Seminar on Sublinear Time Algorithms – Handout 6

Robert Krauthgamer

June 23, 2010

1 Today's topics

- Local partitioning of sparse graphs
- Additive approximation of vertex-cover in planar graphs

Remark: All these results immediately extend to graphs excluding a fixed minor.

2 Open problems

Is there a partition oracle with runtime (or even query complexity) that is polynomial in $1/\varepsilon$?

3 Homework

1. Prove that for every $\varepsilon, d > 0$ there is $k^* = k^*(\varepsilon, d)$ such that every planar G with maximum degree $\leq d$ admits an (ε, k^*)-partition.

2. Extend the algorithms seen in class (the partition oracle and its usage to approximate vertex-cover) to all planar graphs.

   Hint: Use the fact that the average degree is bounded by a constant $\tilde{d}$, and thus most vertices have small degree as well.

3. Adapt the partition oracle seen in class, so as to bound the worst-case (instead of expected) runtime, as follows: the algorithm (oracle) gets as input also $0 < \alpha < 1/2$; it is allowed to return "fail", but this happens with probability $\leq \alpha$ (over the algorithm's coins); the runtime (per query) is bounded in terms of ε,d, and α.

4. Design an algorithm that tests whether an input graph of maximum degree $\leq d$ is planar (or ε^*-far from planarity).

   Hint: apply the above partition oracle (without knowing if the graph is planar), estimate how many edges it removes, and check whether the resulting parts are planar.