Sublinear Time and Space Algorithms 2022B – Problem Set 1

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General instructions: Please keep your answers short and easy to read. You can use results, calculations or notation seen in class without repeating them, unless asked explicitly to redo them.

1. Generalize Morris' Algorithm (seen in class) and its analysis to an algorithm that increment X with probability $\frac{1}{(1+a)^X}$ for fixed $0 < a \leq 1$, and use this (without repetitions) to improve over the space complexity of Algorithm Morris+ for $(1+\epsilon)$-approximation, (seen in class), from polynomial to logarithmic dependence on $1/\epsilon$.

   Hint: Use $a = \Theta(\epsilon^2)$ to achieve space complexity $O(\log \frac{1}{\epsilon} \cdot \log \log m)$.

2. Design a streaming algorithm that at any point m (not known in advance) receives a query $S \subset [n]$ and outputs an estimate of what fraction of items in the stream belong to S, within additive error ϵ. Note that S is given only at query time (not in advance).

   Hint: Maintain $O(1/\epsilon^2)$ random samples and use them to estimate the fraction in S.