Randomized Algorithms 2020-1
Lecture 13

Balls and Bins, Poisson Approximation, Mutli-Choice Paradigm. *

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1 Karp Rabin Pattern Matching

Pattern matching is an important topic in algorithms and there are many books on the subject. The idea of the method is to come up with a 'rolling' hash function: one where given $h(x_1, x_2, \ldots x_m$ it is easy to compute the value on h on a sliding window input, i.e. if we wnat to compute $h(x_2, \ldots x_m, x_{m+1})$, then this can be done from x_1, x_{m+1} and $h(x_1, x_2, \ldots x_m)$.

The hash function can be based on irreducible polynomial of on primes. The analysis asks what is the probability that for $x \neq x'$ and a random prime we have that $x = x' \bmod p$. The answer relies on the Prime number Theorem that states that up to 2^k there are roughly $2^k/k$ primes and for an n bit number, the maximum number of primes that divide it is roughly $n/\log n$.

2 Poisson Approximation, Mutli-Choice Paradigm

Most of the material of this lecture is based on Chapters 5 and 17 in Mitzenmacher-Upfal and the survey by Udi Wieder [3].

The $Left[d]$ process was suggested by Vöcking [2]. The heavy loaded case was analyzed by Berenbrink et al. [1]

References

[1] Petra Berenbrink, Artur Czumaj, Angelika Steger, and Berthold Vöcking, *Balanced allocations: The heavily loaded case*, SIAM J. Comput.,35(6):1350–1385, 2006.

[2] Berthold Vöcking, How asymmetry helps load balancing. J. ACM,50(4):568–589, July 2003.

[3] Udi Wieder, **Hashing, Load Balancing and Multiple Choice**, Foundations and Trends in in Theoretical Computer ScienceVol. 12, No. 3 (2016) 275–378

*These notes summarize the material covered in class, usually skipping proofs, details, examples and so forth, and possibly adding some remarks, or pointers. In the interest of brevity, most references and credits were omitted.