Homework #7

Due: Jan. 13, 2015

1. Questions 2,3 of Homework #5 of M. Cross class (ref. (**C**) on the web-site).

2. Let Σ_N consist of all sequences of natural numbers $\{0,1,2,..,N-1\}$. Let σ denote the shift map on these sequences.

   (a) Find $CardPer_k(\sigma)$: the number of the periodic points of σ of period k.

   (b) Show that σ has a dense orbit.

   (c) Consider the map: $x_{n+1} = 3x_n$ mod 1. Prove that the map is chaotic (hint: use the symbolic dynamics on Σ_3). Prove that the middle-third Cantor set Λ is invariant under the map and that the map has a dense orbit on Λ (hint: use the subset of Σ_3 of sequences containing only the symbols $\{0,2\}$).

3. Questions 2,3 of section 4.13 of Meiss book (M, pg. 159)

4. **Bonus questions (nice questions on 1d maps):**

   - Construct numerically, by iterating initial conditions and leaving out the transients, the bifurcation diagram for:

     (a) The quadratic map: $x_{n+1} = rx_n(1-x_n) \qquad x_n \in [0,1]$ for $r \in [0,4]$

     (b) The sine map: $x_{n+1} = r \sin \pi x_n \qquad x_n \in [0,1]$ for $r \in [0,1]$

     (c) Let r_n denote the nth period doubling bifurcation in the doubling bifurcation sequence. For both maps, find numerically, for as large an n as you can, the ratio: $\delta_n = \frac{r_n - r_{n-1}}{r_{n+1} - r_n}$. Can you see convergence to the Universal Feigenbaum constant $\delta = 4.669201..$? (can you derive more sophisticated ways to find δ_n?). Read about the universality of the period doubling bifurcation.

   - Questions 4 of Homework #5 of M. Cross class