Sublinear Time and Space Algorithms 2018B – Problem Set 5

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Due: July 8, 2018

General instructions: Please keep your answers short and easy to read. You can use results, calculations or notation seen in class without repeating them, unless asked explicitly to redo them.

1. Prove that coresets for a monotone cost function C satisfy the Merge and Reduce properties. Prove that coresets for C^{MEB} satisfy also the Disjoint Union property.

2. Design a 1-coreset (i.e., exact and not approximate) for Minimum Enclosing Ball in $\mathbb{R}^d$ under ℓ_∞ norm, i.e., the cost function is

   $$C_P^\infty(x) = \max_{p \in P} \|p - x\|_\infty.$$

   What is the size of your coreset (as a function of d)? Does this cost function satisfy the the Merge and Reduce properties? And the Disjoint Union property?

3. Prove that every randomized streaming algorithm that determines whether an input graph on the vertex set $V = [n]$ is connected, requires storage of $\Omega(n)$ bits.

   For full credit, prove it for undirected graphs without parallel edges, and under edge insertions only (no deletions).

   Hint: use reduction from Indexing.

Extra credit:

4. Design for the equality function a simultaneous protocol with private randomness that uses $O(\sqrt{n})$ bits of communication.

   Hint: use a code (a collection of 2^n strings of length $O(n)$, every two of which have a large Hamming distance).

5. Prove the every algorithm that returns the exact median of a stream of odd length n, requires storage of $\Omega(n)$ bits, even if the algorithm is randomized and items come from a domain of size $O(n)$.

   Hint: use reduction from Indexing.