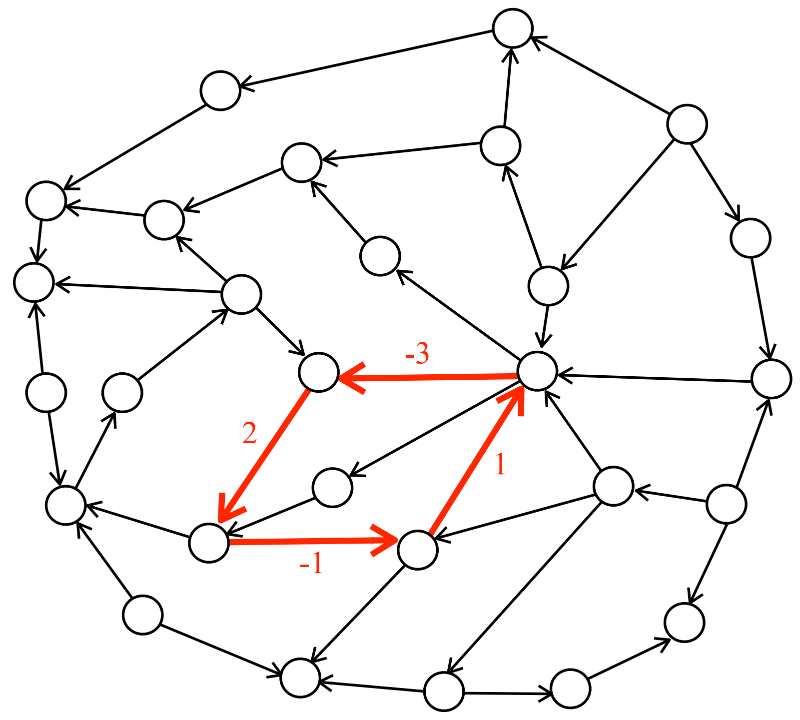
Planar Negative k-Cycle

Paweł Gawrychowski,
Shay Mozes,
Oren Weimann



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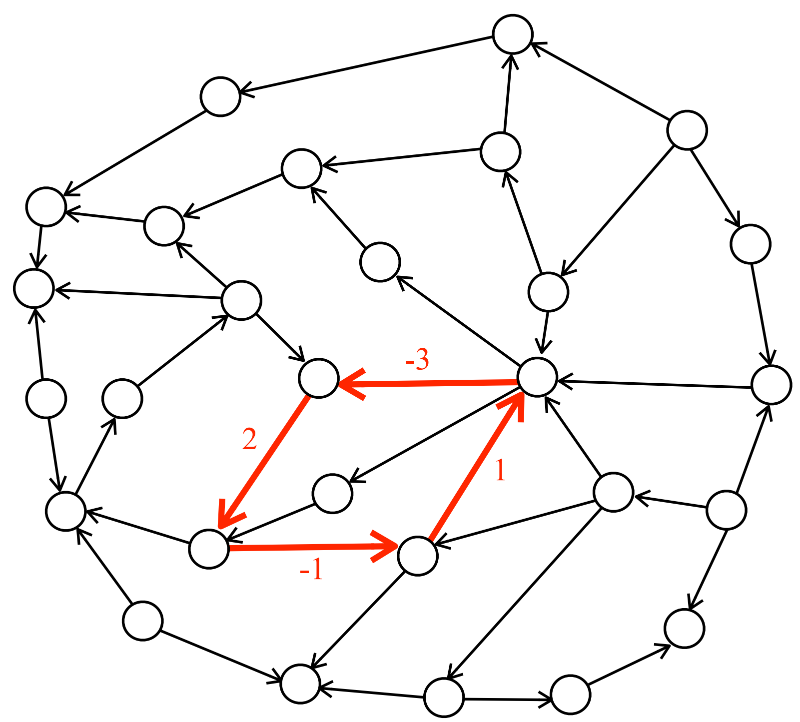
Given a directed edge-weighted graph G,
Does G contain a negative-weight cycle with at most k edges?



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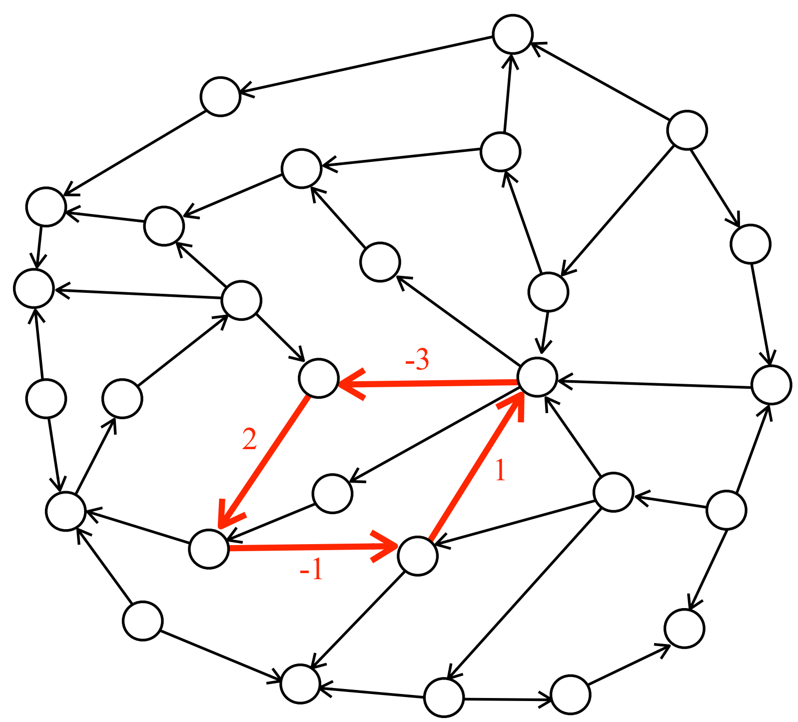

$k \geq 4$

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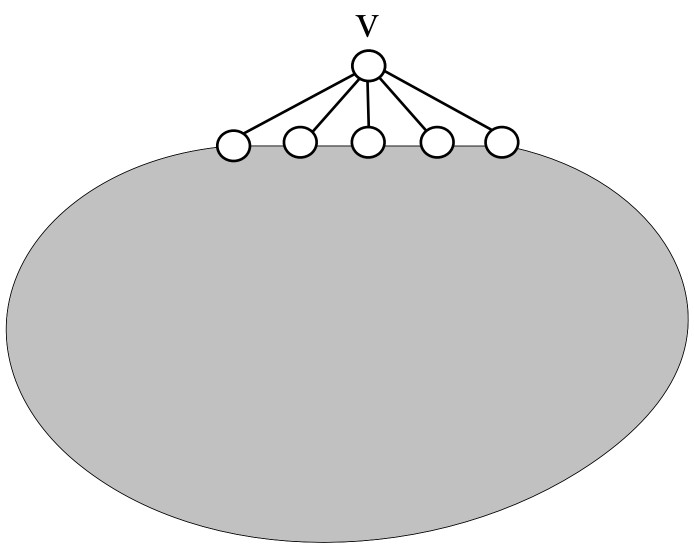

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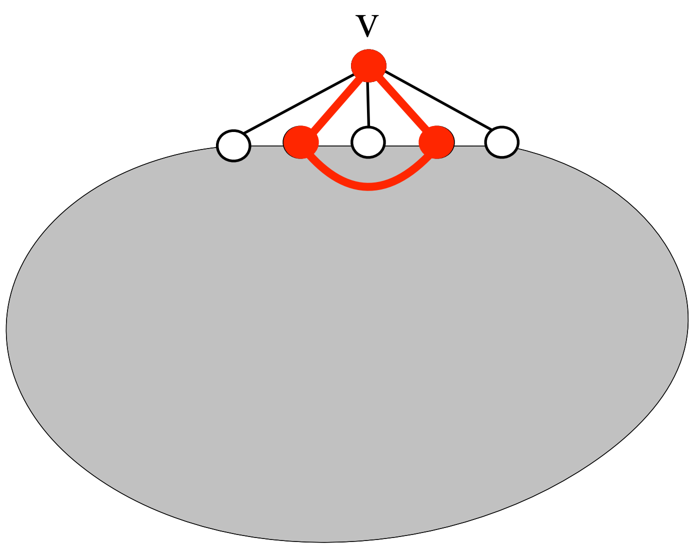

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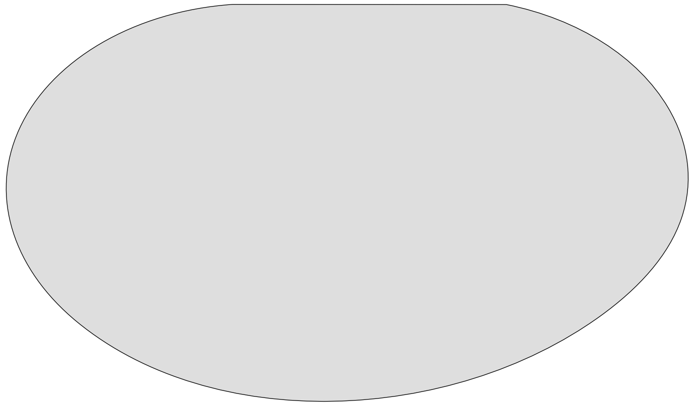

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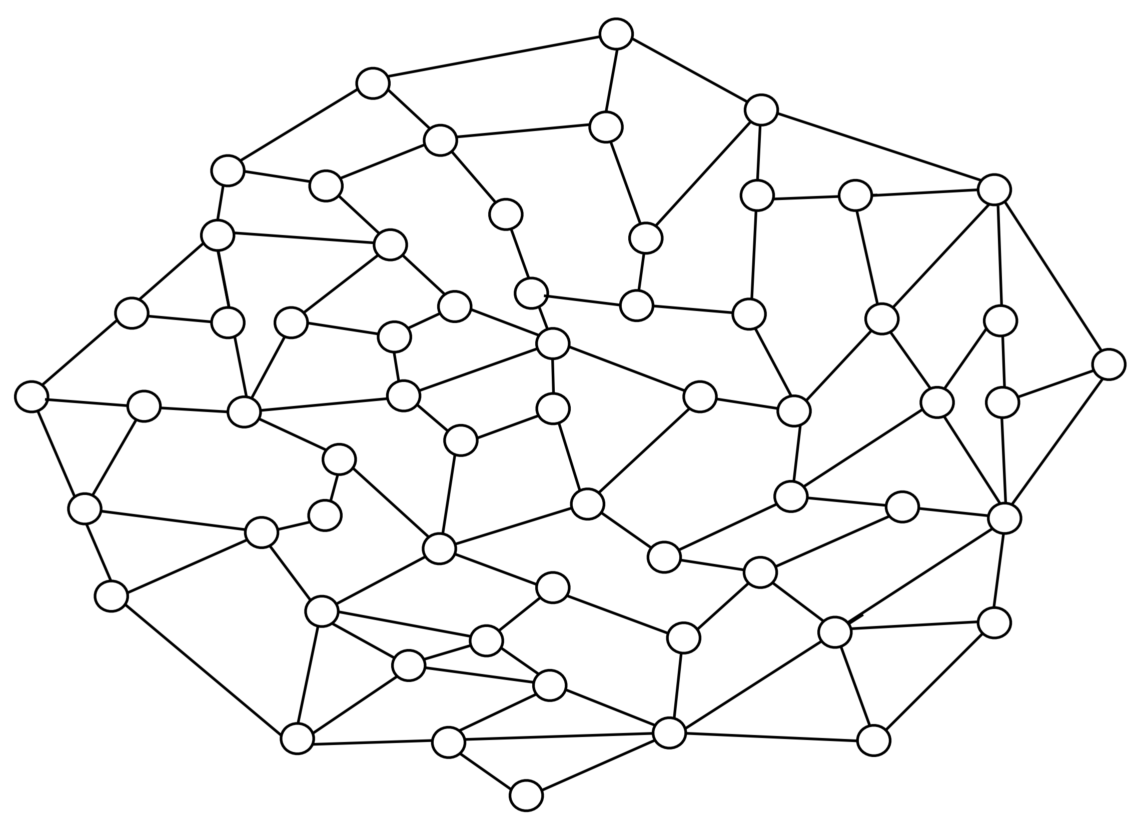

An Õ(nk²) Algorithm

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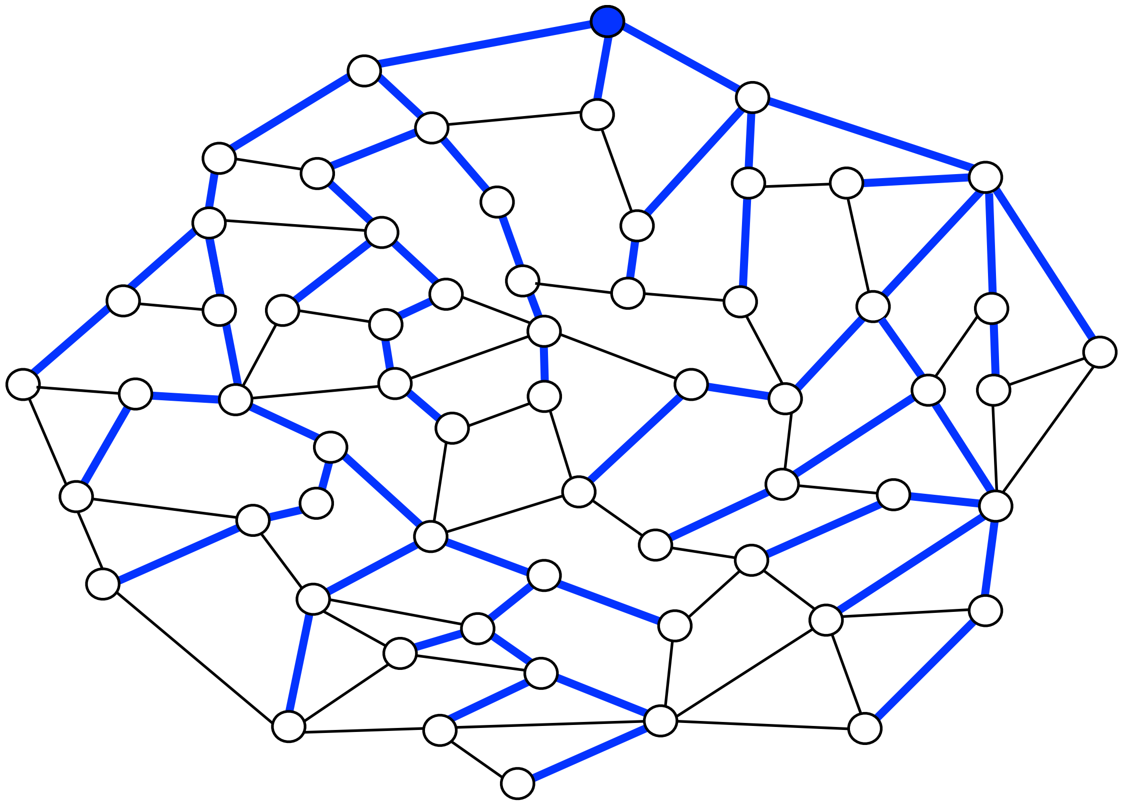

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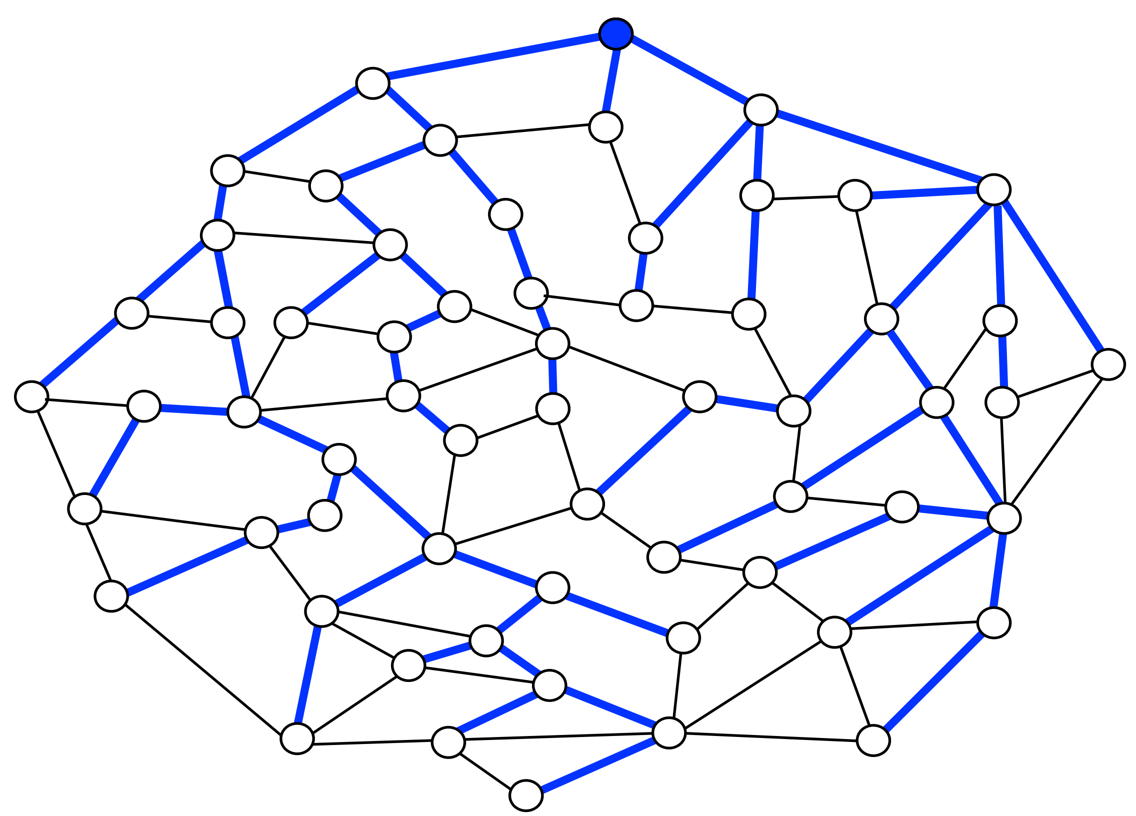

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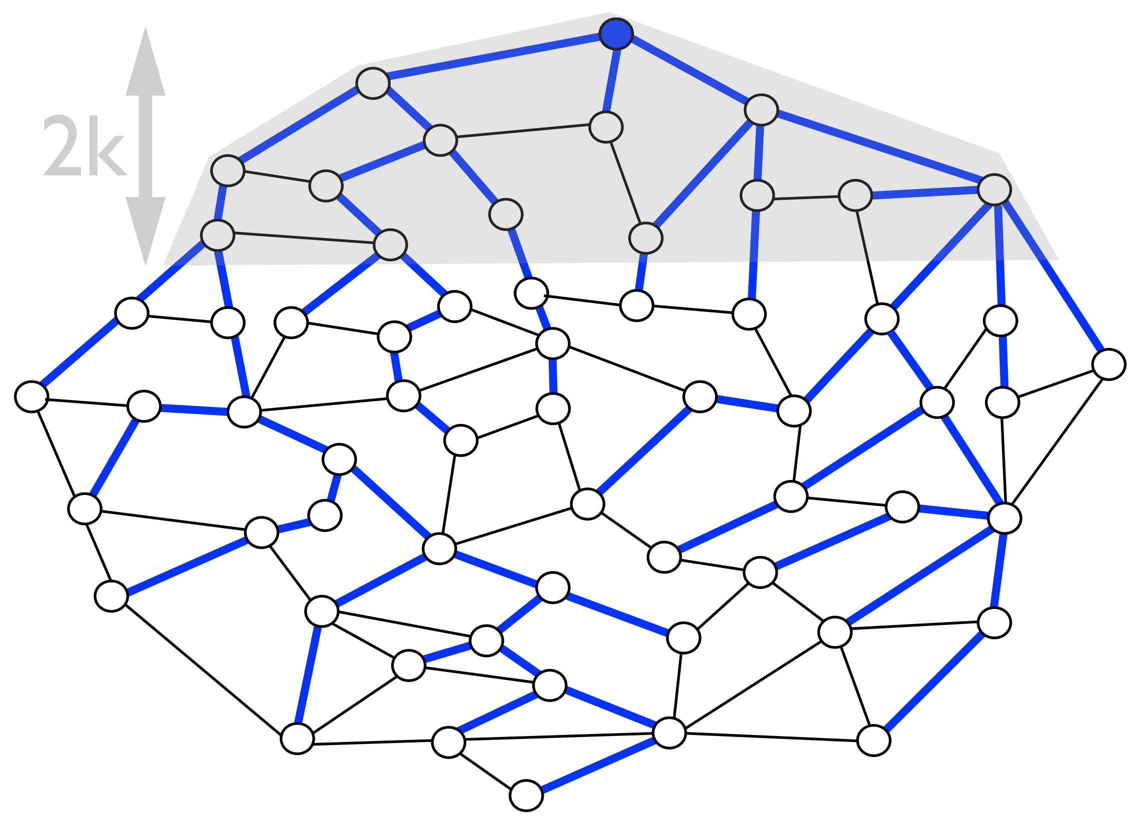

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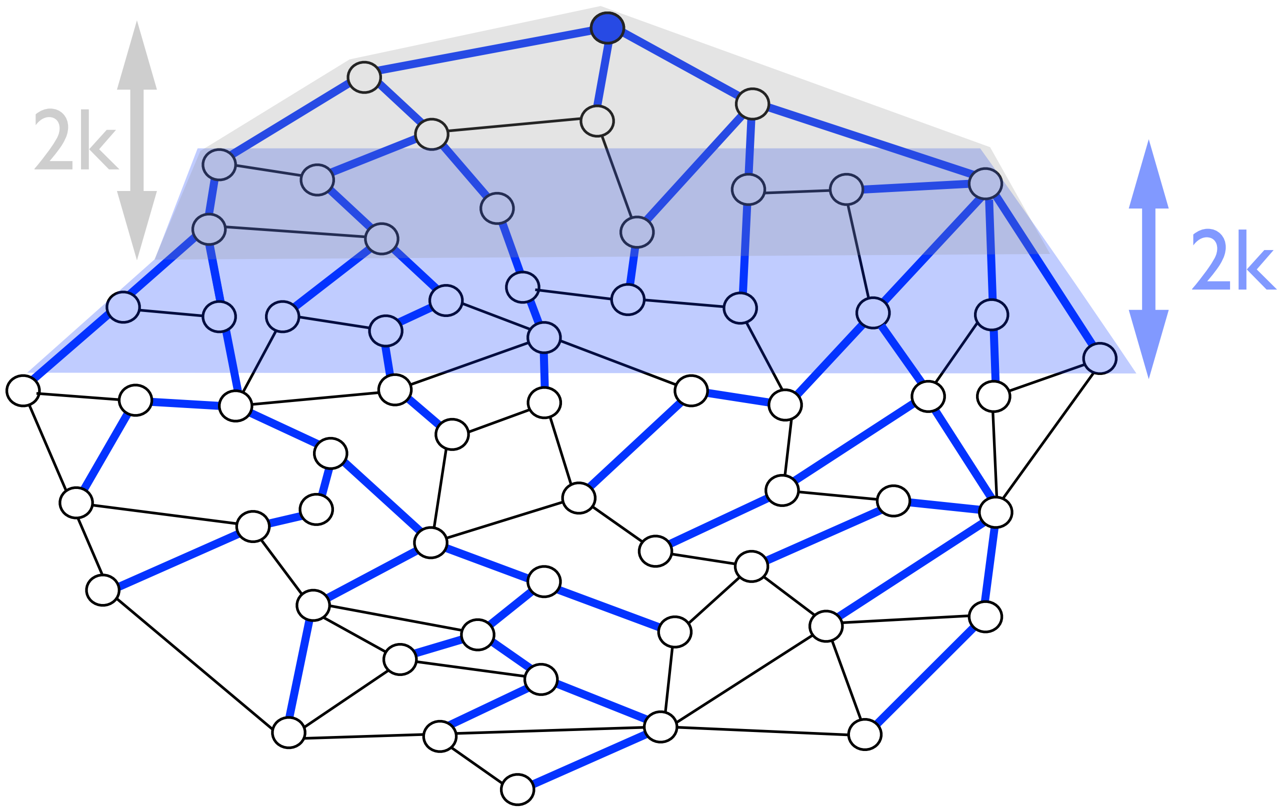

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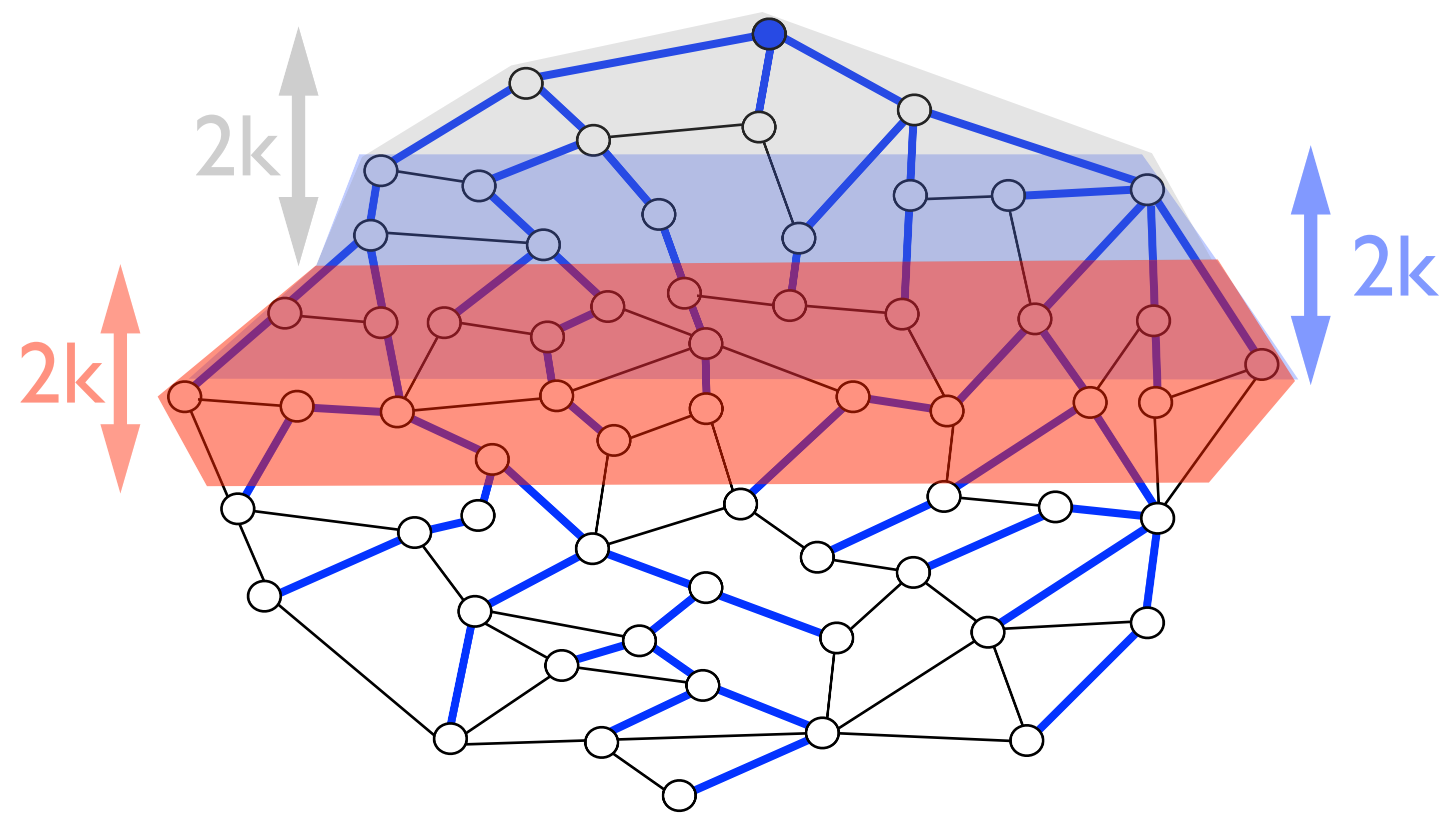

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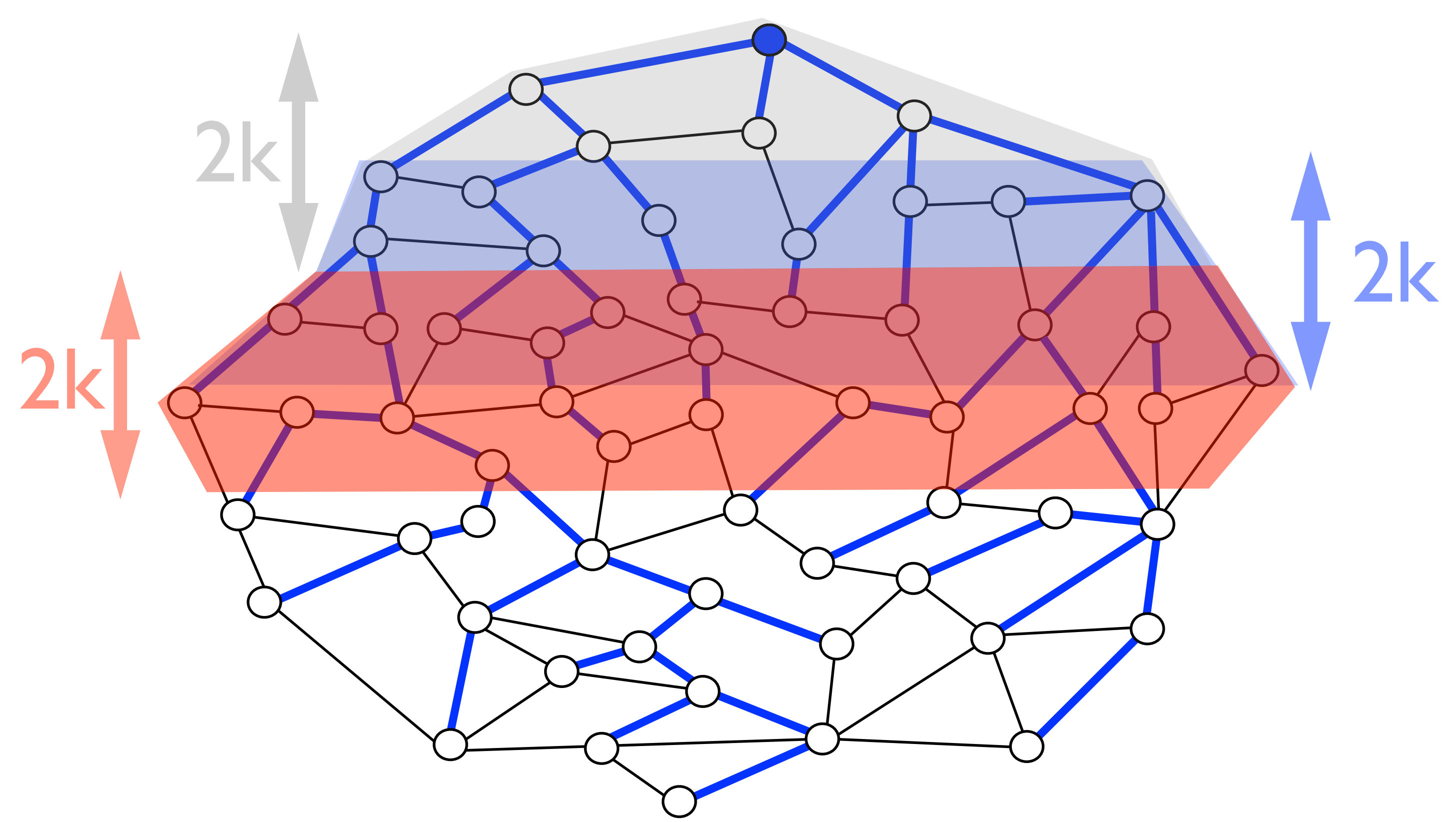

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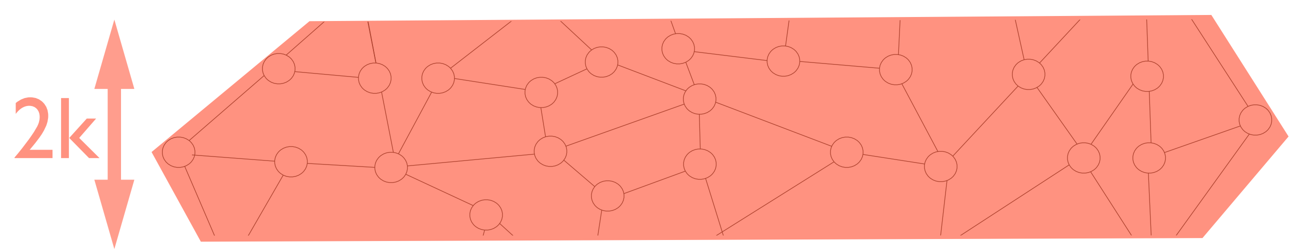

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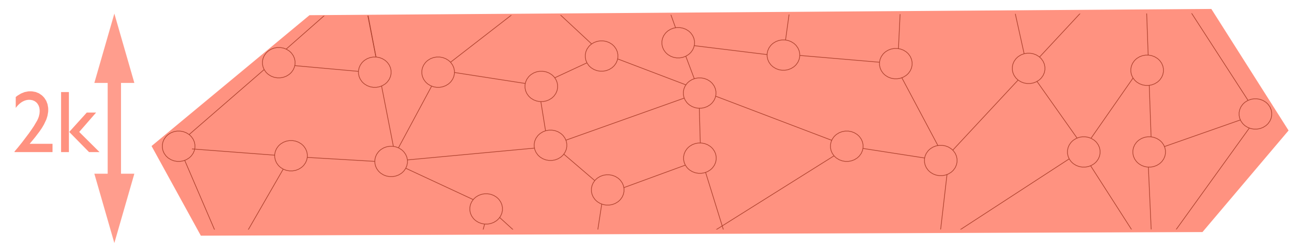

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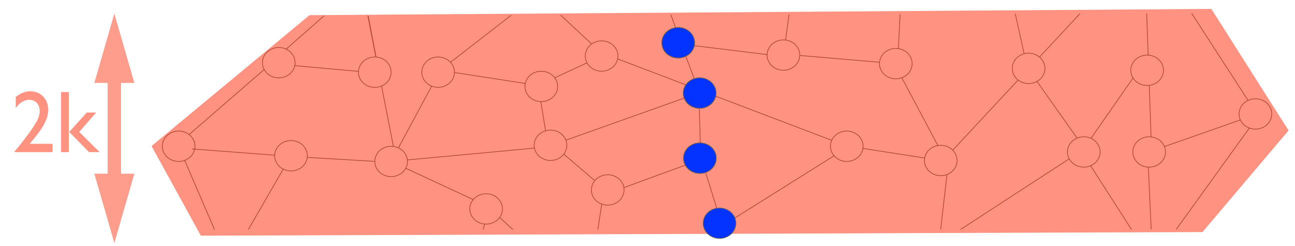

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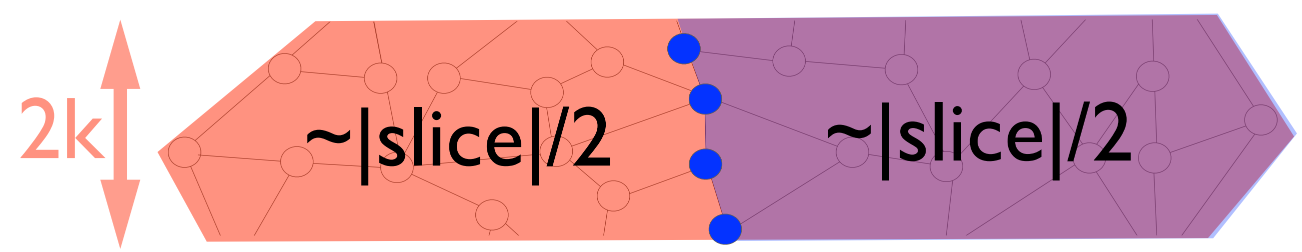

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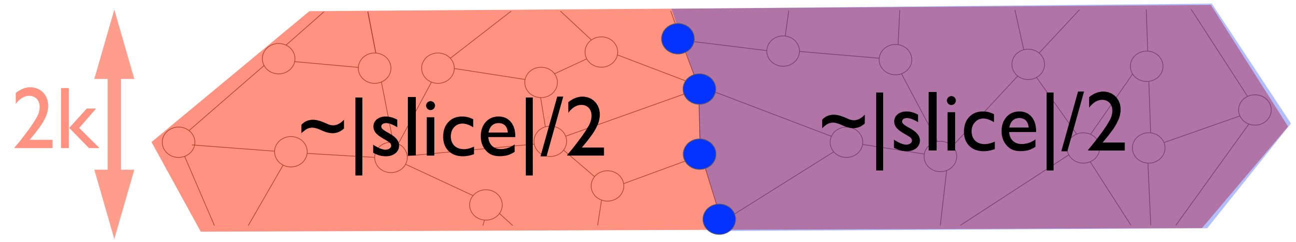

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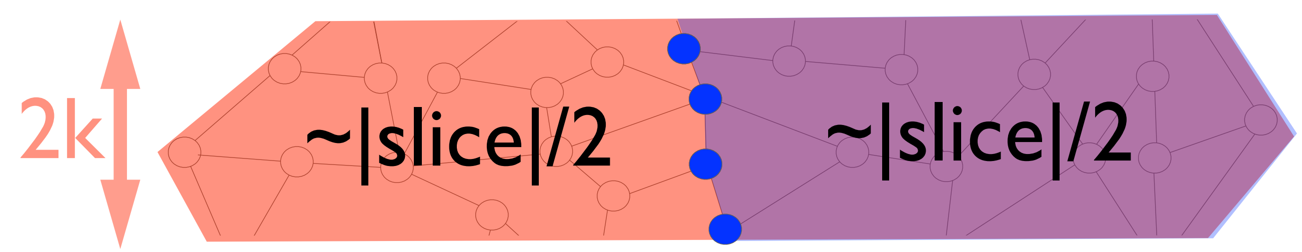

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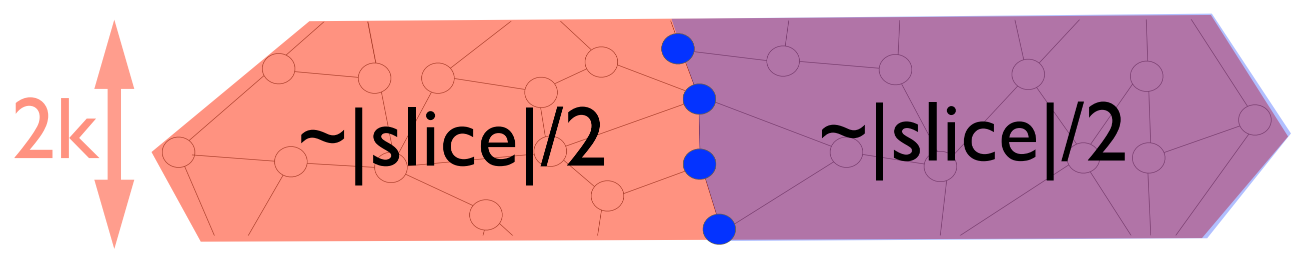

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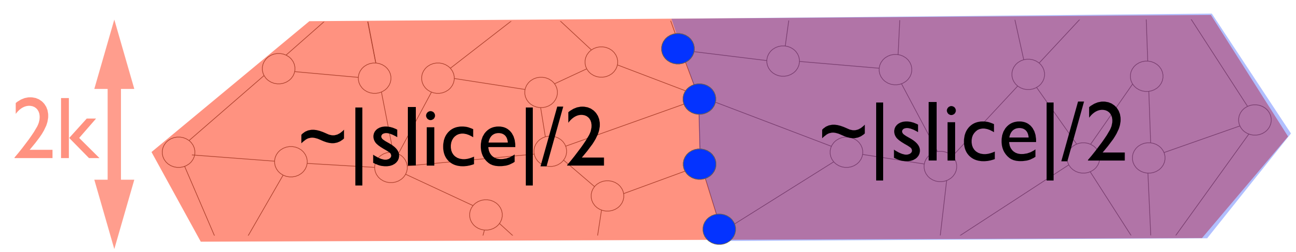

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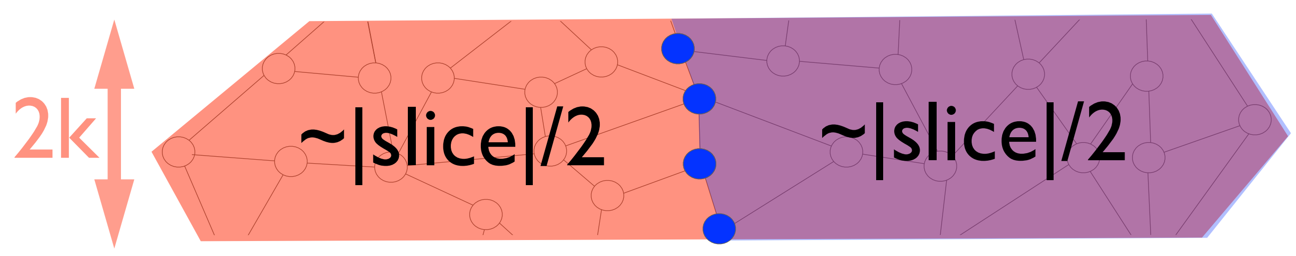

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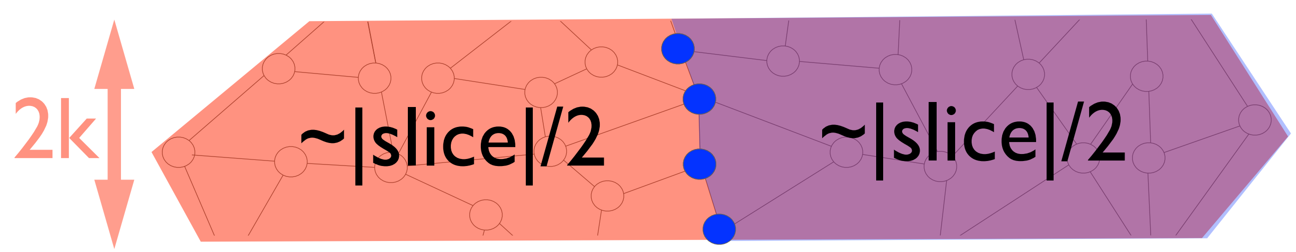

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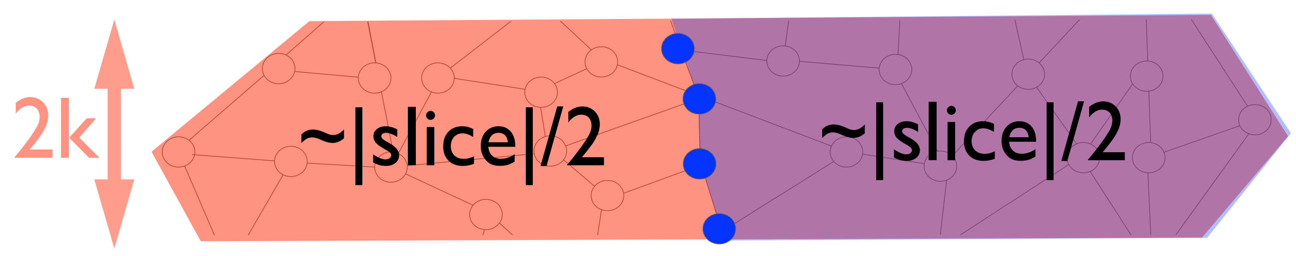

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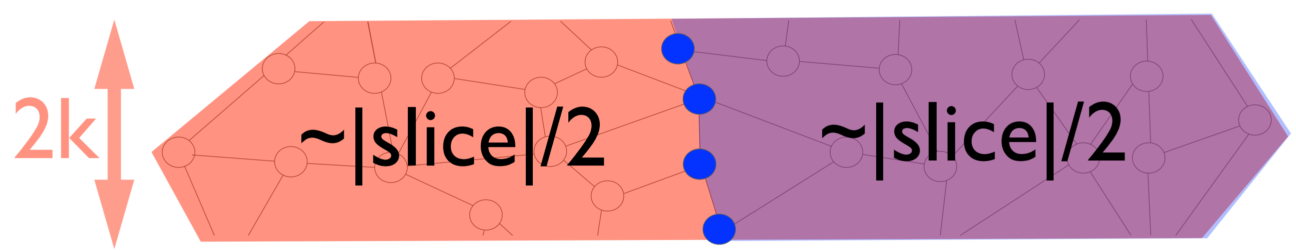

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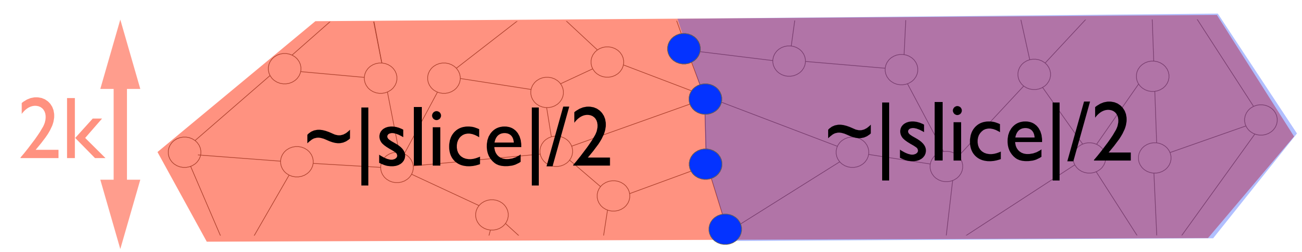

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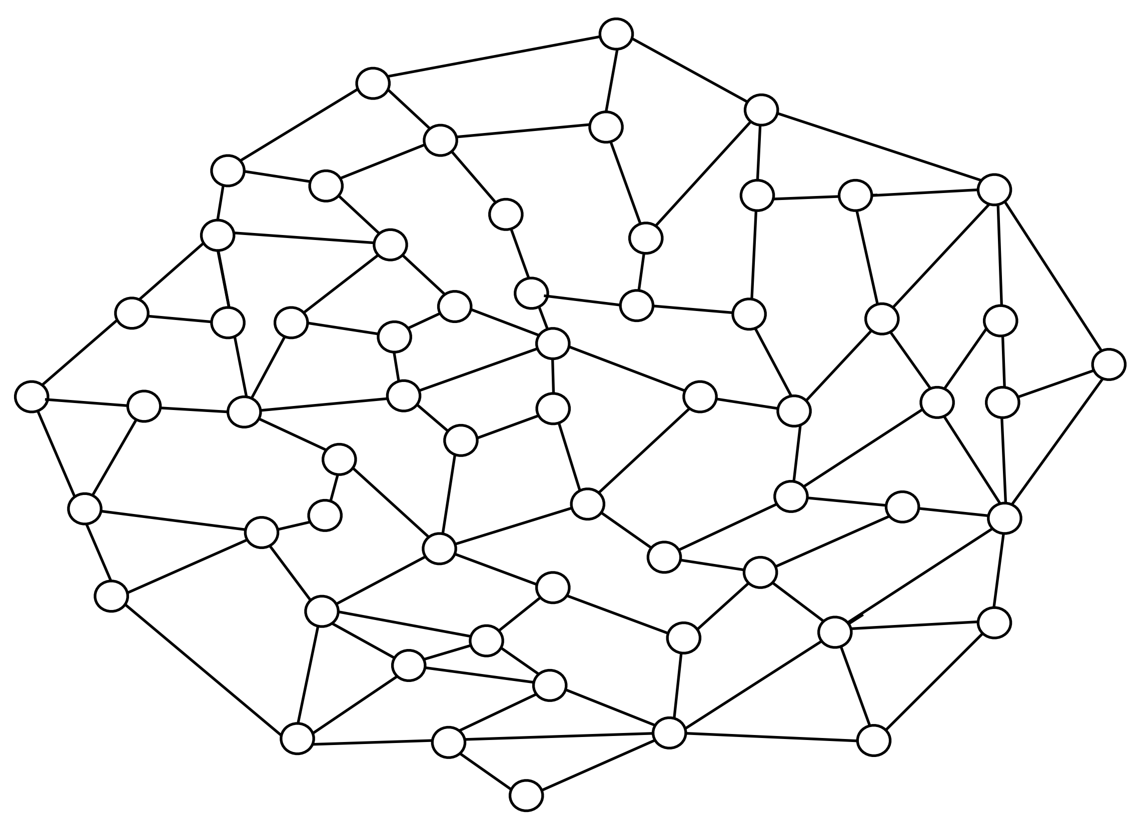

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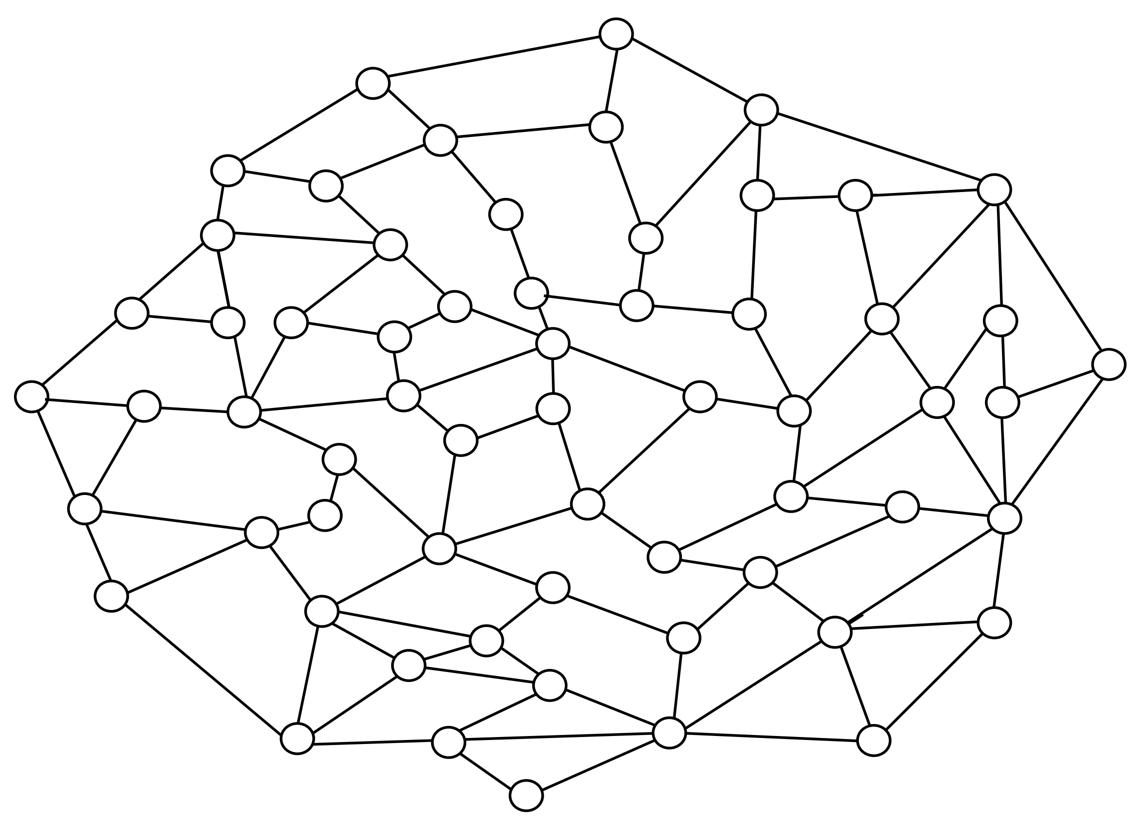

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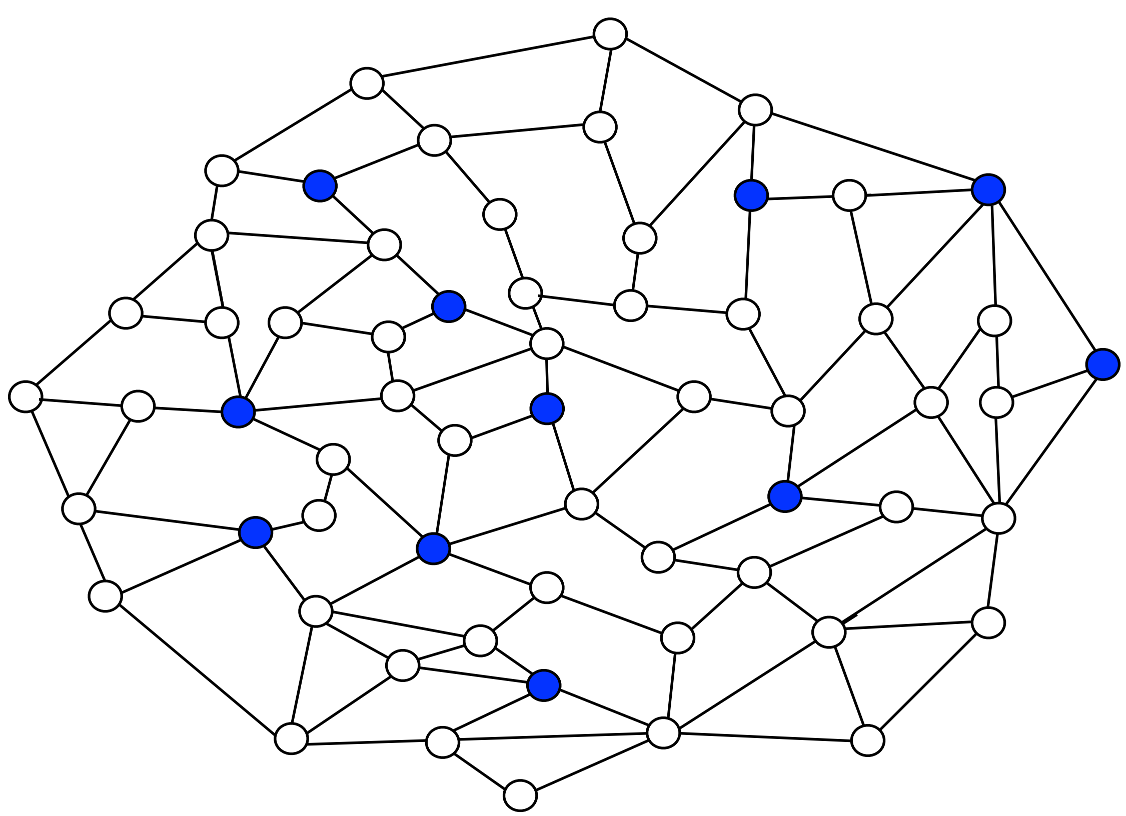

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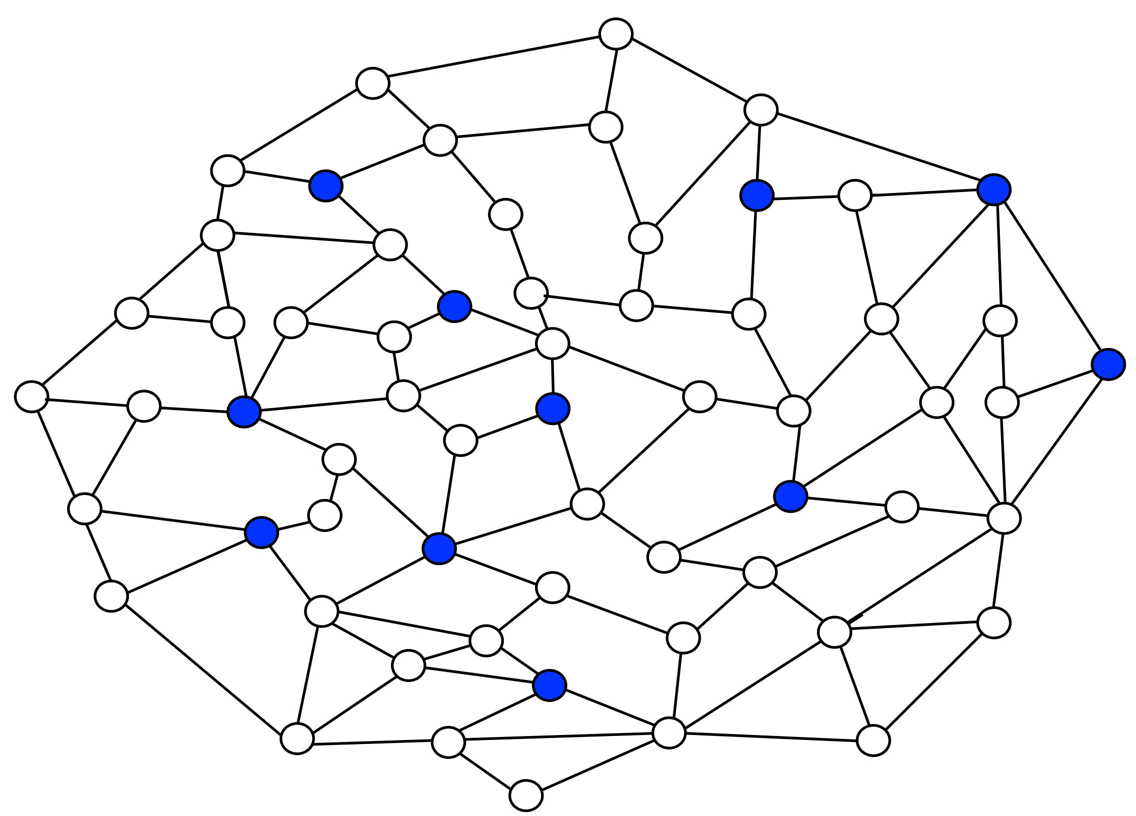

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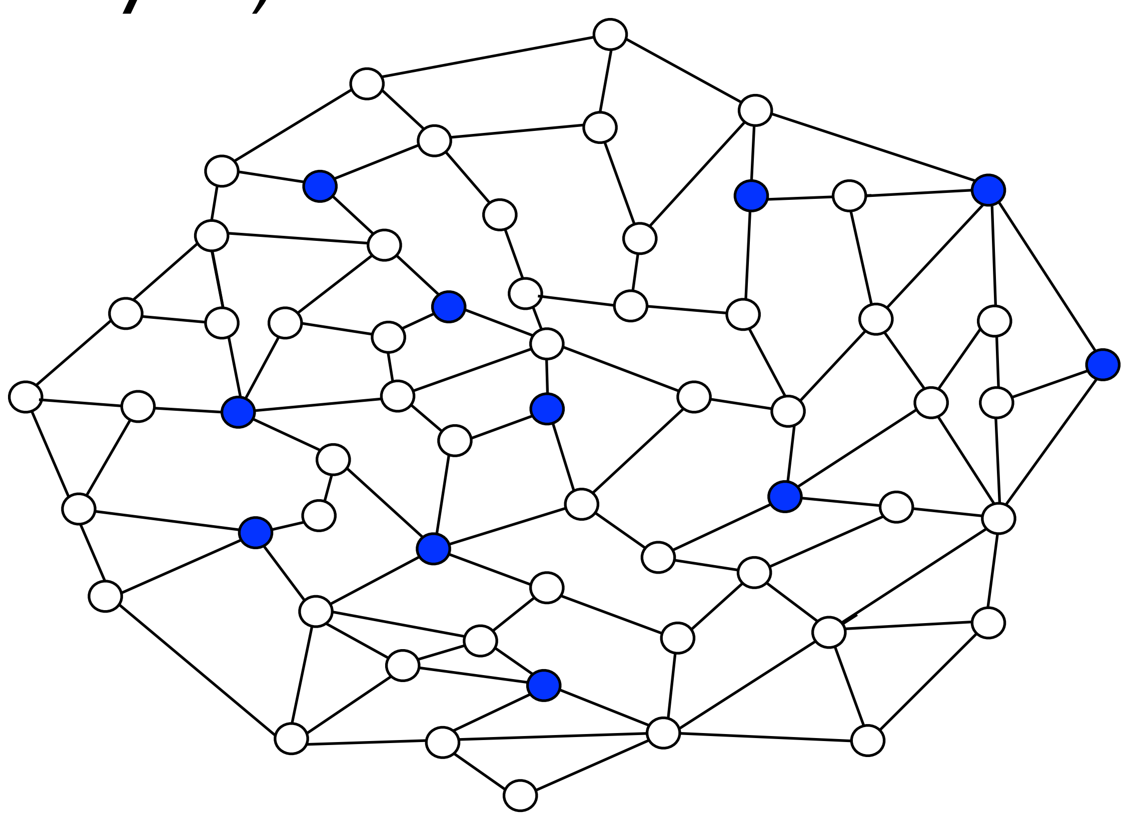

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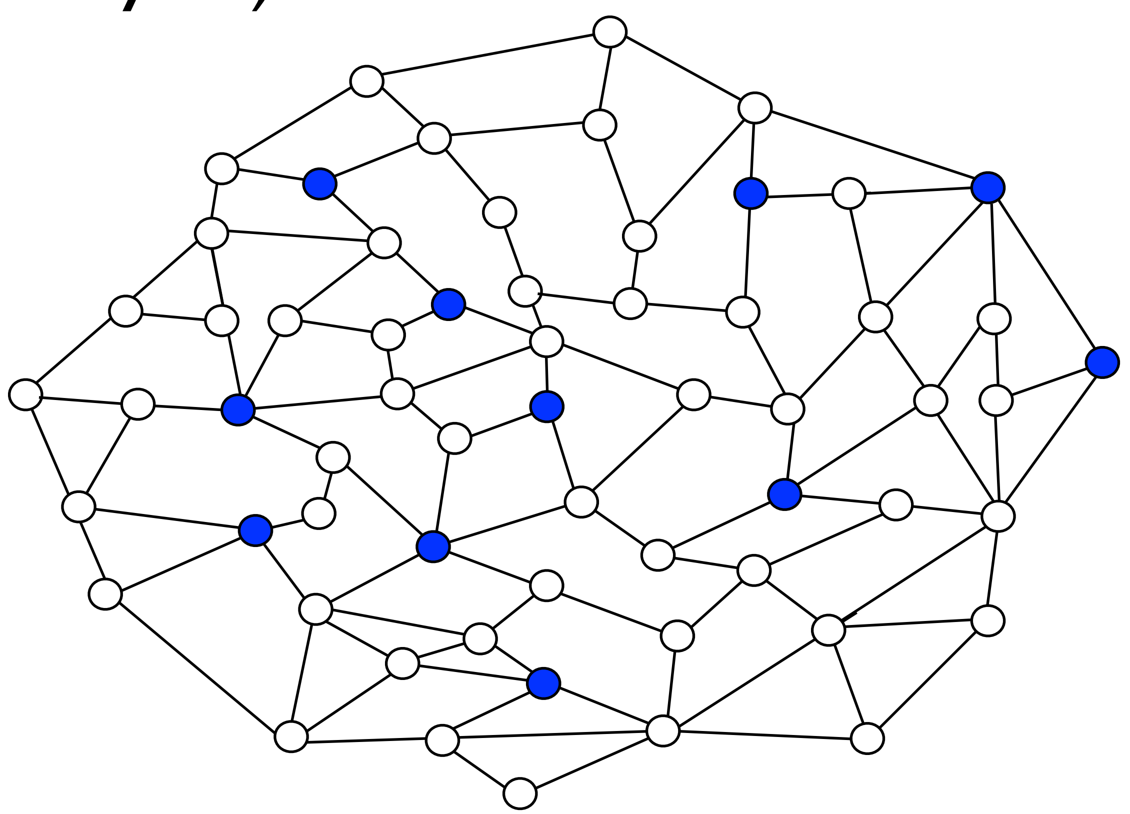

Time = $(n/k) \cdot O(n \cdot k)$

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The first non-trivial tight bound for a problem in planar graphs

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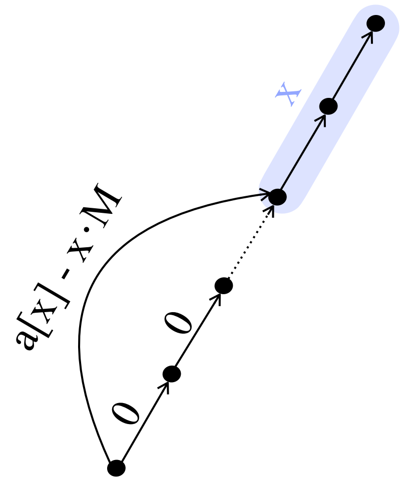

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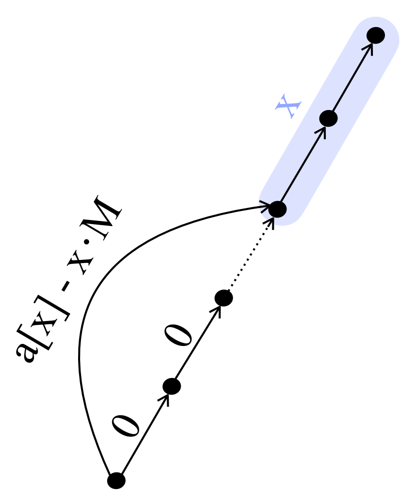

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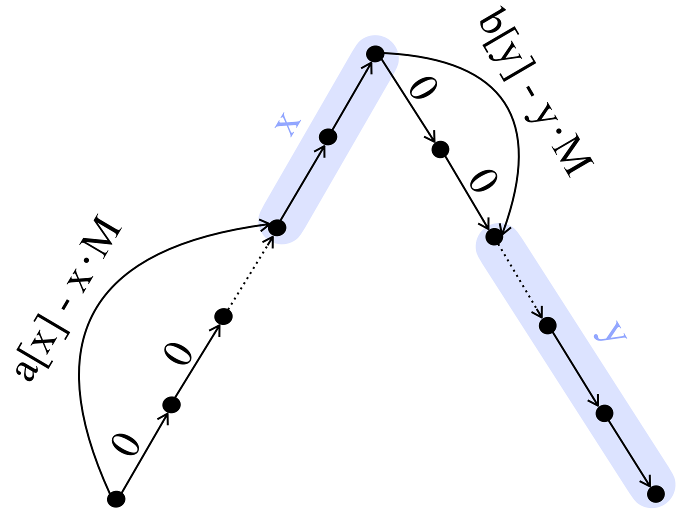

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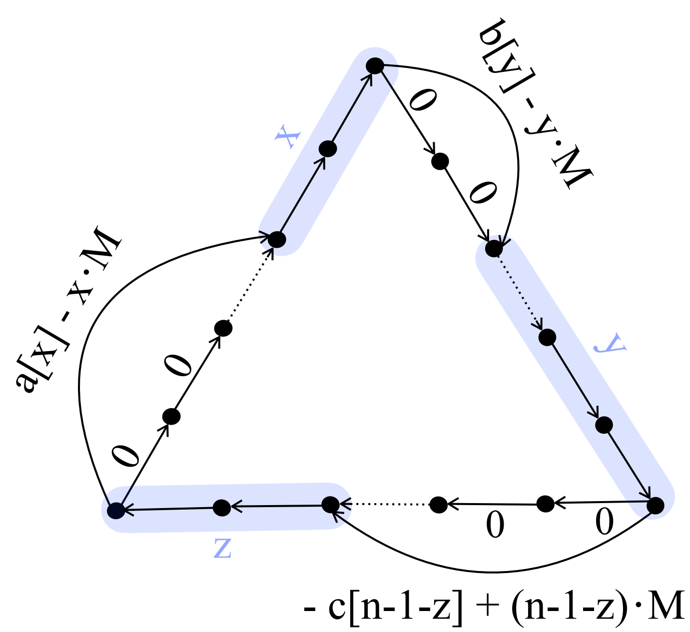
For $k = \Theta(n)$, Negative k-Cycle cannot be solved in $O(n^{2-\varepsilon})$ time

Simple proof (inspired by [AbboudCohen-AddadKlein 2020]):



Conjectures

min-plus convolution Conjecture
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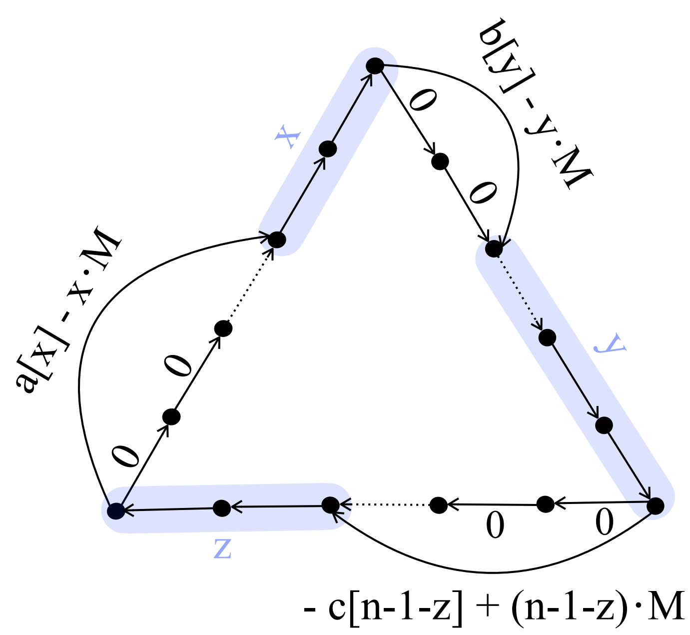

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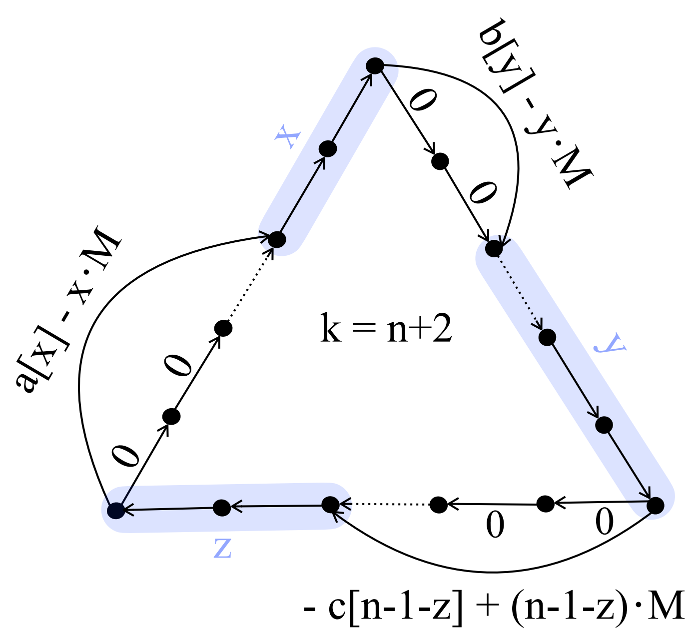

Conjectures

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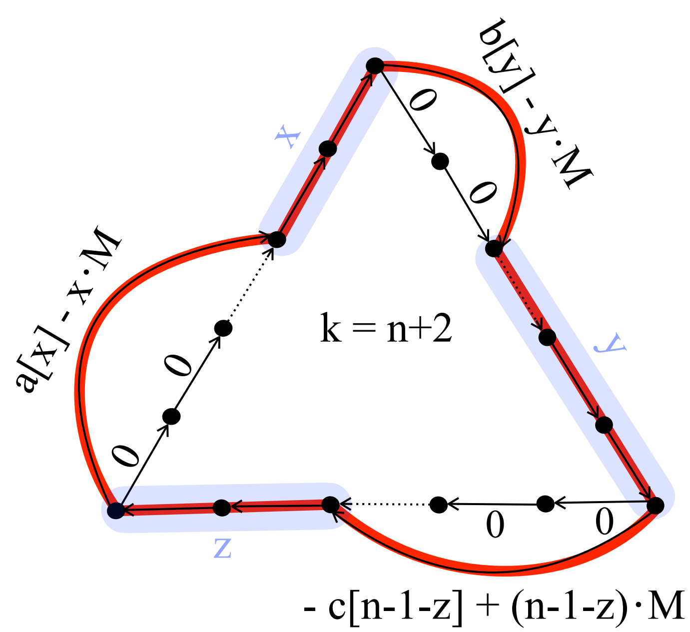

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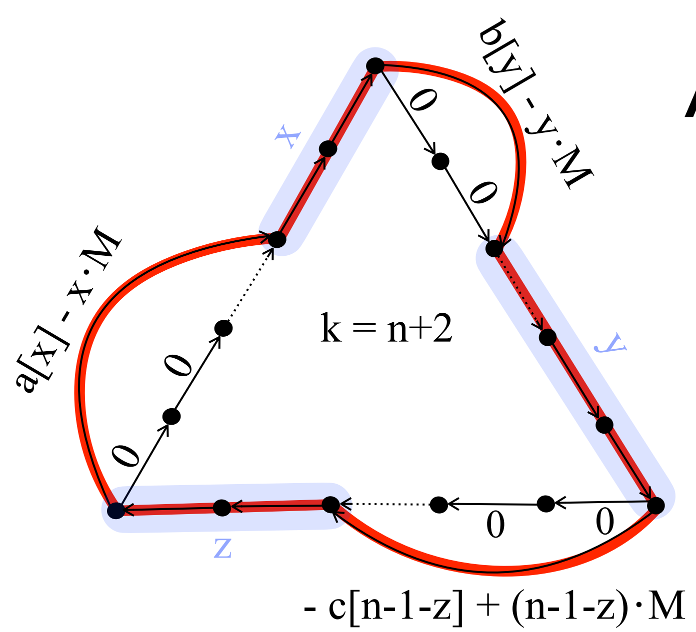

At most n+2 edges: $x + y + z + 3 \leq n + 2$

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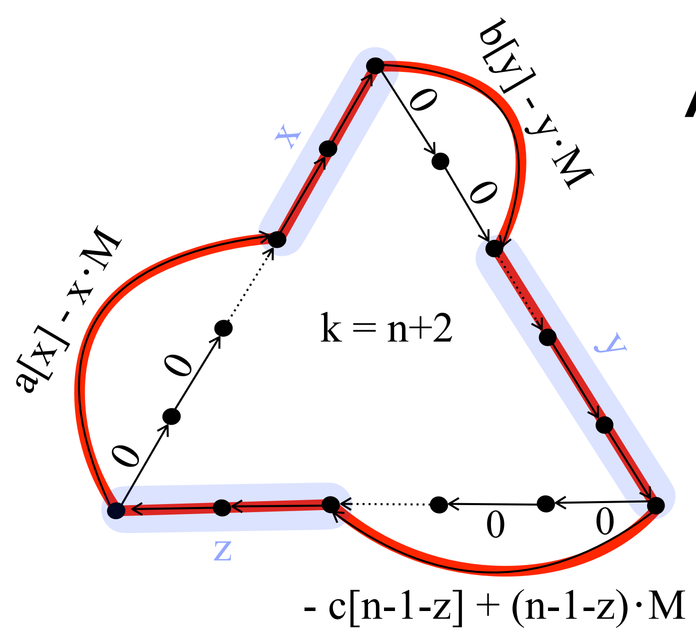

At most n+2 edges: $x + y + z \leq n - 1$

Conjectures

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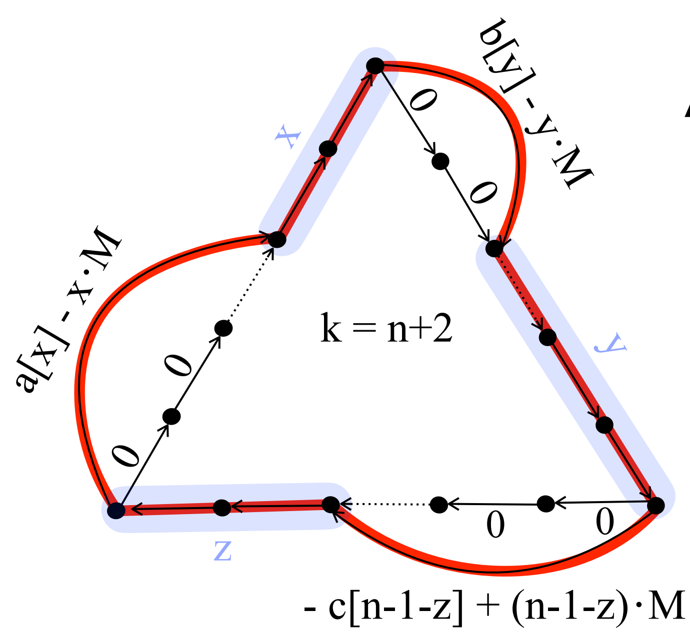

At most n+2 edges: $x + y + z \leq n - 1$

Negative: $a[x] + b[y] - c[n-1-z] - x \cdot M - y \cdot M + (n-1-z) \cdot M < 0$

Conjectures

min-plus convolution Conjecture
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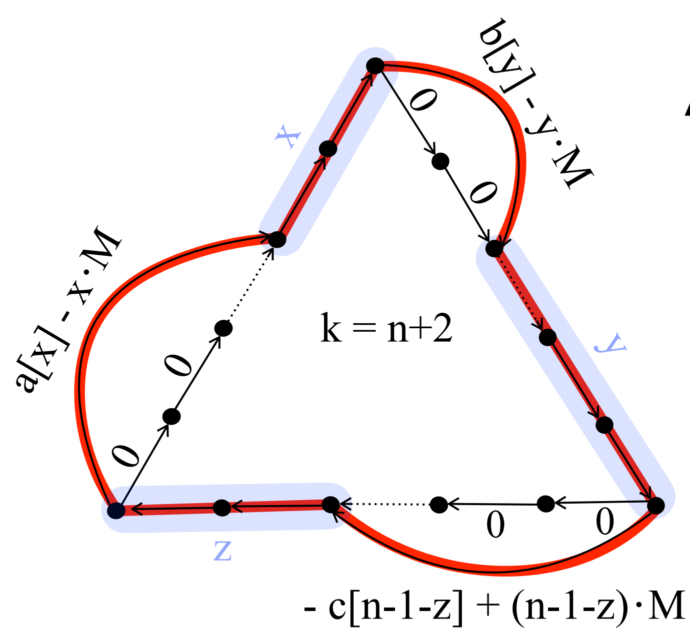

At most n+2 edges: $x + y + z = n - 1$

Negative: $a[x] + b[y] - c[n-1-z] - x\cdot M - y\cdot M + (n-1-z)\cdot M < 0$

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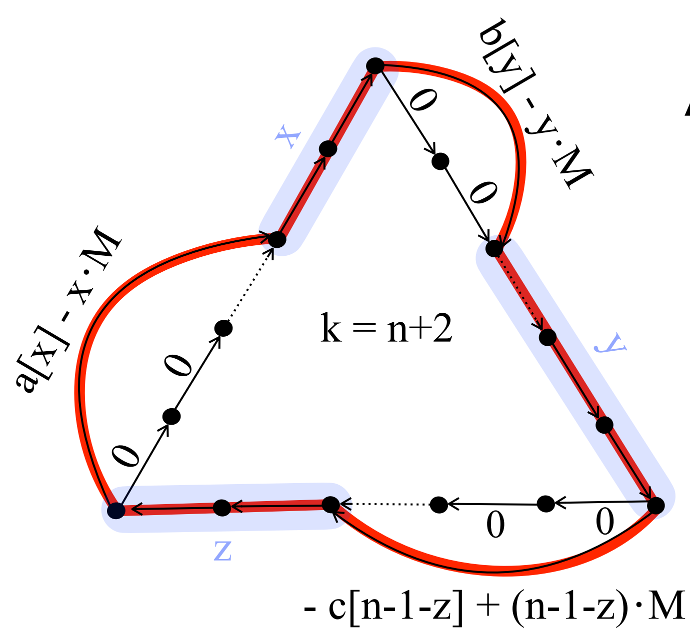

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Iff: $a[x]+b[y] < c[z]$ for some x,y, and z=x+y

Conjectures

min-plus convolution

Given n-length sequences a,b,c whose entries are integers,
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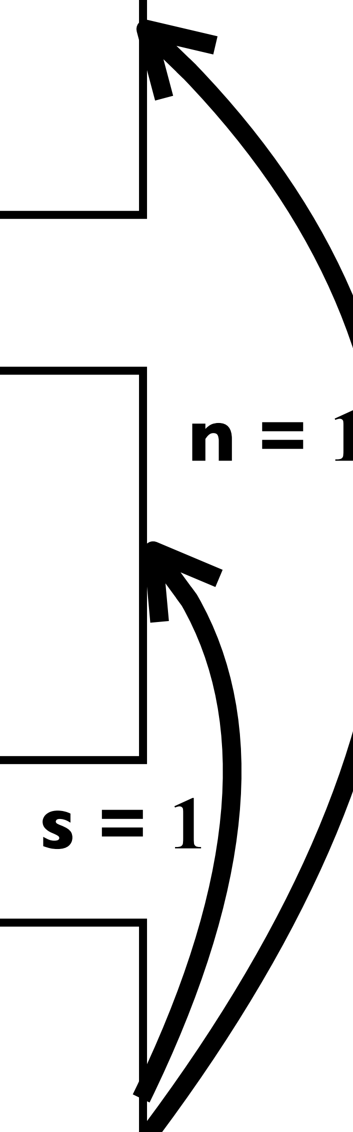

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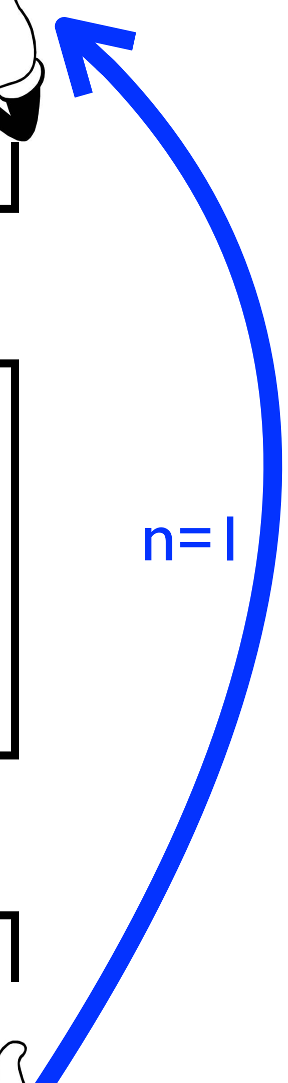

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Bremner et al. 2014

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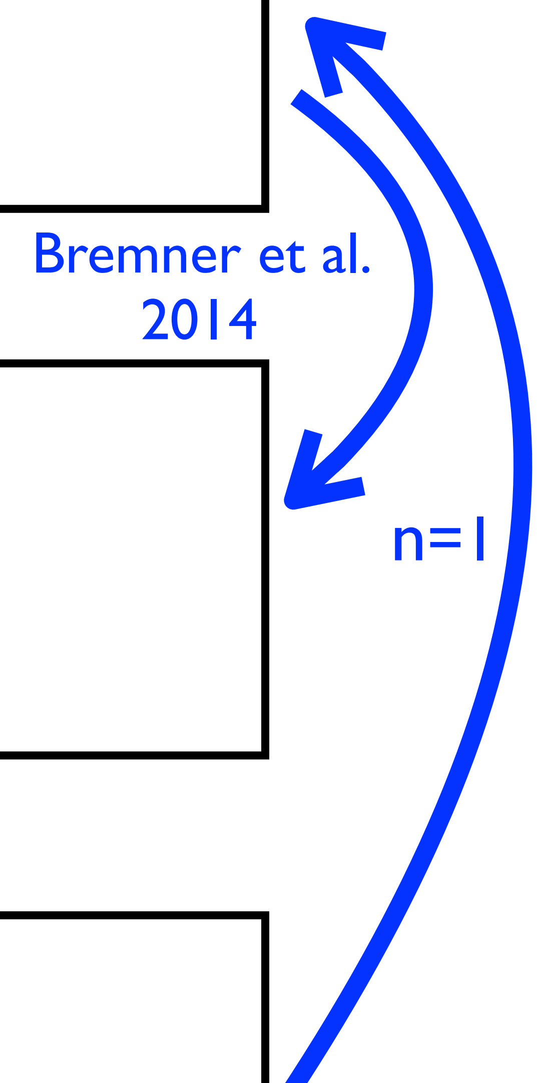

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Planar Negative k-Cycle

min-plus multiplication-convolution Conjecture
Cannot be solved in $O(n^3 s^{2-\varepsilon})$ time
Given n x n matrices A,B,C whose entries are s-length sequences, $\mathbf{s=n^{\alpha}}$
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Assuming the **min-plus convolution** conjecture:

For $k \leq n^{1/3}$ there is no algorithm polynomially faster than $O(nk^2)$
For $k > n^{1/3}$ there is no algorithm polynomially faster than $O(n^{1.5} k^{0.5})$

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adjusting parameters

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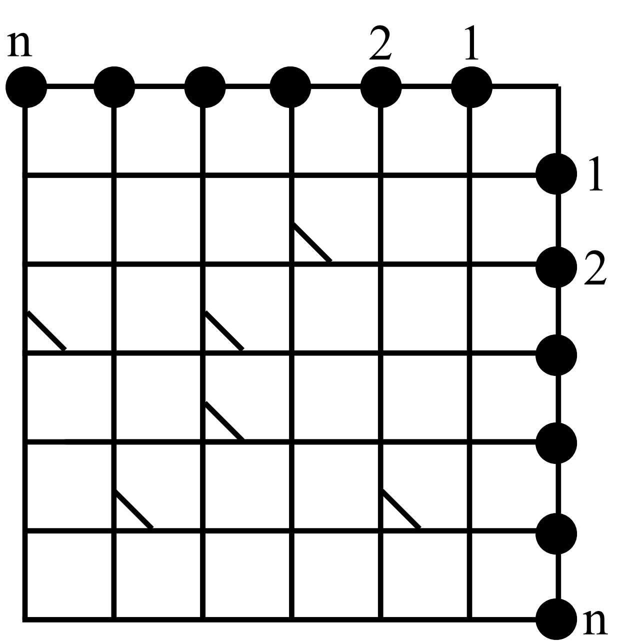
Lower bound for **distance labeling:** [GavoillePelegPrennesRaz 2001]



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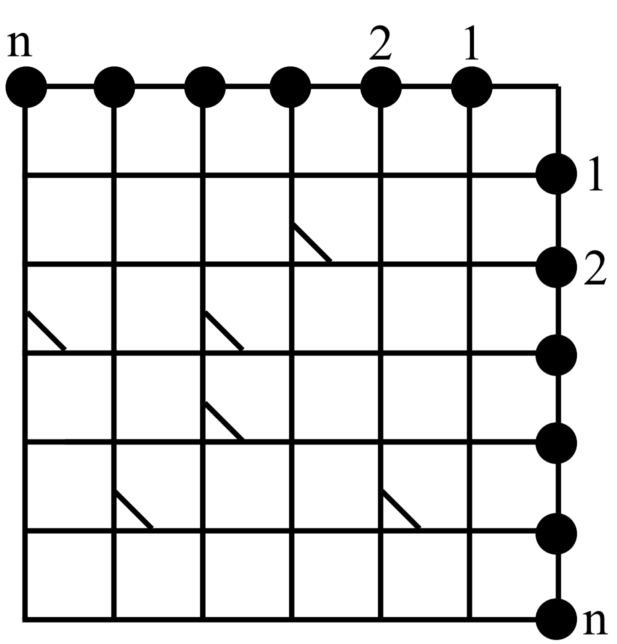

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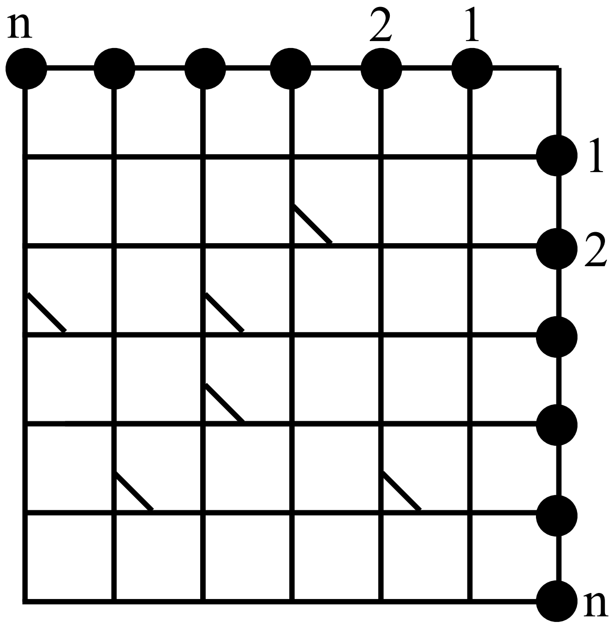

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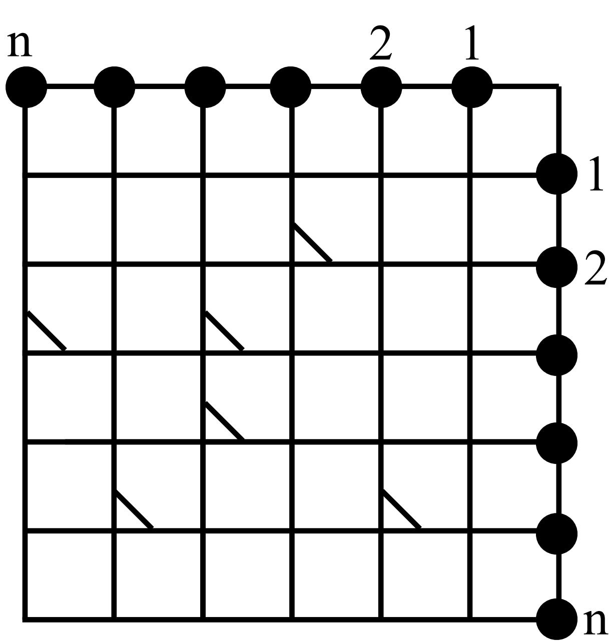

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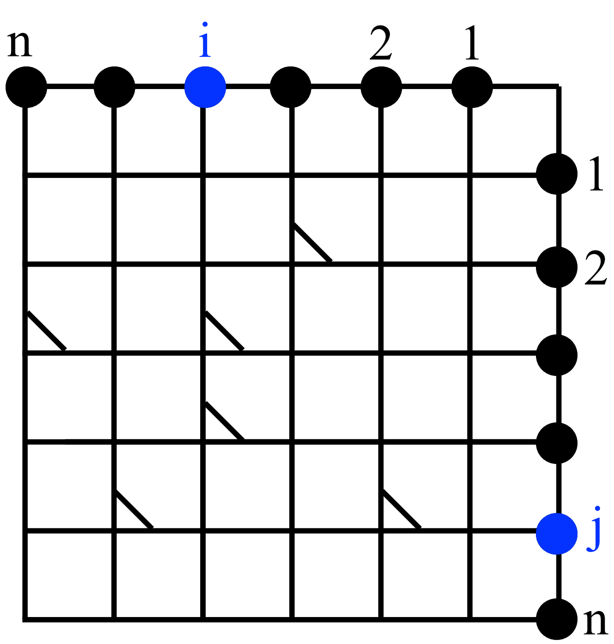

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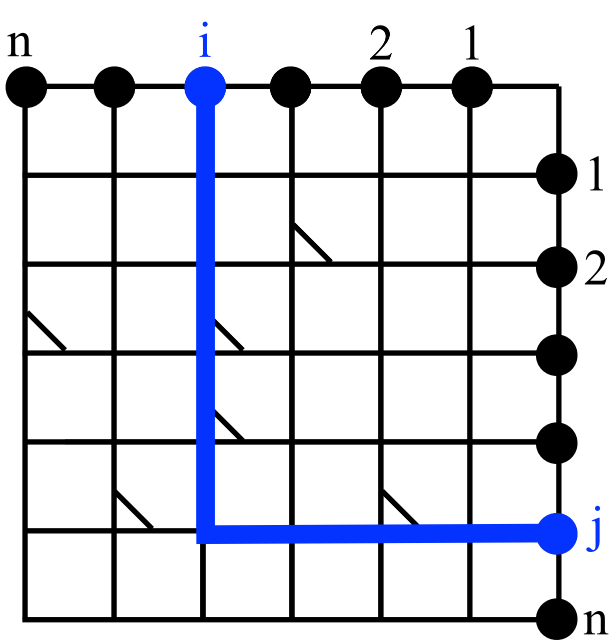

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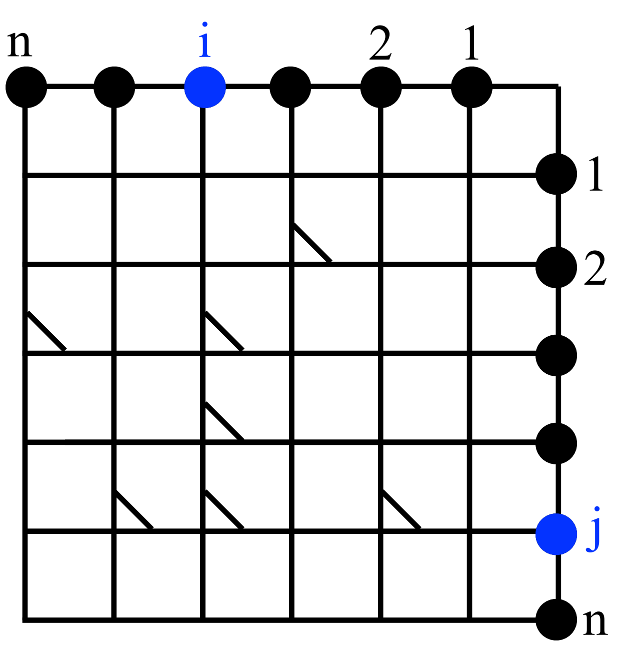

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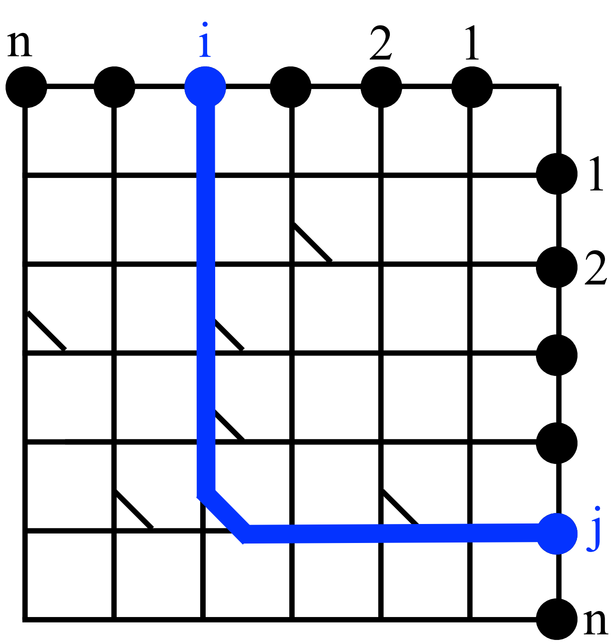

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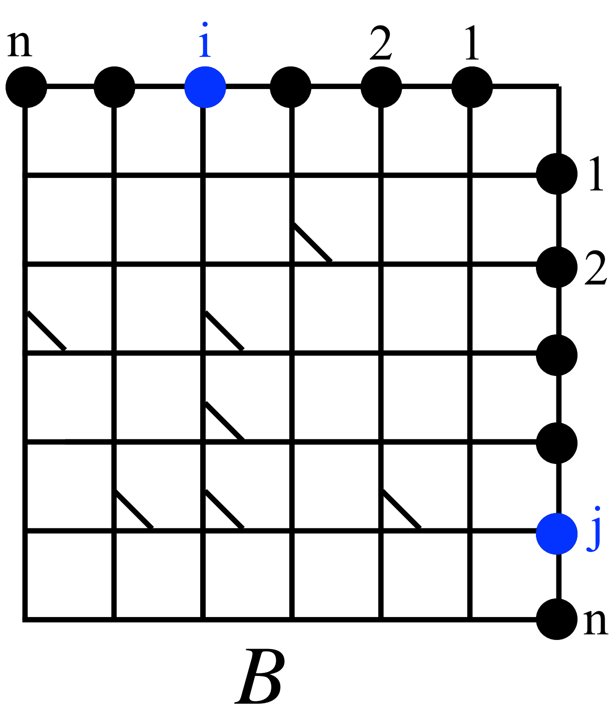
- encode an *n*-by-*n* boolean matrix B using the shortcuts



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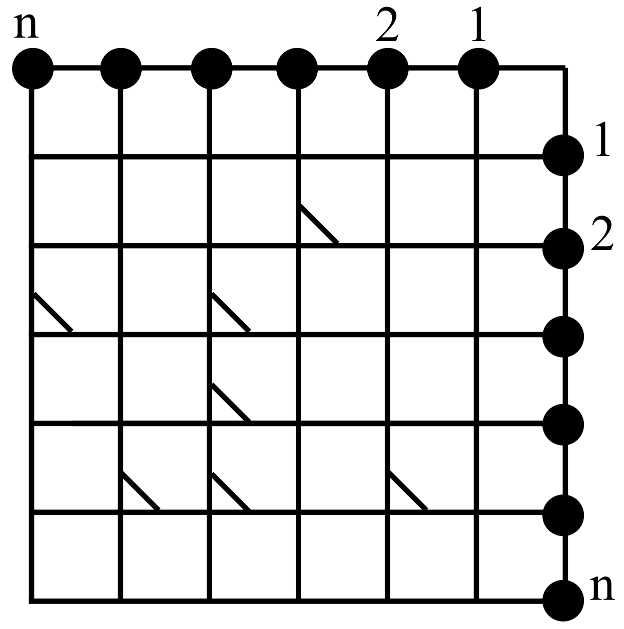
Lower bound for **dynamic distance oracles:** [AbboudDahlgaard 2016]



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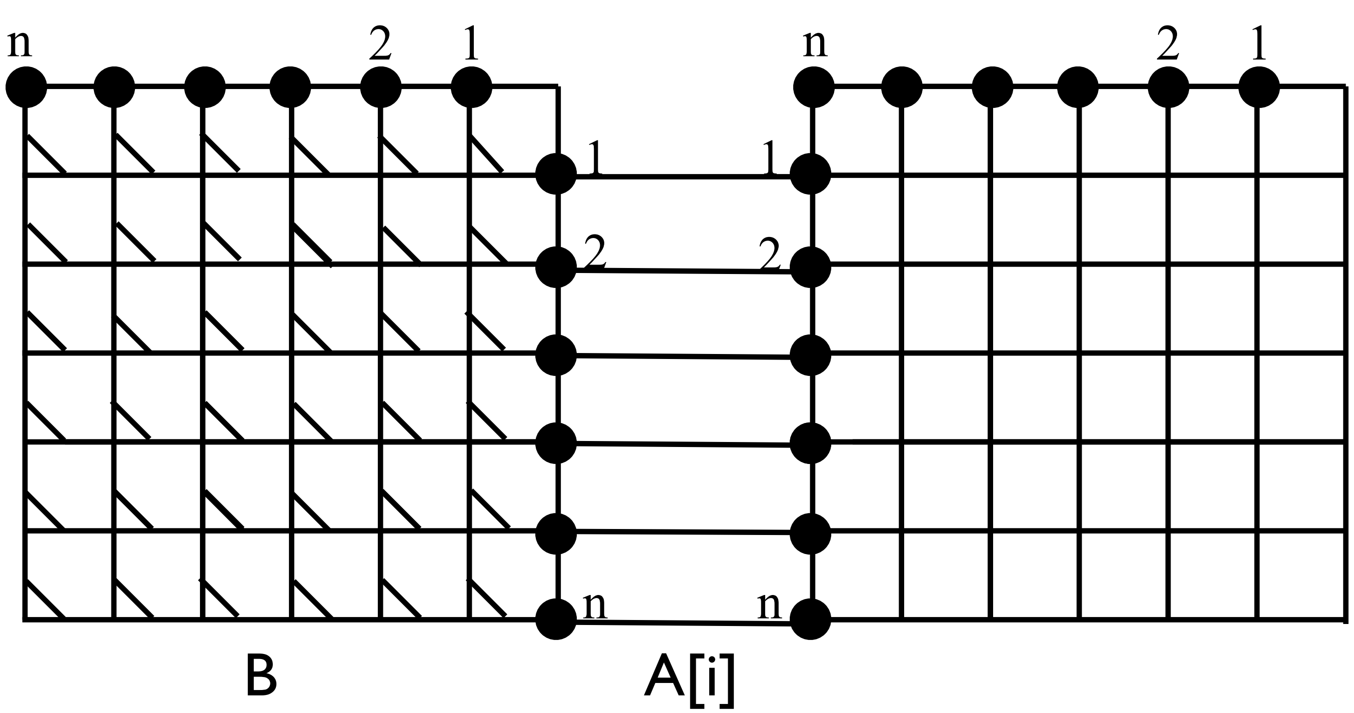

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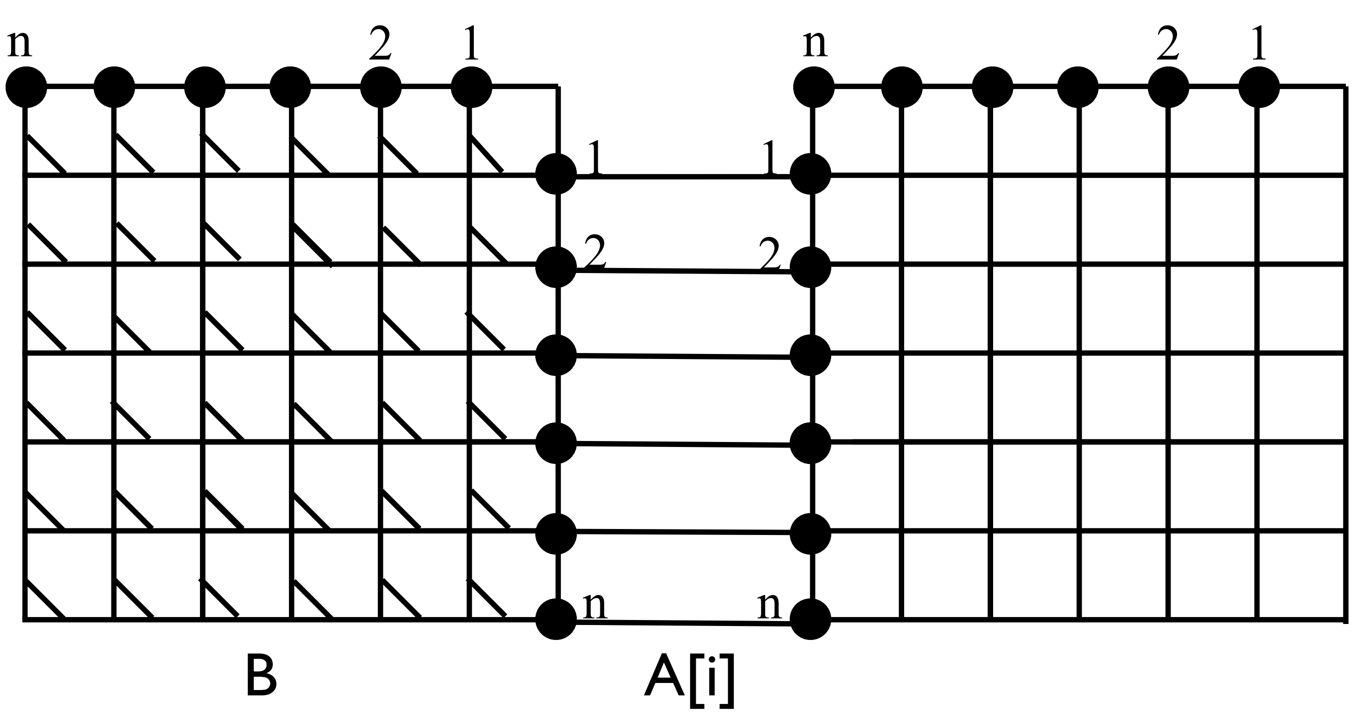

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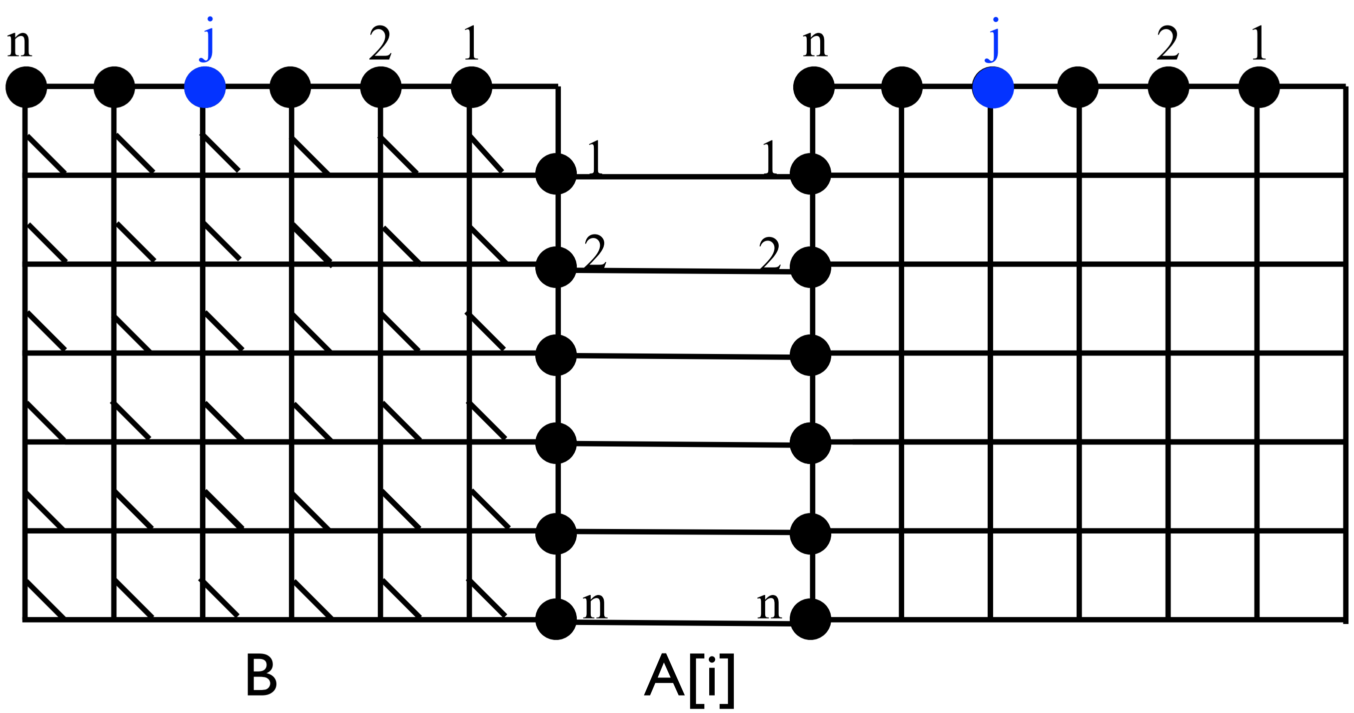

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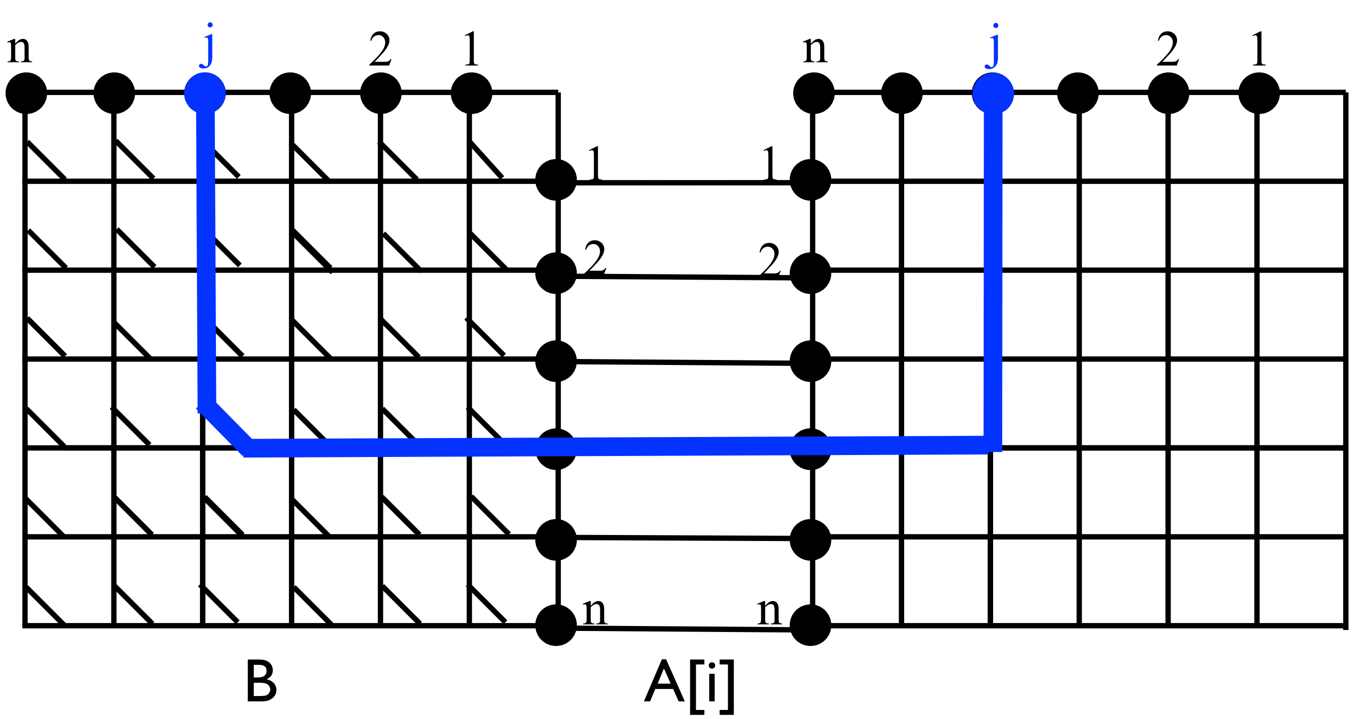

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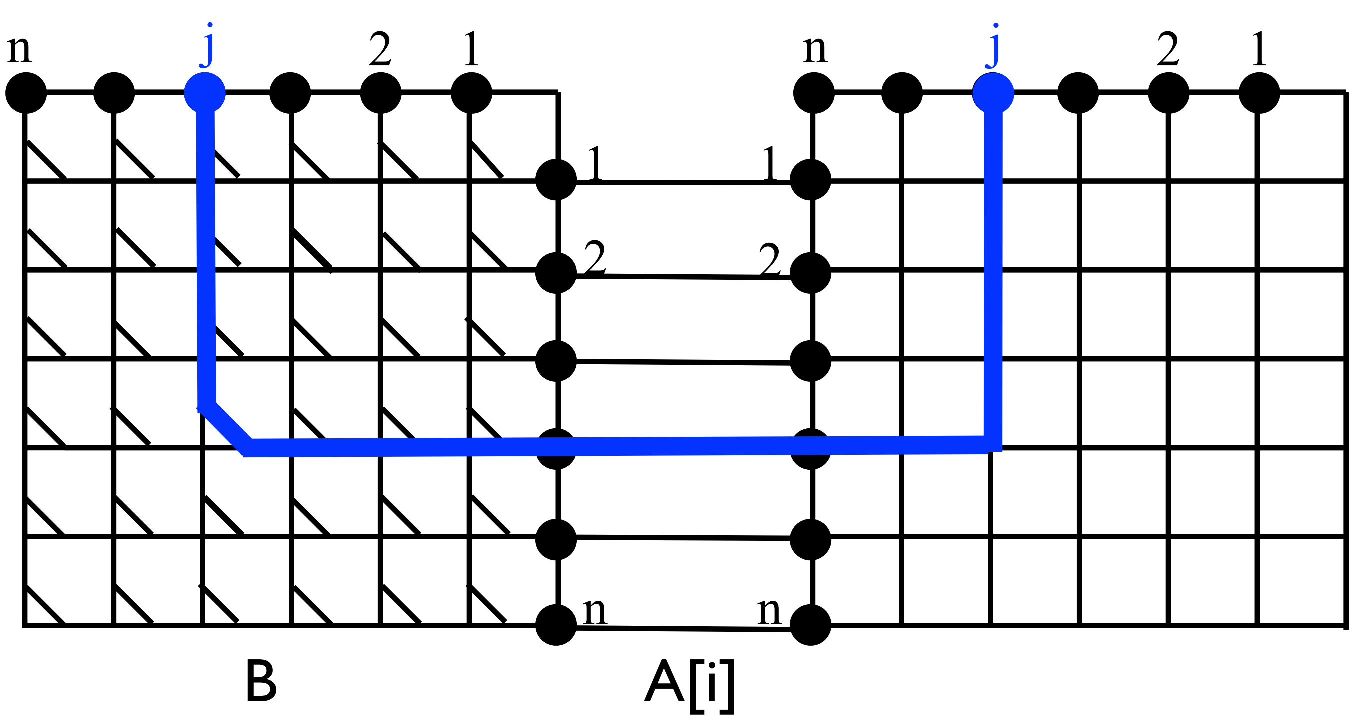

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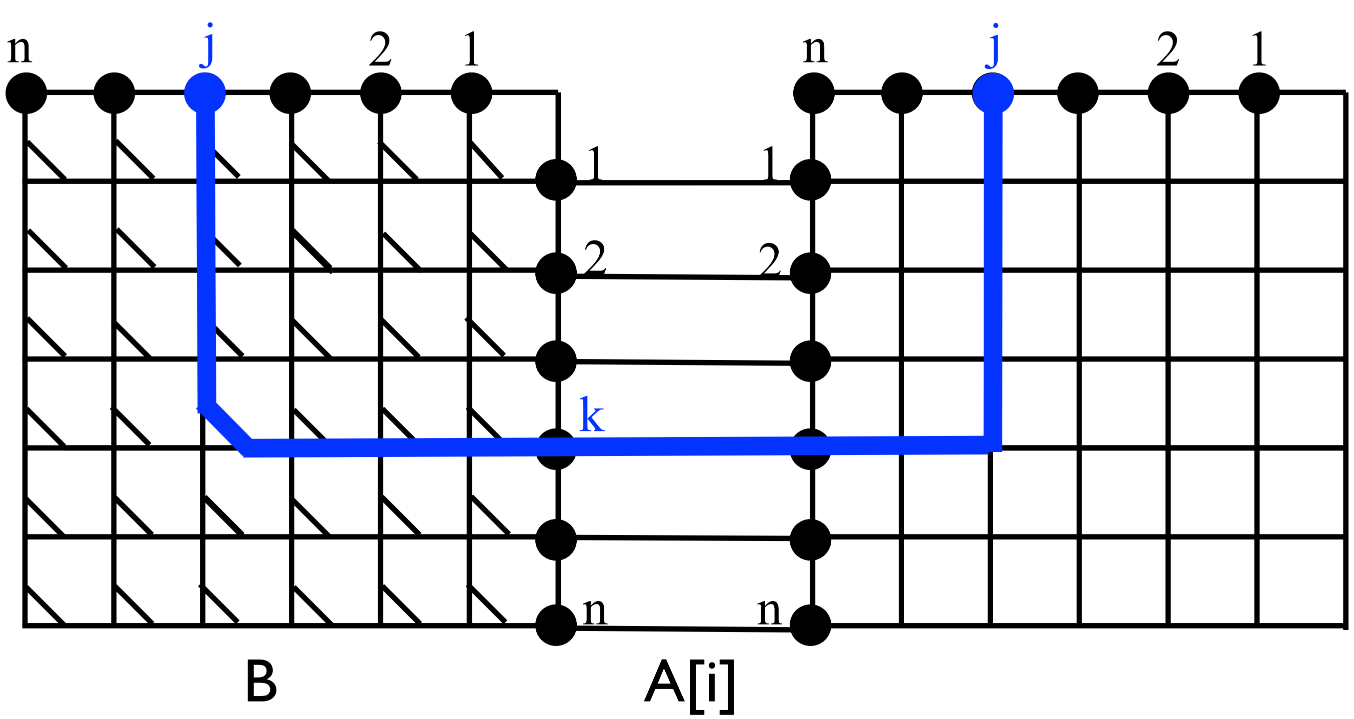

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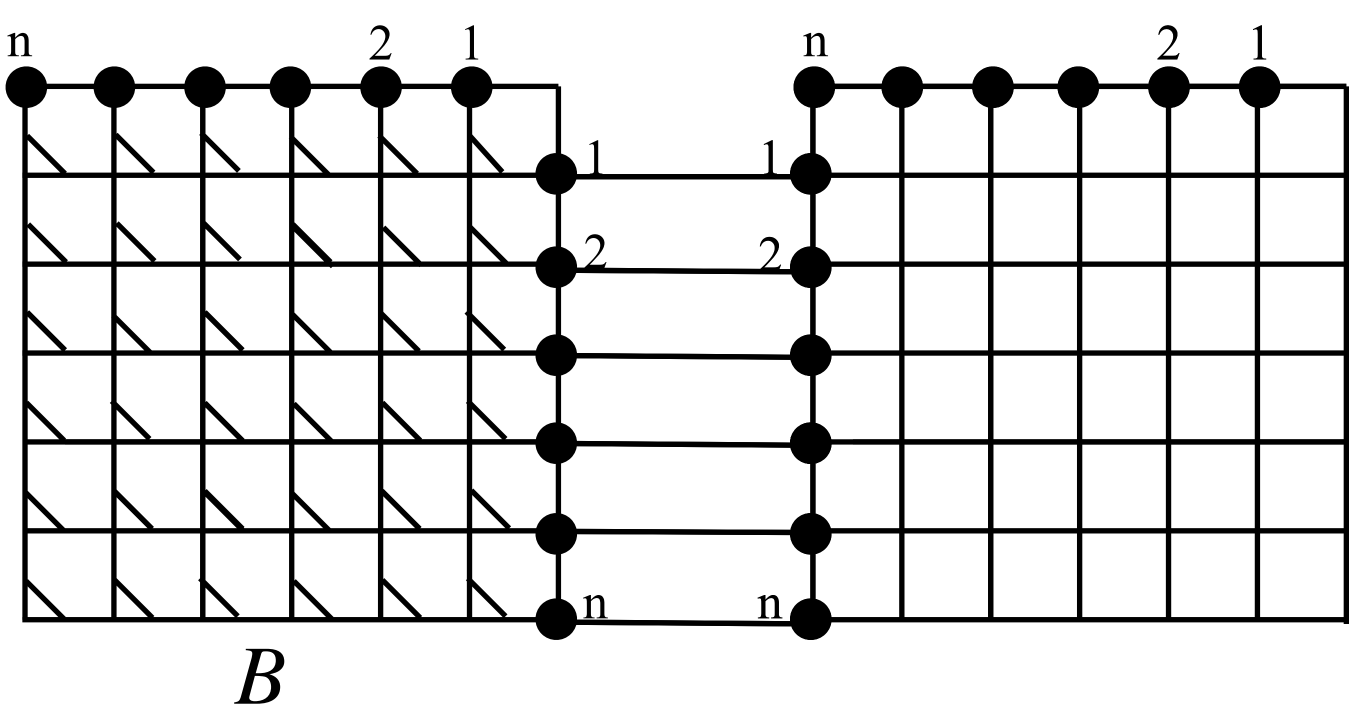

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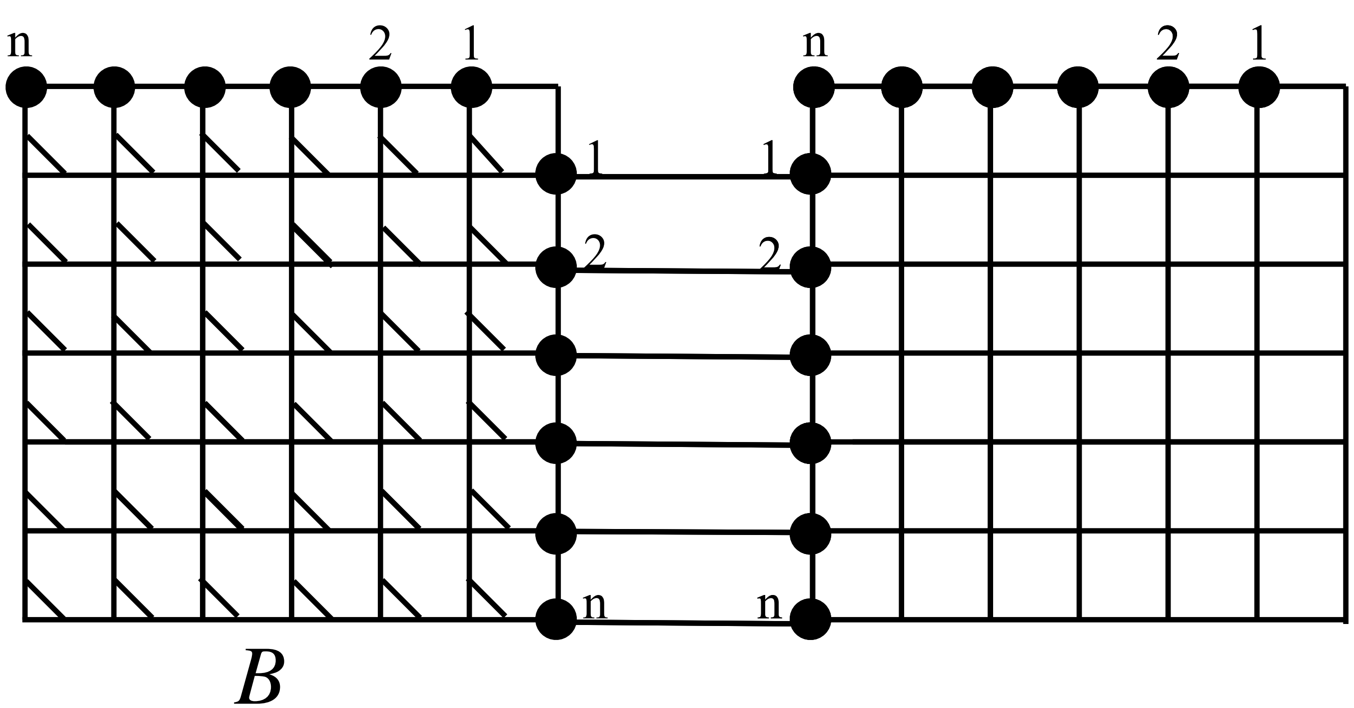

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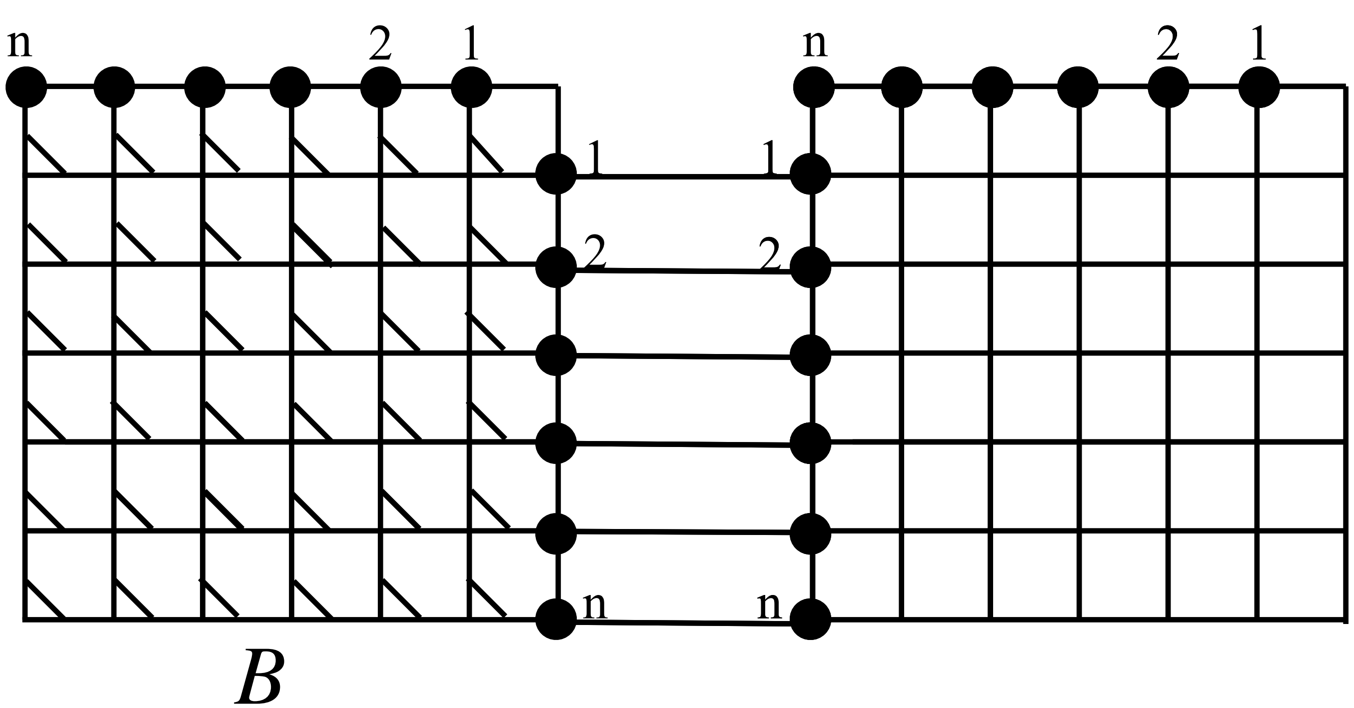

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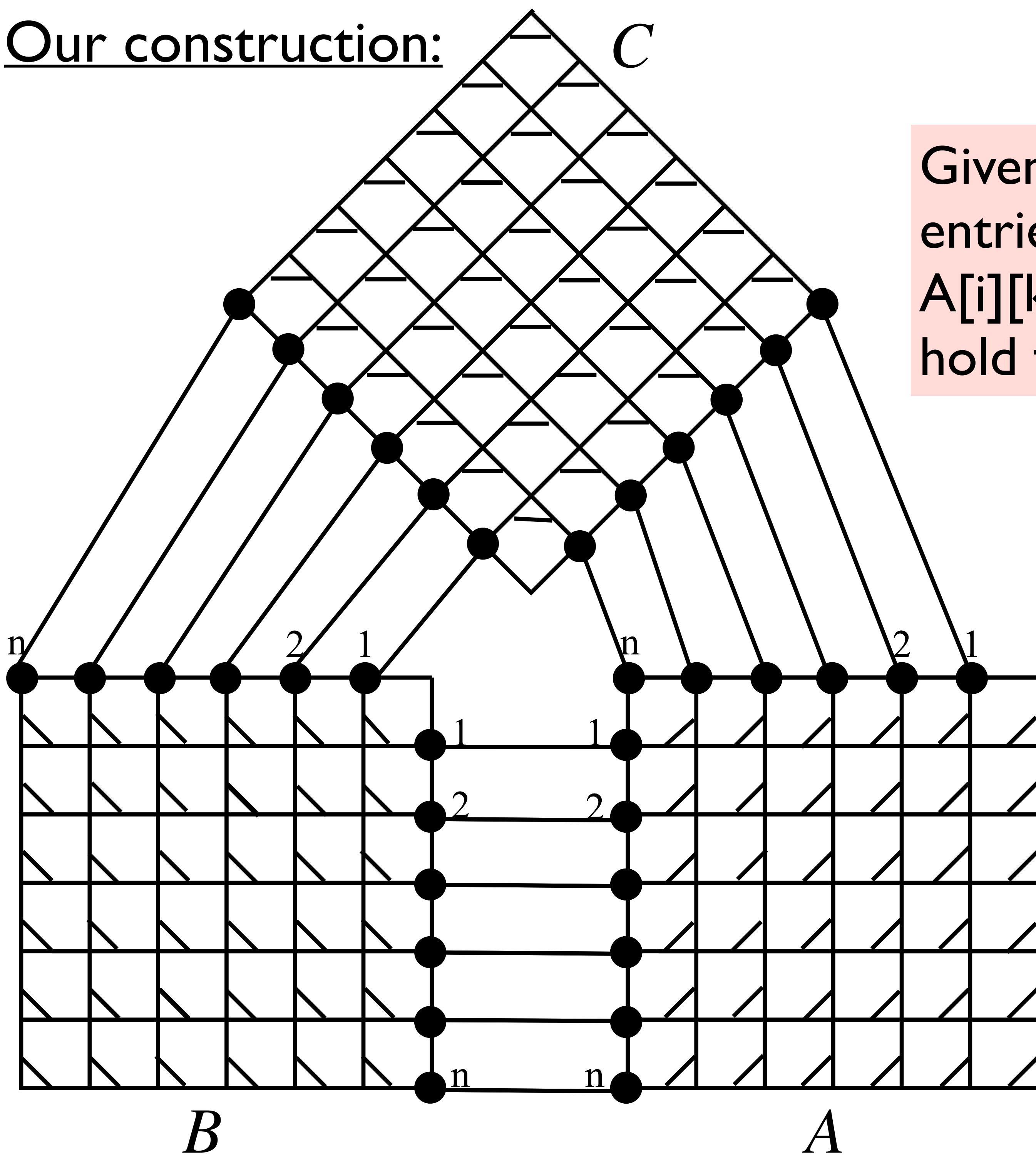

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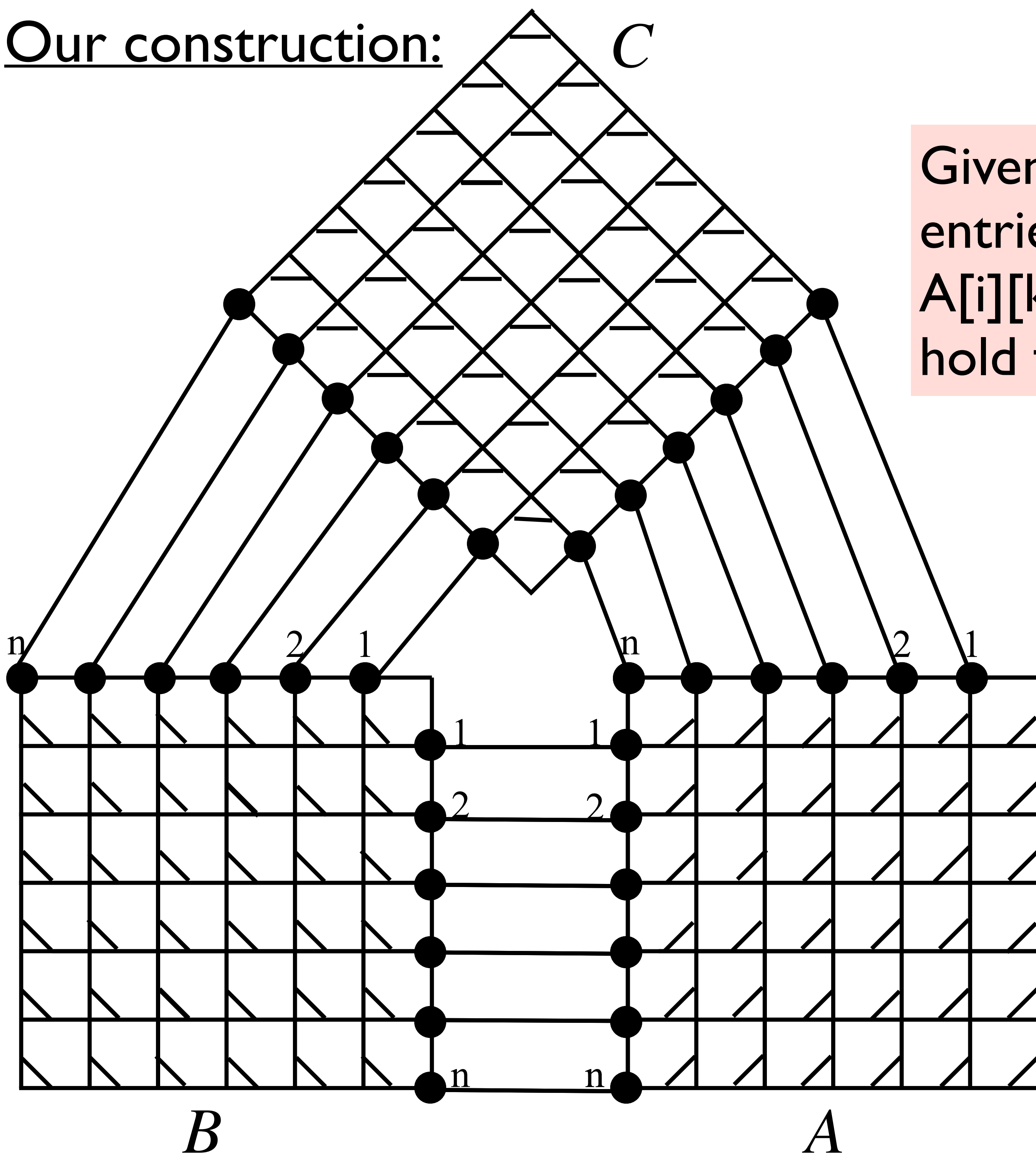

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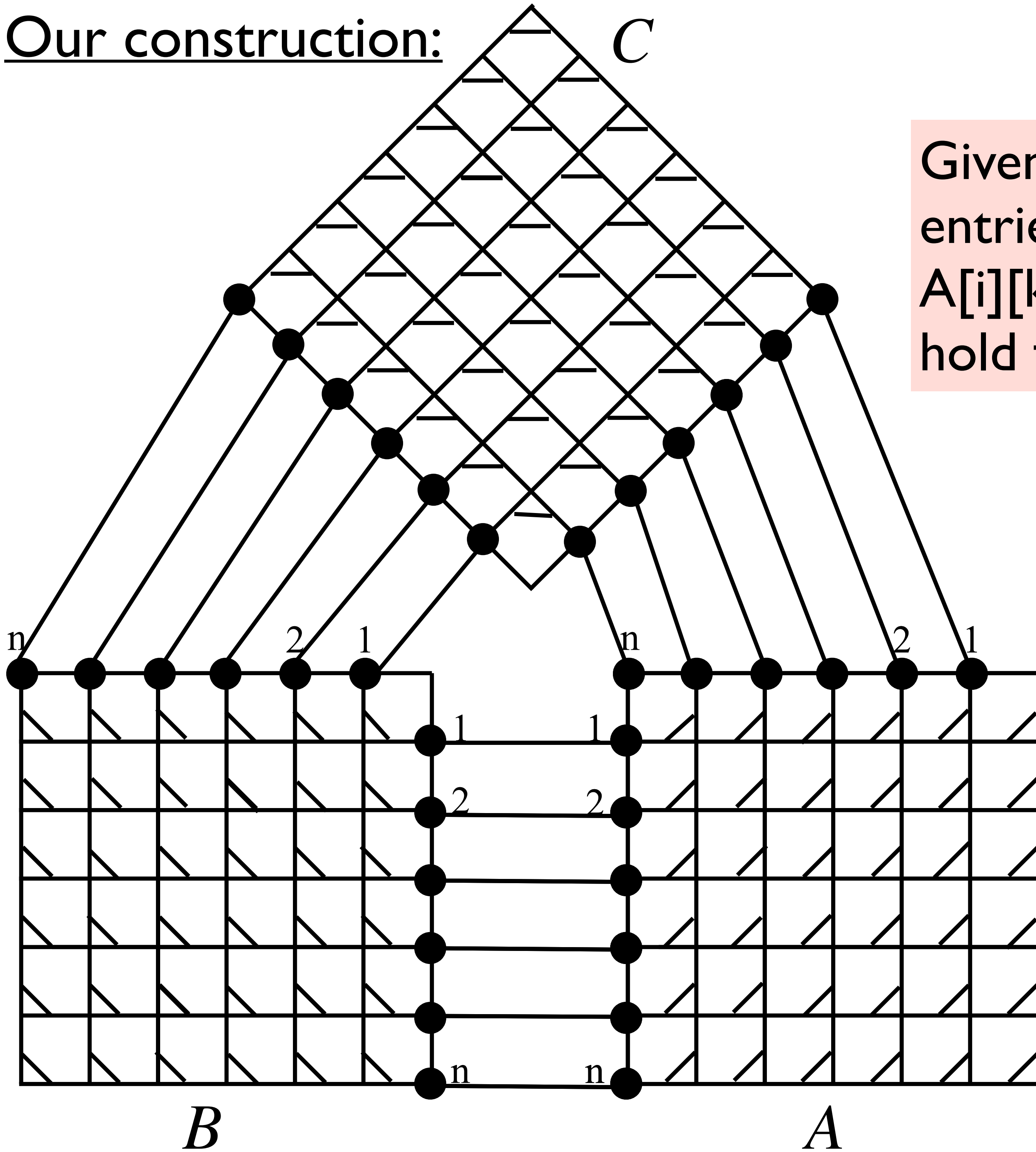

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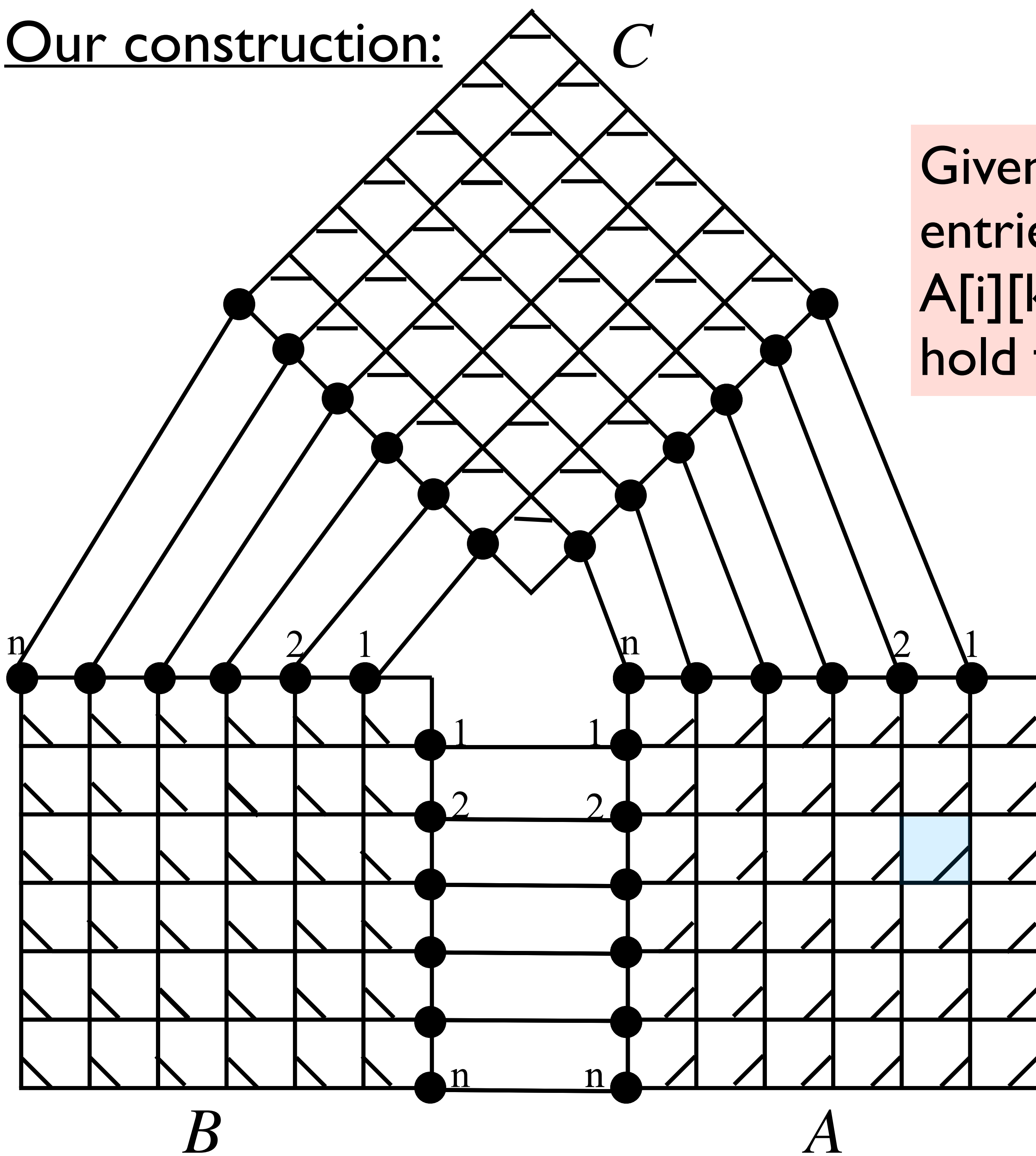

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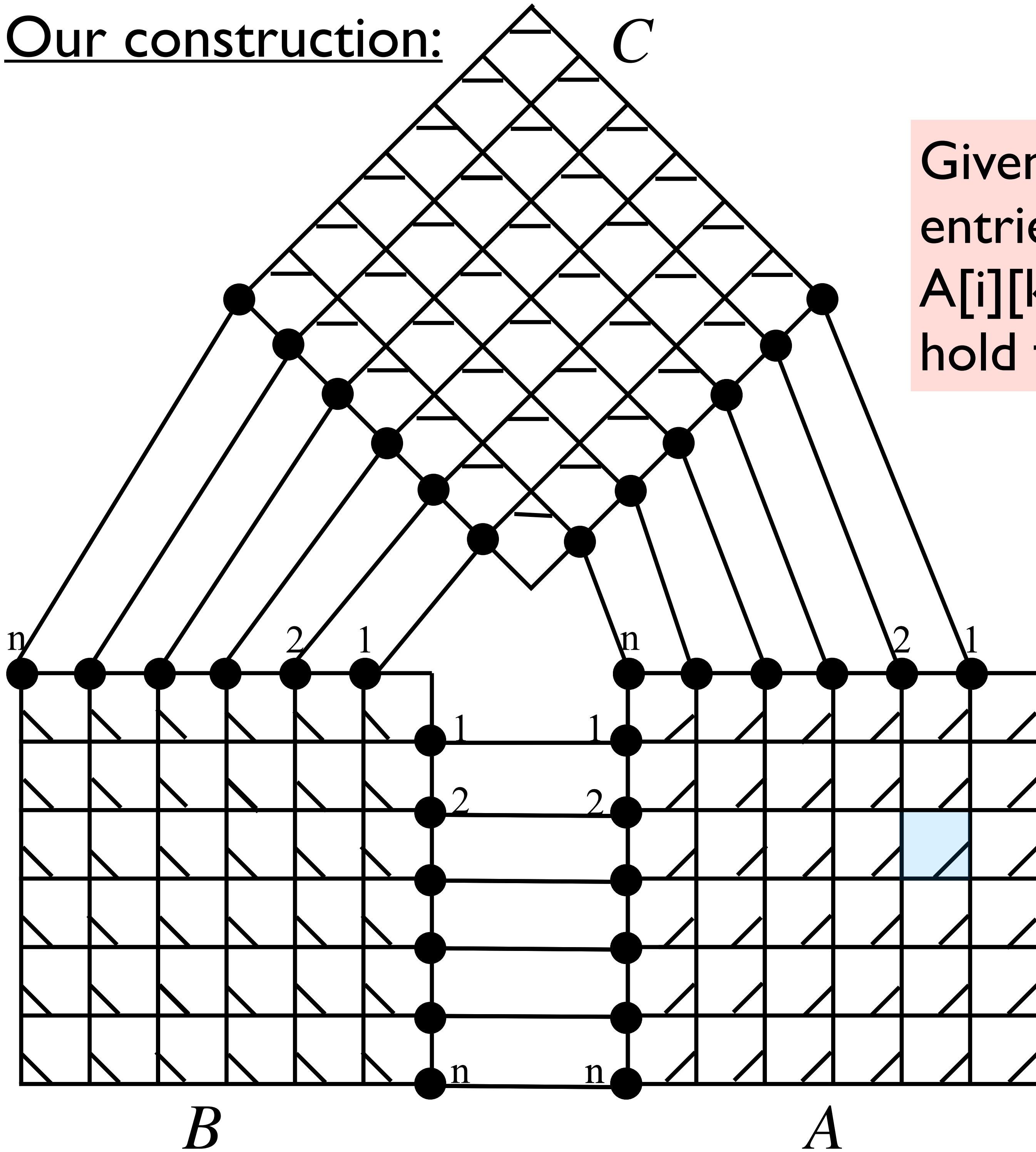

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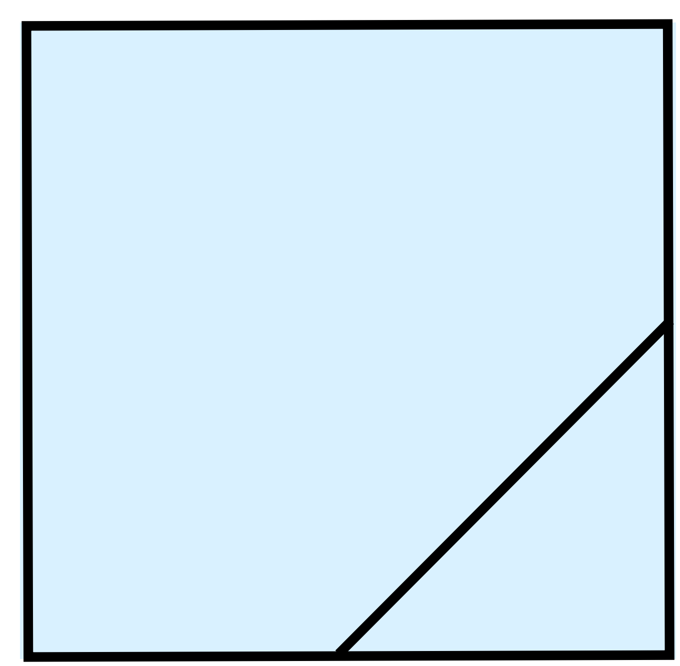

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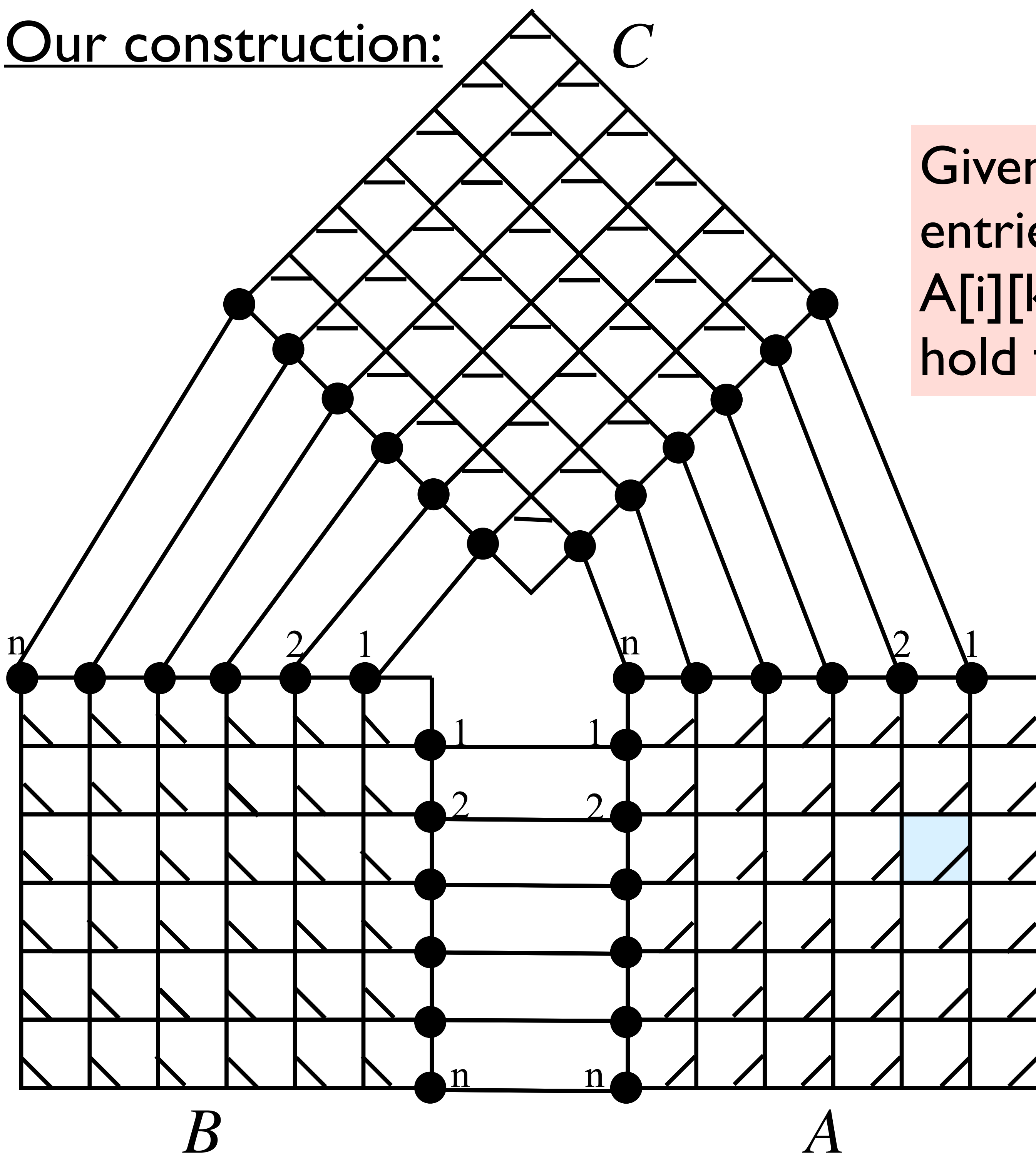

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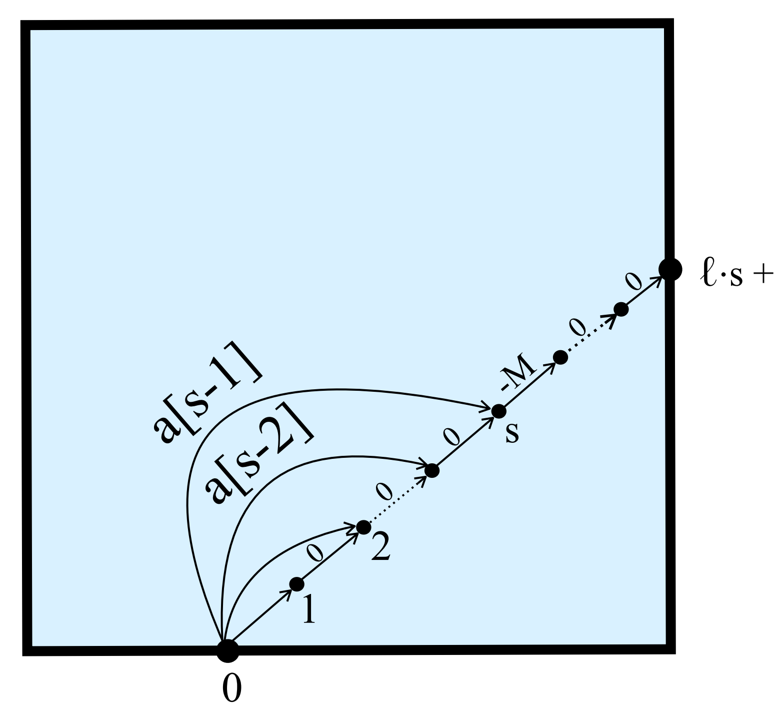

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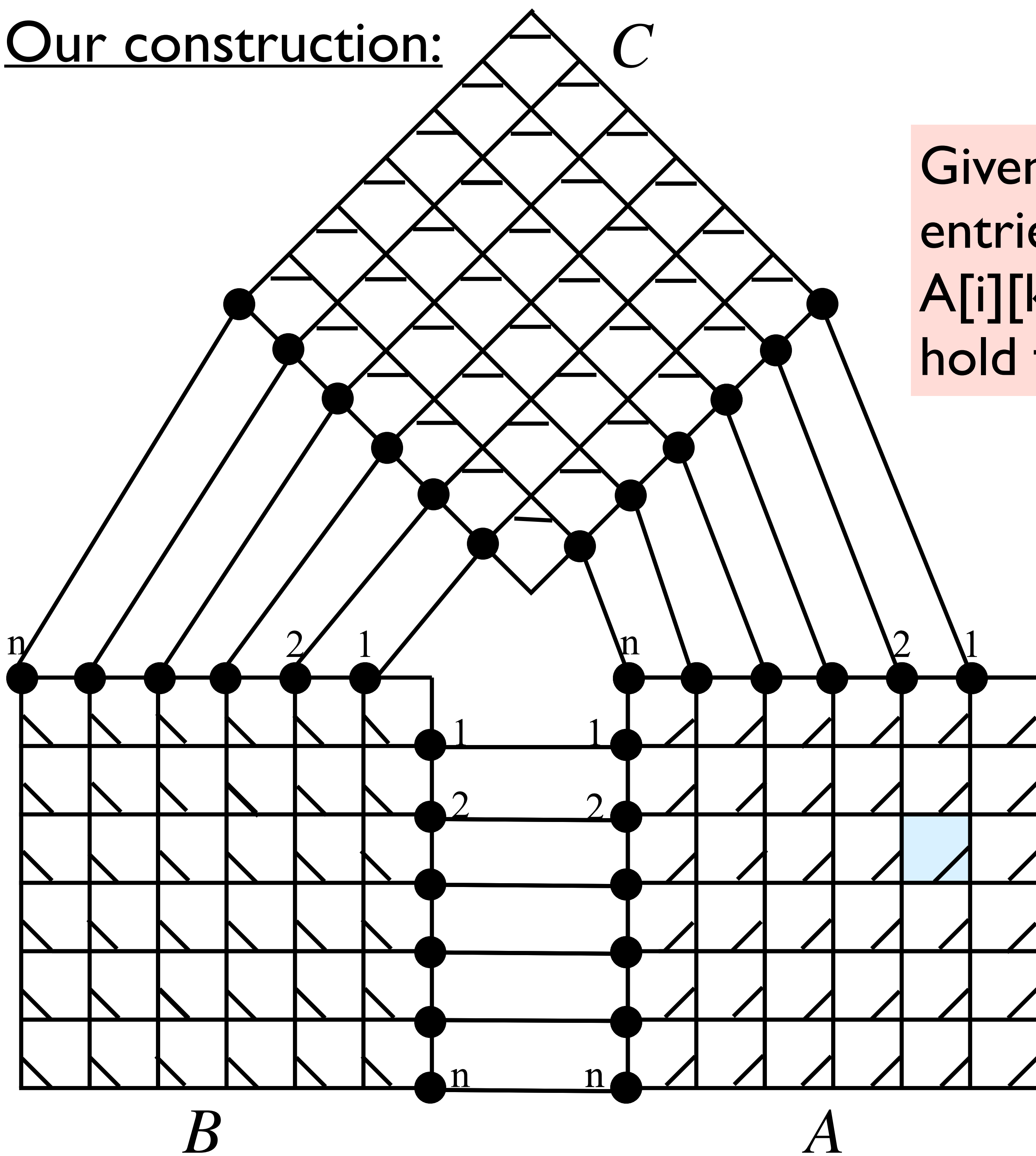

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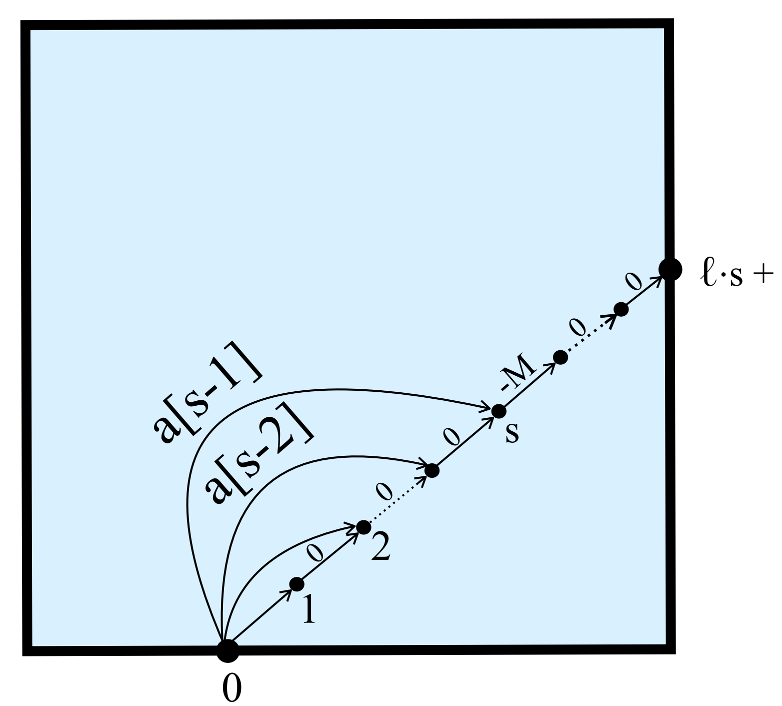
Sequence Gadget (a,ℓ)



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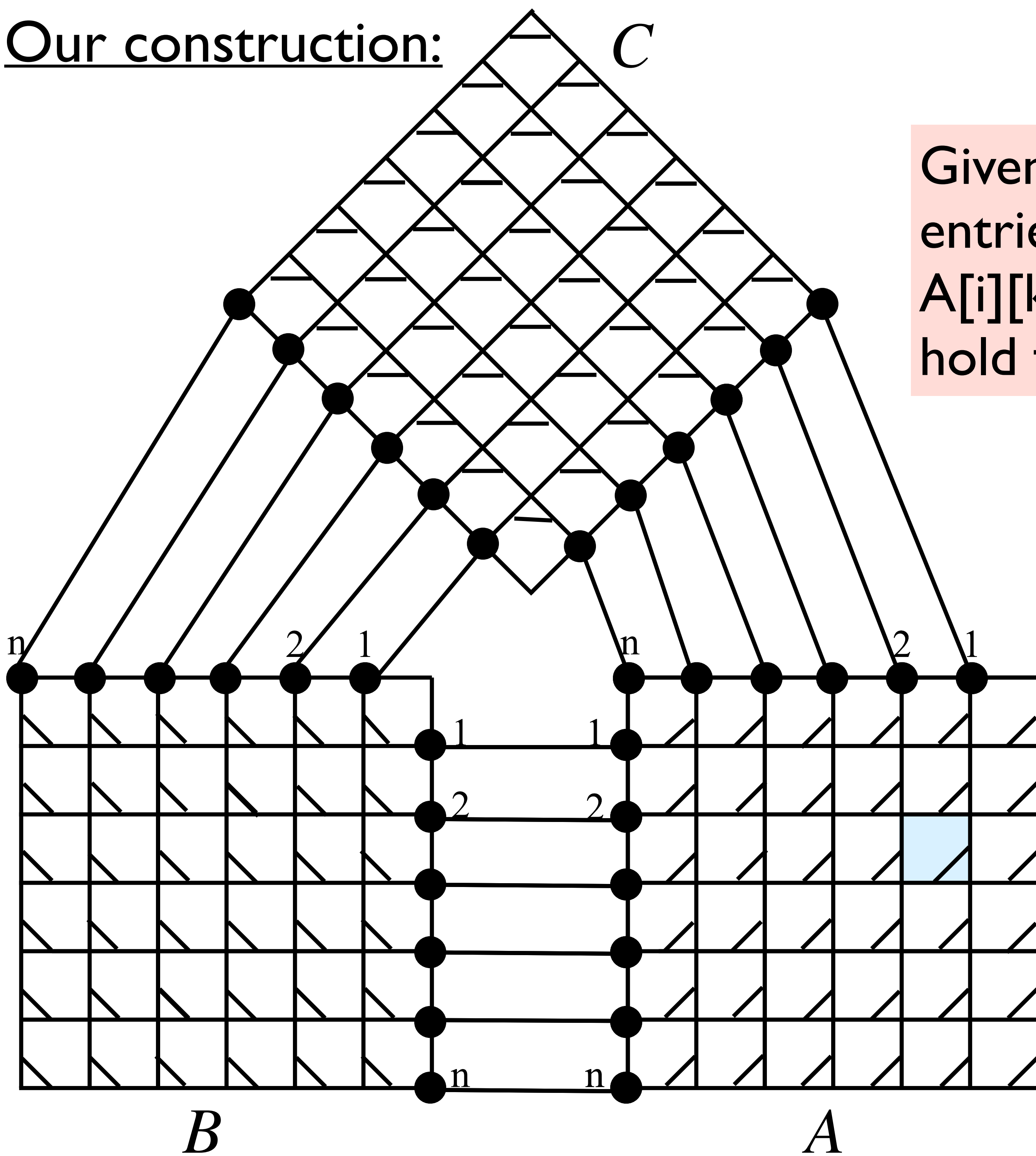

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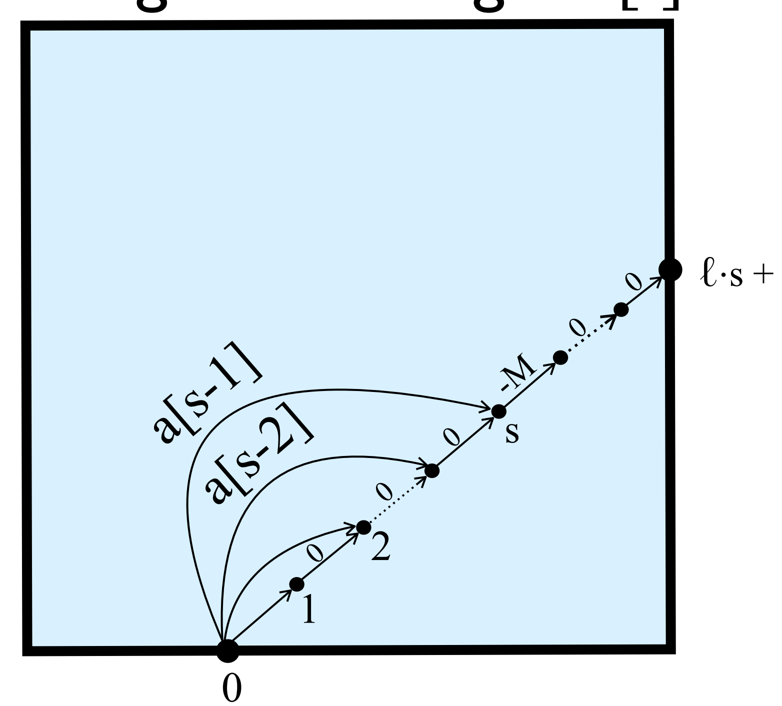
$\forall i$ there's a source-to-sink path with $\ell \cdot s - i + 2$ edges and weight $a[i] - M$



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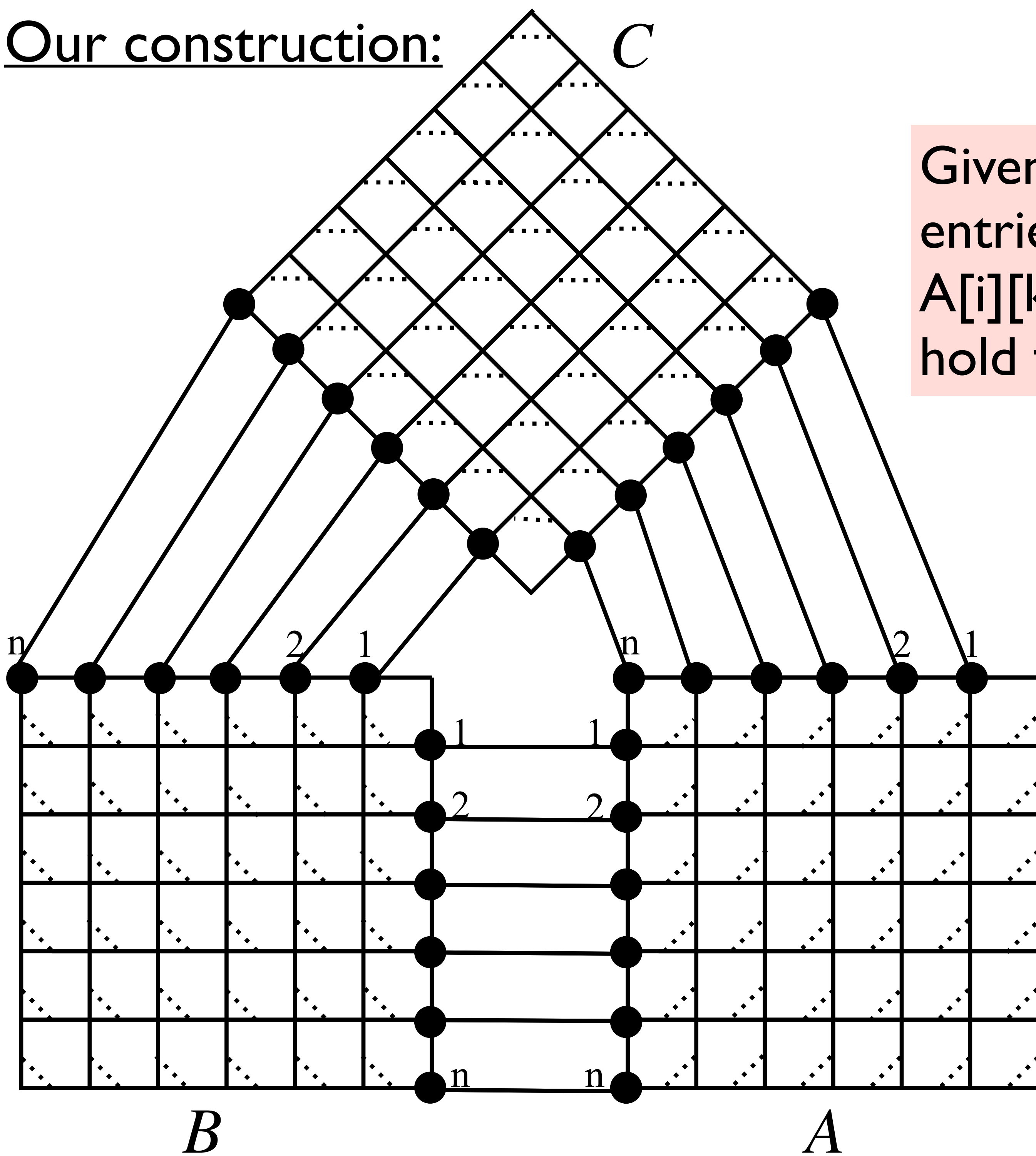

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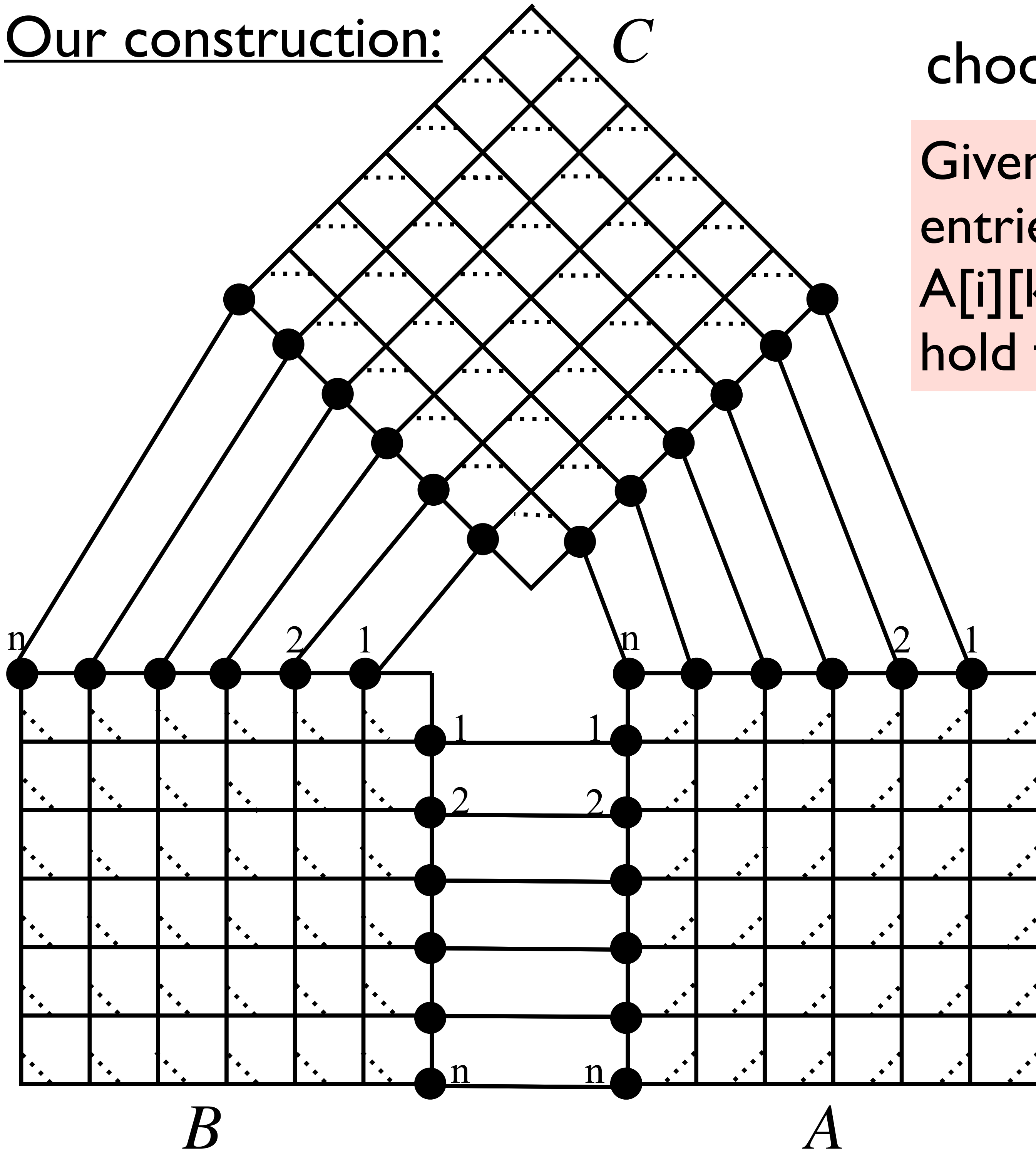


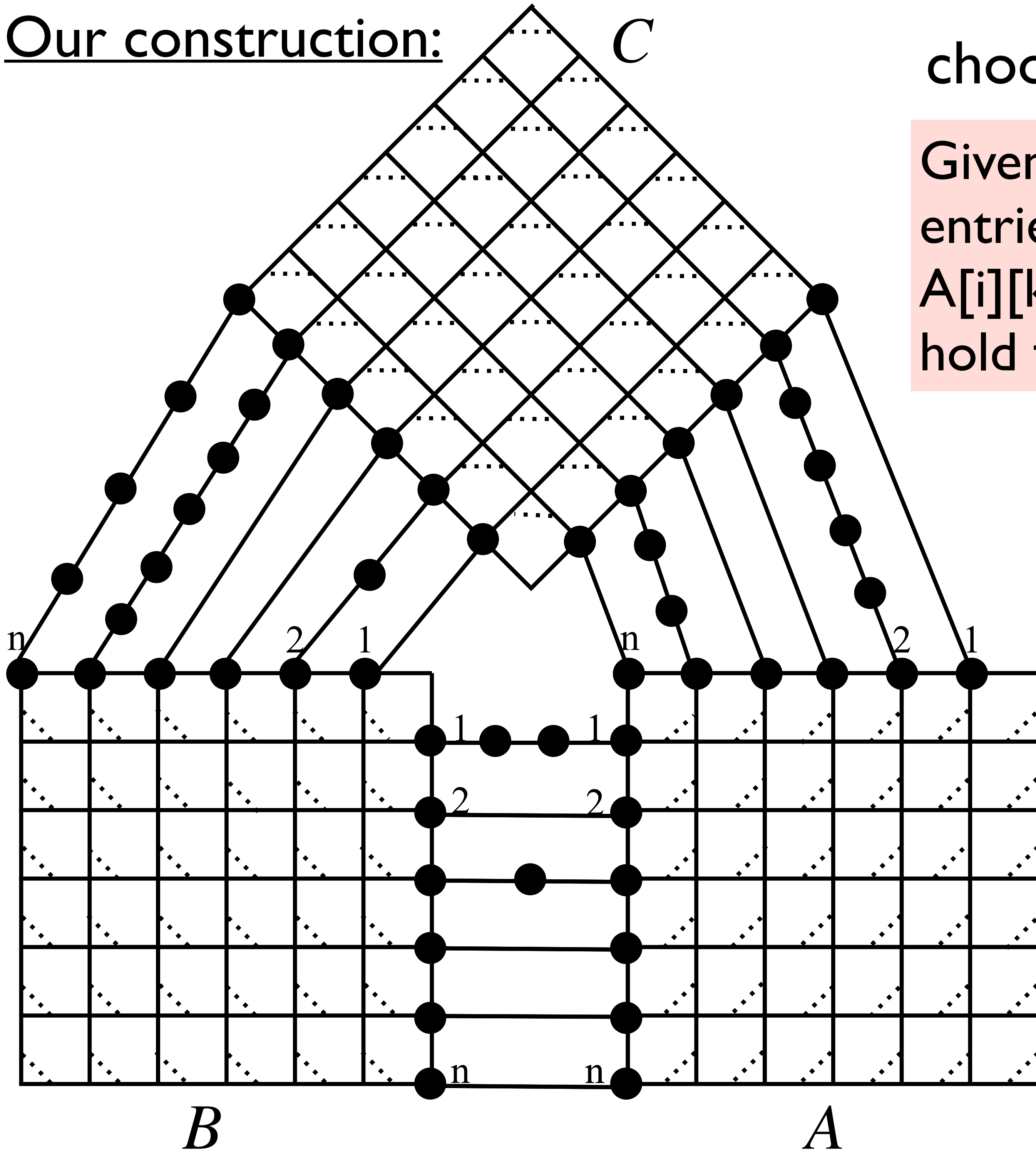
choose edge weights / subdivisions

Given n x n matrices A,B,C whose entries are s-length sequences, does $A[i][k][x] + B[k][j][y] + C[i][j][z] < 0$ hold for some i,j,k,x,y, and z=x+y

Theorem:

In linear time we can reduce **min-plus multiplication-convolution** to **Planar Negative-k-Cycle** on $O(n^2 s)$ vertices and $k = O(n + s)$.

Our construction:



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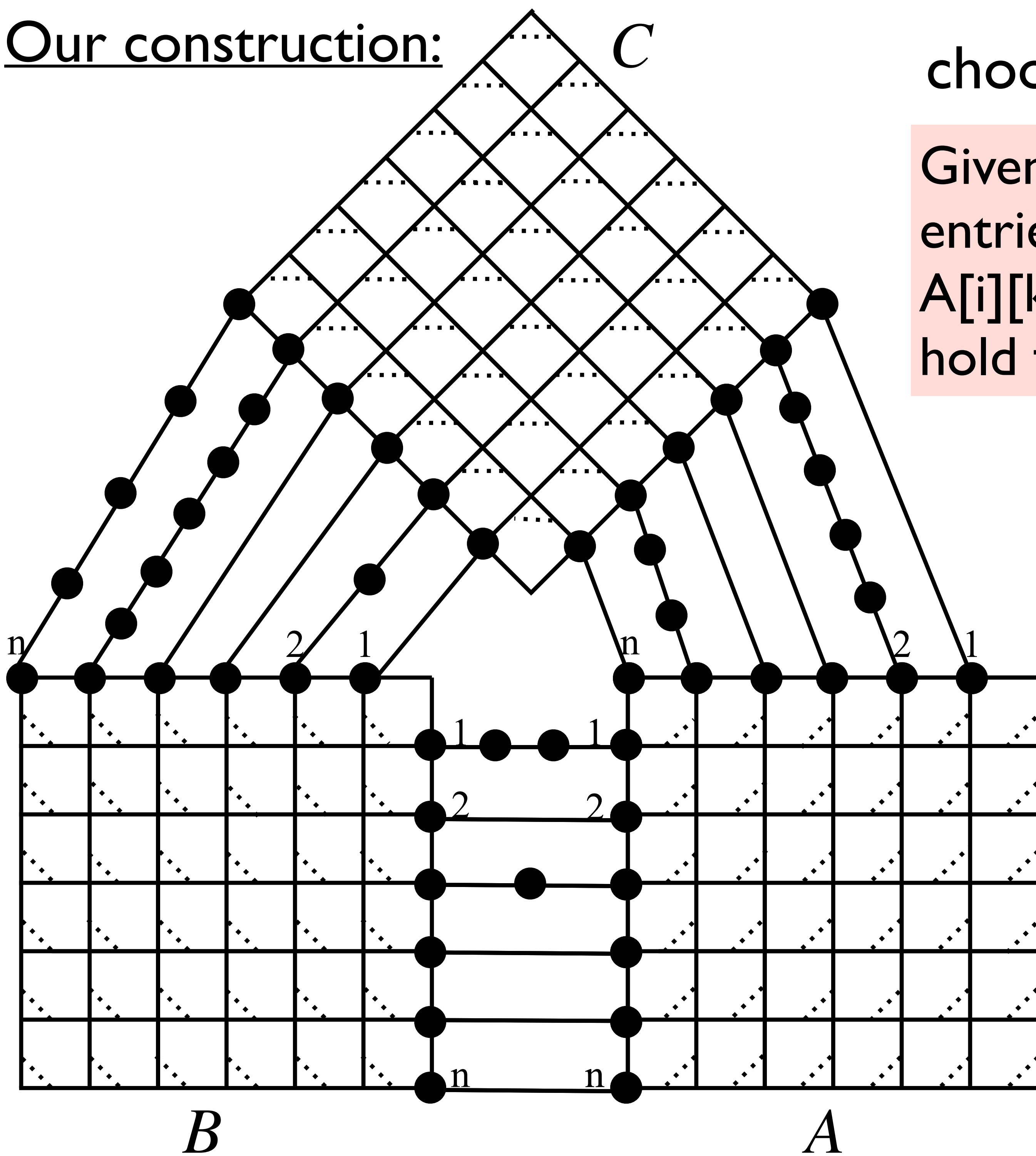

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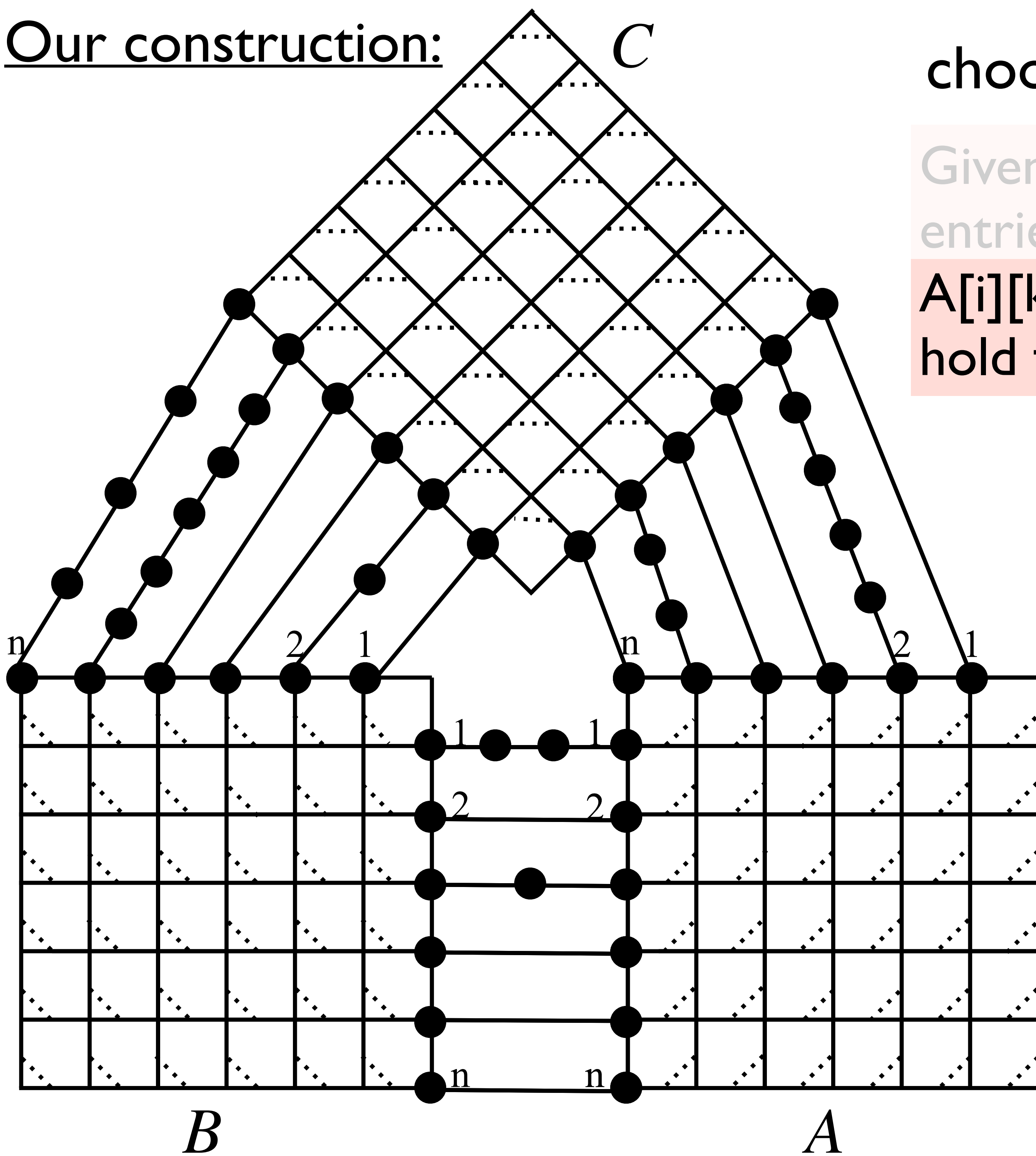

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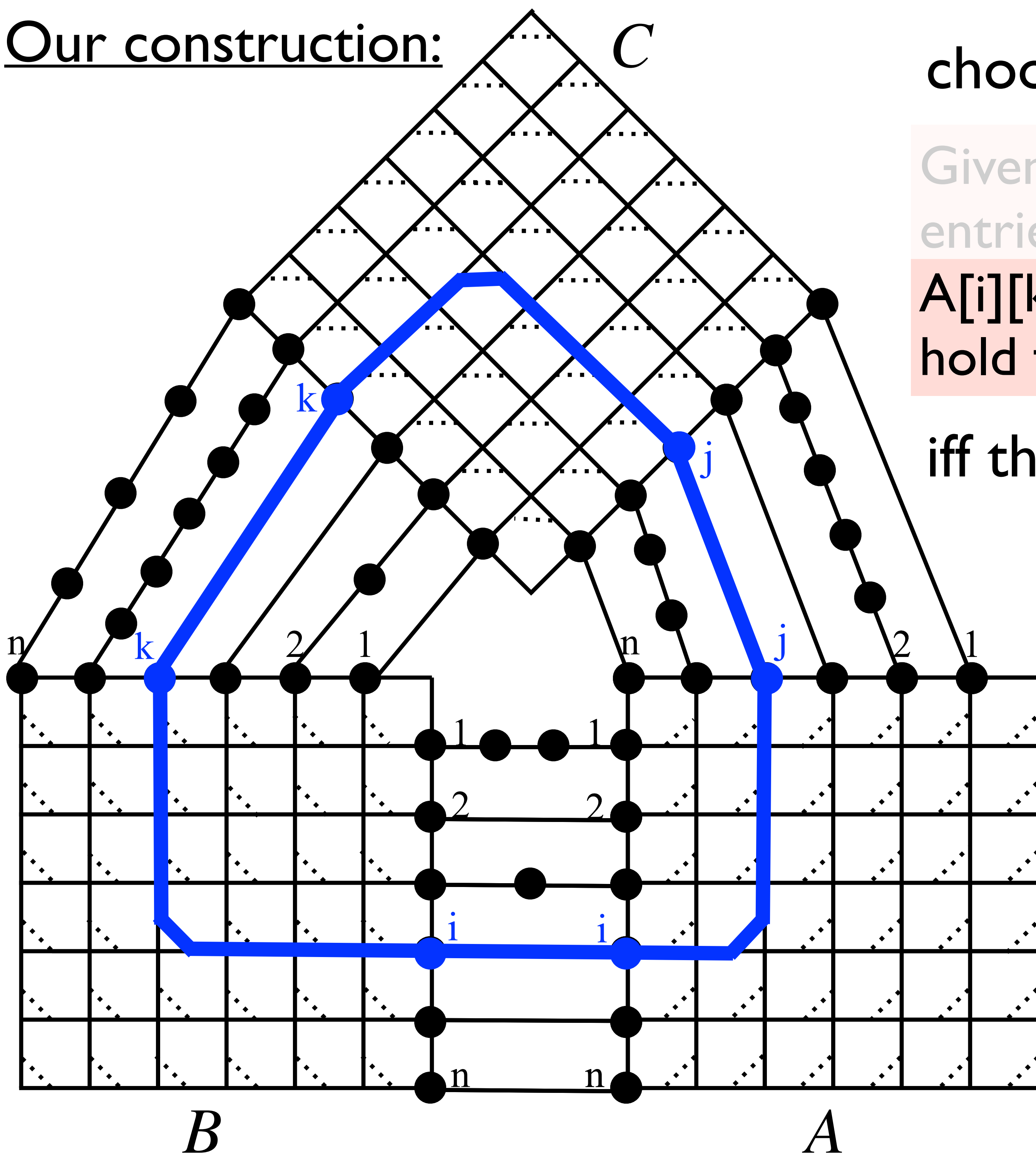

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iff there is a negative k-cycle

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Thank You!