Homework #6

Due: Jan 6, 2015

1. Show that the map:

$$F_\mu : x_{n+1} = x_n + \mu - x_n^2 \qquad x_n \in R \tag{1}$$

undergoes a saddle-node bifurcation at $(x,\mu) = (0,0)$; show that for $\mu < 0$ it has no fixed points whereas for $\mu > 0$ it has two fixed points, one stable and one unstable. Draw the bifurcation diagram for F_μ.

2. Consider a general one dimensional family of maps $F_\mu(x) : R \to R$. Find conditions under which $F_\mu(x)$ undergoes a saddle-node bifurcation at some value $(x,\mu) = (x^*,\mu^*)$: define $G(x,\mu) = F_\mu(x) - x$, and using the implicit function theorem find conditions under which $G(x,\mu) = 0$ has a unique parabola like solution $G(x,\mu(x))$ for $\mu > \mu^*$ (and $|\mu - \mu^*|$ and $|x - x^*|$ small). Verify that (1) indeed satisfies these conditions.

3. Show that the flow:

$$f_\mu : \dot{x} = \mu x - x^2 \qquad x \in R$$

undergoes a transcritical bifurcation at $(x,\mu) = (0,0)$; show that the stability of the two fixed points is interchanged at the origin. Draw the bifurcation diagram for f_μ. Such a bifurcation occurs when symmetry/modeling constraints imply that $x = 0$ is always a solution.

Optional: You may want to verify that by breaking this symmetry assumption we are back to the saddle-node case. You may want to derive general conditions for a flow f_μ having this symmetry to have a transcritical bifurcation.

4. Consider the initial value problem

$$\dot{y} = \lambda y, \quad y(0) = 1, \quad \lambda = -50.$$

For $t = [0,1]$ display (y,t) graph for both the analytic solution and the numerical solution by solving with:

(a) Explicit Euler and RK4 methods for different time steps h. What h should be, for the method to converge?

(b) “ode45" and “ode15s" functions in Matlab. Try to increase $|\lambda|$ and t and describe the difference between them.

5. Consider the tent map: $x_{n+1} = F(x_n)$

$$F(x) = \begin{cases} ax & x < 1/2 \\ a(1-x) & x > 1/2 \end{cases} \qquad x \in [0,1], \quad a \in [0,2]. \tag{2}$$

(a) Find the fixed points of F and their stability.

(b) Show graphically convergence to a stable fixed point and divergence from an unstable fixed point (choose appropriate a values).

(c) Draw the graph of $F^2(x)$ for $a = 2$. How many fixed points it has? How about $F^n(x)$?