Seminar on Sublinear Time Algorithms – Handout 3

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1 Today's topics

- Property testing framework
- Testing monotonicity of a list
- Testing homomorphism of a function

We will also briefly discuss how to read and present a paper.

2 Open problems

The tester we saw in class for monotonicity of a list can be extended to a sublinear algorithm that estimates the distance from monotonictity, i.e. the smallest ε for which the input list is ε-far from monotone. Roughly speaking, this algorithm achieves approximation 2, and runs in time $O(\frac{1}{\varepsilon}\log n)$. It is not known whether approximation better than 2 is possible (in sublinear time, for constant $\varepsilon > 0$).

3 Homework

1. (a) Prove that a function $f : \mathbb{Z}_n \to \mathbb{Z}_n$ is a homomorphism if and only if it satifies $f(x)+f(1) = f(x+1)$ for all x.

   (b) Show that there exists a function $f : \mathbb{Z}_n \to \mathbb{Z}_n$ that then satifies $f(x) + f(1) = f(x+1)$ for $1 - o(1)$ fraction of x's, yet it is 1/100-far from a homomorphism.

2. Show an algorithm that tests whether a function $f : [n] \to [n]$ is a bijection (i.e. permutation). That is, given black-box access to f, the algorithm should determine whether f is a bijection or ε-far from a bijection.

   Hint: The running time should be roughly $\sqrt{n}$.

   Remark: One can also think of f as a list $f(1), \ldots, f(n)$.