Seminar on Sublinear Time Algorithms – Handout 2

Robert Krauthgamer

March 24, 2010

1 Today's topics

- Minimum spanning trees in sparse graphs
- Maximal and maximum matching

2 Open problems

- The MST approximation implies approximation within factor $2+\varepsilon$ for the traveling salesman problem (TSP). Can you do better?

3 Homework

1. Let G be a connected graph with edge weights from $\{1,\ldots,W\}$, and let G_i be the subgraph of G consisting of all edges with weight at most i. Prove that the cost of a minimum spanning tree can be written as

$$\mathrm{MST}(G) = n - W + \sum_{i=1}^{W-1} c_i,$$

   where c_i is the number of connected components in G_i.

   Hint: Recall Kruskal's MST algorithm.

2. Let G be an undirected graph. Recall a matching is called *maximal* if it is with respect to containment, and is called *maximum* if it has the largeest possible size. Prove that every maximal matching has size which is at least half that of a maximum matching.