Advanced Algorithms 2012A – Problem Set 3

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In class we discussed the flow/cut gaps, first for a single-commodity, then multi-commodity flow/multicut, and finally conccurrent-flow/sparset-cut.

1. Write for concurrent flow an LP formulation that has a polynomial size. (The one seen in class uses exponentially many variables.)

   Remark: There is no need to prove your LP is equivalent to the one in class, but make sure (to yourself) it correctly solves the concurrent flow problem.

2. Recall that in the definition of the Sparset-Cut problem, the goal is to find a subset of edge $E' \subseteq E$ that minimizes a certain ratio. Prove that there is always (meaning in every instance of the problem) an optimal solution which is the cut $(A, \bar{A})$ for some vertex-subset $A \subset V$.

   Hint: Start with an arbitrary optimal E', and consider the connected components of $G \setminus E'$.

3. Prove that Sparse-Cut can be solved (exactly) in polynomial-time on trees, i.e. when the input graph G is a tree.

   Hint: First prove that there is always an optimal solution consisting of a single edge.

Extra credit:

4. Multiway cut is a special case of multicut, in which the demand pairs $\{s_i, t_i\}_{i \in [k]}$ are exactly all pairs of vertices in a set $T \subset V$ (called terminals); in particular $k = \binom{|T|}{2}$.

   Prove that in this special case of multiway cut, the flow/cut gap is at most 2 (or any other constant).

   Hint: Adapt the argument seen in class for single-commodity, where we cut at a radius $R > 0$ chosen at random from a uniform distribution. As usual, it is instructive (but not necessary) to assume that every edge (u, v) has very small length.