Homework #1

Due: 3/11/20

1. (Strogatz 2.4) Use linearization to classify the fixed points of the following systems. Use graphical approach to explain your result, and use this approach when linearized stability fails:

$$\dot{x} = ax - x^3 \text{ for all possible values of } a \tag{1}$$

$$\dot{x} = x(1-x)(2-x) \tag{2}$$

$$\dot{x} = x^2(6-x) \tag{3}$$

$$\dot{x} = \ln x \tag{4}$$

$$\dot{x} = 1 - e^{-x^2} \tag{5}$$

2. Consider a Hamiltonian system:

$$\begin{cases} \frac{dx}{dt} = \frac{\partial H(x,p)}{\partial p} \\ \frac{dp}{dt} = -\frac{\partial H(x,p)}{\partial x} \end{cases} \tag{6}$$

   (a) Prove that $\frac{dH}{dt}\,(x(t), p(t)) = 0$ where $(x(t), p(t))$ denotes a solution to (6).

   (b) What is the dynamical significance of the level sets of the Hamiltonian $H(x, p) = h$?

   (c) Consider the Hamiltonian of a particle in a double well potential:

   $$H(x,p) = \frac{1}{2}p^2 - \frac{1}{2}x^2 + \frac{1}{4}x^4, \quad (x,p) \in R^1 \times R^1$$

   draw the level curves of H in the (x, p) plane (the phase space), add time arrows, and explain the different types of motion of the particle.

   (d) Think: how would the above answers change if $H = H(x, p, t)$?

3. Meiss, pg 24, Q. 3.
4. List at least three (and up to 5) most important concepts learned in your reading (Chapter 1 of Meiss and/or additional material, see below).
5. List at least one result/notion which you find significant to you (related to area of your interest/difficult/non-intuitive/other).
6. Bonus I: Read chapter 3 of Meiss and solve additional ex. on pg 23-27.
7. Bonus II: find a research paper with a simple model in your field of interest (look for "toy model of .."/ "populations dynamics model of .."/ "bifurcations in ..") and describe the model: what are the dependent and independent variables, what effects where neglected and how was this justified.