# From the Tropics to the Poles in Forty Days

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## ABSTRACT

Field experiments show that the poleward velocity of high-altitude weather balloons may, on rare occasions, be much higher than the observed poleward winds, while their eastward velocity is much slower than the observed eastward winds. Considering a simple physical model of horizontal particle's motion in the atmosphere, which includes a realistic model of the pressure field, it is shown that the existence of a nearly flat parabolic resonance in the model gives rise to such flights of balloons on the observed timescales even though the associated atmospheric pressure field does not support large poleward velocities.

## 1. Introduction

The dynamics of weather balloons is of vast interest for meteorological and for environmental studies, inducing expensive large-scale field experiments [e.g., the TWERL experiment (Julian et al. 1977) and the planned 1999 STRATEOLE experiment, with an estimated cost of several million dollars]. One question that is addressed in these experiments is the stability/instability of the balloons in the meridional (i.e., north–south) direction. Indeed, observations on the meridional excursion of high-altitude (10–15 km above sea level) constant-level balloons launched in middle and low latitudes have shown that in the course of their flight these balloons can have extended periods (in particular exceeding 1 day) during which the flight is directed entirely poleward. During these periods the balloons move with a meridional speed of over 30 kt (about 15 m s$^{-1}$) and with nearly zero zonal (i.e., east–west) speed (Julian et al. 1977; Levanon and Julian 1977). While these meridional squirts were encountered only in about 1% of the balloons and thus are by no means representative of the mean balloons trajectories, their existence poses several fundamental difficulties. First, these infrequent Lagrangian findings seem to contradict the Eulerian observations and the results obtained from Eulerian general circulation models (GCMs) of the atmosphere on the global winds, which together show that the prevailing winds flowing around the globe are mainly zonal. There have been no other reports on observations of either such high meridional winds or of such low zonal winds except for those resulting from monitoring constant-level balloon trajectories. Another related issue, resulting from the balloons' meridional rather than zonal flight, is the pressure field associated with this velocity. It is well known that, to a very good approximation, the velocity of air parcels in the planetary atmosphere is determined by a balance, called geostrophy, between the local pressure gradient force acting on it and the Coriolis force. The latter results from the rotation of the earth acting to produce a force perpendicular to the parcel's velocity relative to the rotating earth. According to classic theory this balance is established in about 1 day from the application of an initial pressure gradient. Thus one expects to find a pressure field with a strong zonal gradient to support the observed meridional velocity of these exceptional poleward flights. By calculating the pressure

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field for the time when the meridional squirts took place, it became obvious that no such zonal gradients of pressure existed so that the velocity as inferred from the balloons' flight could not have been in geostrophic balance with the pressure.

Here, we propose that a Lagrangian mechanism may explain the above-mentioned observations of exceptional poleward squirts. Traditionally, two simple models have been used for describing the motion of particles in the atmosphere. One model proposes that the geostrophic wind, which provides a first-order steady (i.e., time independent) approximation to the observed air velocity in the planetary atmosphere for a given pressure field, carries the balloons as Lagrangian particles. Perturbing the geostrophic flow by adding time-dependent components can yield the chaotic transport. Yet, as mentioned above, such models have not resulted in an explanation of the large-scale meridional squirts. At the other extreme lies the simplest time-dependent model of a particle flow in the atmosphere—the inertial motion—where no pressure gradient, or other forces, exists and the Coriolis force causes an acceleration of air parcels. In such a model the effect of the atmospheric pressure variation is considered small; thus the particle motion may be treated as nearly inertial; see Paldor and Killworth (1988), Paldor and Boss (1992), and Rom-Kedar et al. (1997). One way of bridging the gap between these two simplified models is to introduce into the time-dependent inertial model the steady, meridional pressure gradient corresponding to the zonal geostrophic winds. Then the equilibrium solutions of the new time-dependent model correspond exactly to the steady geostrophic winds, and nearby solutions correspond to oscillations about geostrophy. Moreover, in such a model it is natural to introduce additional perturbation terms in the pressure gradient, which is, in addition, time and zonally dependent. Physically, such a model proposes that the weather balloon is driven by the atmosphere via the atmospheric pressure field exerted upon it, while the particle's velocity is determined by balancing this pressure gradient with the Coriolis force that results from its own motion. Thus, one obtains the following near-geostrophic model for the eastward and northward velocity components ($u$, $\upsilon$) and the rate of change of the longitude and latitude coordinates ($\lambda$, $\phi$) of a particle in the atmosphere:

$$\frac{d\lambda}{dt} = \frac{u}{\cos\phi},$$

$$\frac{du}{dt} = \upsilon \sin\phi\left(1+\frac{u}{\cos\phi}\right) - k\varepsilon\frac{A(\phi)}{\cos\phi}\cos(k\lambda-\sigma t),$$

$$\frac{d\phi}{dt} = \upsilon, \tag{1}$$

$$\frac{d\upsilon}{dt} = -u\sin\phi\left(1+\frac{u}{\cos\phi}\right) - B'(\phi) - \varepsilon A'(\phi)\sin(k\lambda-\sigma t).$$

These equations are written in nondimensional form, where the length is normalized by the radius of the earth and time by 12 h/$2\pi$ (velocity scale is thus $10^3$ m s$^{-1}$ and energy $10^6$ m$^2$ s$^{-2}$). Thus, the relevant atmospheric velocities are of $O(0.01)$ corresponding to dimensional velocities of order 10 m s$^{-1}$. The Coriolis force, on a global scale, is the first term in the equations for $\dot{u}$, $\dot{\upsilon}$ that includes the convergence of longitudes at the poles term $u/\cos\phi$ resulting from the spherical geometry. Here, $B(\phi)$ and $A(\phi)$ represent the latitude-dependent amplitudes of the constant pressure term and the traveling pressure wave, respectively. The constant $\varepsilon$ is smaller than 1, while $k$ and $\sigma$ are the wavenumber and frequency of the zonally traveling wave whose phase speed, $c$, equals $\sigma/k$.

## 2. The near-geostrophic model

For steady, zonally independent pressure fields ($\varepsilon = 0$), the corresponding motion of weather balloons (i.e., particles with small kinetic energy) is very simple: for midlatitudes it always corresponds to small oscillations around a mean value $\bar{\phi}$ and a drift in longitude, with mean zonal velocity

$$\frac{1}{2}\sqrt{1-\frac{8B'(\bar{\phi})}{\sin(2\bar{\phi})}} - \frac{1}{2}.$$

To obtain our new realistic model, we take a steady pressure term that is monotonically decreasing poleward, allowing strong zonal jets to appear near $\phi_0$:

$$B(\phi) = \beta\tanh\frac{\phi^2-\phi_0^2}{\alpha},$$

$$\beta = -0.001,$$

$$\phi_0 = 30° \approx 0.5236, \quad (2)$$

$$\alpha = 0.05.$$

Such a form fits well with the *U.S. Standard Atmosphere, 1976* data on the geopotential of the 150 mB, creating a zonally propagating jet at $\phi = 30° \pm 7°$ with a maximal eastward velocity of 25 m $s^{-1}$.

Near the equator, three types of balloon motions may appear: the typical motions correspond to either small oscillations restricted to one hemisphere (as for the midlatitude case), or to equator-crossing excursions. The special initial conditions, lying on the boundary between these two regions that produce the typical behaviors, correspond to an asymptotic convergence to the equator (homoclinic motion). The maximal latitude to which all such near-equatorial motions may reach is *bounded* and *small* for the initial velocities associated with weather balloons. Thus, with this simple model, north–south large-scale motion of equatorial particles occurs only for particles with large initial velocities. For example, an equatorial particle with small initial poleward velocity may reach Antarctica (60°S) only if it has an initial westward velocity of order 250 m $s^{-1}$, which is two orders of magnitude faster than the observed equatorial velocities in the atmosphere.

Zonally traveling pressure waves ($\varepsilon \neq 0$) that perturb the geopotential surface from its mean axis-symmetrical shape cause some of the particles to move chaotically (Paldor and Killworth 1988; Paldor and Boss 1992). Such zonal waves correspond to variation in the geopotential height of the isobaric surface of $O(\varepsilon)$ in nondimensional units, corresponding to height variations of order $10^5\,\varepsilon$ in meters. Thus, the observed amplitude of zonal waves of about 100 m (Holton 1975) corresponds to $\varepsilon = O(0.001)$ in our model. The analysis (Rom-Kedar et al. 1997) [performed for $B(\phi) = 0$] shows that depending on the wave speed $c \equiv \sigma/k$, the wavenumber $k$, and the initial conditions, such zonal waves may create many different types of chaotic motions, some of which are highly nonuniform in the zonal direction. Nonetheless, the existence of KAM tori (Arnold 1988) prevents, in most cases, the envelope of the chaotic solutions from deviating too much from the corresponding unperturbed motion. The exception to this rule appears when a flat, or nearly flat, parabolic resonance occurs (Rom-Kedar 1997). The flat parabolic resonance appears in the near-geostrophic model when the total pressure field consists of a zonally standing wave multiplied by an arbitrary latitude-dependent coefficient [i.e., $B'(\phi) \equiv 0$, $\sigma = 0$]. Notice that in this case, the mean drift near the elliptic equilibria vanishes for all $\bar{\phi}$. In this case, even the slightest $\varepsilon$ value [e.g., $\varepsilon = O(10^{-4})$] can cause rare squirts extending from the equator vicinity to the poles; see Fig. 1a. Mathematically, this situation corresponds to the existence of a singular energy surface on which no KAM tori survive the perturbation. Any situation that is close to the above, for example, our realistic situation that includes small wave speeds and $B'(\phi) \ll 1$, results in a nearly flat parabolic resonance for which the instability exists in a somewhat reduced form. This phenomenon was demonstrated for an artificial function $B(\phi)$ in Rom-Kedar (1996). Here, we show that the model with the realistic value of $\varepsilon$ and the realistic form of $B(\phi)$ gives rise to a similar phenomenon and, moreover, that the resulting timescale for the balloons' squirts fits the field observations well (Julian et al. 1977; Levanon and Julian 1977).

To better understand the mathematical origin of this phenomenon, Eq. (1) is transformed to the new canonical coordinates ($\phi$, $\upsilon$, $D$, $\Lambda$), where $D$ is the angular momentum[1] [$D = \cos\phi(1/2\cos\phi + u)$] and $\Lambda$ is the longitude coordinate moved into the traveling pressure wave frame ($\Lambda = \lambda - ct$). In these new coordinates the system (1) is transformed into a canonical Hamiltonian system:

$$\frac{d\Lambda}{dt} = \frac{D}{\cos^2\phi} - \left(c + \frac{1}{2}\right), \quad (3a)$$

$$\frac{dD}{dt} = -k\varepsilon A(\phi)\cos(k\Lambda), \quad (3b)$$

$$\frac{d\phi}{dt} = \upsilon, \quad (3c)$$

$$\frac{d\upsilon}{dt} = \frac{1}{8}\sin(2\phi)\left(1 - \frac{4D^2}{\cos^4\phi}\right) - B'(\phi) - \varepsilon A'(\phi)\sin(k\Lambda). \quad (3d)$$

Notice that $\dot{D} = O(\varepsilon)$, thus to zeroth order the angular momentum $D$ is indeed a constant of motion. Linear

[1]Notice that in our previous publications (Rom-Kedar 1997; Rom-Kedar et al. 1997) we have used $\mathrm{D} = 2D$, $\Lambda = 1/2\Lambda$.

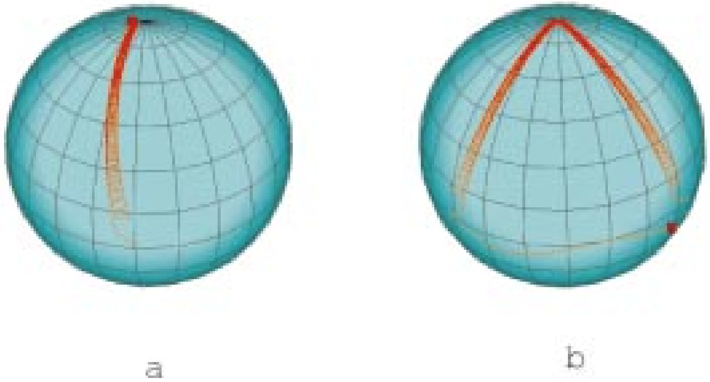


FIG. 1. Meridional squirts for the near-inertial model with zonally standing waves. Evolution of one initial condition in the $(\phi, \Lambda)$ plane. $A(\phi) = \cos(\phi)$. $B(\phi) \equiv 0$. $\varepsilon = 0.0004$, $k = 3$, $c = 0$. $u_0 = 0.0038125$, $\upsilon_0 = 0$, $\phi_0 = 0.087266 = 5°$, $\lambda_0 = 2.3712 = 135.9°$. (a) $t = 500 \approx 40$ days, (b) $t = 1750$.

stability of the near-equatorial motion w.r.t. small north–south deviations [i.e., for $\varepsilon = 0$, fix $D$ and consider, in the $(\phi, \upsilon)$ plane, the linear stability of the origin[2]] reveals that there exists a

$$D = D_p \equiv \sqrt{\frac{1}{4} - B''(0)}$$

value such that for $|D| > D_p$ it is linearly stable (i.e., center since the system is Hamiltonian), whereas for $|D| < D_p$ the motion is linearly unstable. Indeed, for $|D| < D_p$, there exist a homoclinic loop in the $(\phi, \upsilon)$ plane, emanating from the origin and extending up to latitude around $\pm$ arccos $(2|D|)$. This loop separates initial conditions that oscillate about a mean zonal motion restricted to one hemisphere [for $B'(\phi) \equiv 0$ this corresponds to the well-known westward-migrating inertial oscillations on a $\beta$ plane] from those traveling between the two hemispheres. At $D = D_p$ the equatorial motion is parabolic; namely, both the frequency of the oscillation and the growth rate in the $(\phi, \upsilon)$ plane vanish.

Notice that near-equatorial weather balloons correspond to $\phi \approx 0$, $u$, $\upsilon = O(0.01)$; hence these have initial angular momentum $D \approx 1/2$. Moreover, it follows from (2) that $B''(0) \ll 1$, hence $D_p \approx 1/2$. Therefore, it follows that *weather balloons launched near the equator correspond to initial conditions that are close to the parabolic region in phase space.*

[2]For simplicity of the presentation $B(\phi)$ and $A(\phi)$ are henceforth assumed to be even. Similar expressions may be found for the nonsymmetric case, where the fixed point in the $(\phi, \upsilon)$ plane is shifted from the origin.

For most values of $c$ (i.e., except those values where resonance occurs), by KAM theorem, the motion near the elliptic points hardly changes under small [w.r.t. the typical size of (3b) and (3d); i.e., $\varepsilon = o(0.01)$] perturbations. On the other hand, the homoclinic motion produces chaotic behavior. Usually, the width of the chaotic zone near the homoclinic loop is of order $\varepsilon$. Moreover, it is exponentially small in $c$ and in 1/distance from the parabolic point. Thus, for most values of $c$ and for small values of $\varepsilon$, near-equatorial weather balloons do not rise to high latitudes on any timescale; see, for example, Figs. 2b,c (similar results appear in the near-inertial model).

However, notice from (3) that for any given $c$ value the motion of the particle along the equator coincides with the traveling wave motion for $D = D_c \equiv c + 1/2$ [i.e., $\dot{\Lambda}(\phi = \upsilon = 0, D_c; c) = 0$]. Then, in the moving frame, any initial condition on the equator with $D = D_c$ is a fixed point of (3) for $\varepsilon = 0$. This situation is highly degenerate. If it happens for $|D_c| > D_p$, in which case the equator is stable w.r.t. north–south deviations, then a strong elliptic resonance occurs, leading to zonally localized motion near the equator. The width (in $D$) of such an elliptic resonance is of order $\varepsilon^{1/2}$. If $|D_c| < D_p$, then a hyperbolic resonance occurs (Haller and Wiggins 1995); the motion is chaotic near the homoclinic loop that extends up to latitude $\pm$ arccos $2|D_c|$, with the chaotic zone being of order $\varepsilon^{1/2}$ [unlike the usual $O(\varepsilon)$ width associated with the regular homoclinic chaotic zone]. The parabolic resonance occurs whenever the fixed motion along the equator occurs exactly where the equator is parabolic w.r.t. north–south motion; that is, when $D_c = D_p$. This occurs for a specific pressure wave zonal velocity $c = c_p$:

$$c_p = D_p - \frac{1}{2} = \frac{1}{2}\left(\sqrt{1 - 4B''(0)} - 1\right) \approx 0.001. \quad (4)$$

In this case, the motion of some of the particles is dramatically changed.

In Figs. 1a,b and 2a we demonstrate the possible effect of this phenomenon by showing the trajectories of specific initial conditions; in Fig. 1a $B(\phi) \equiv 0$, and it is demonstrated that in this case even extremely small zonal dependence ($\varepsilon = 10^{-4}$) may create squirts extending close to the poles, since this case corresponds to a flat parabolic resonance. Moreover, we show that the trajectories may be localized in longitude even after very long integration time. In Fig. 2a we show a trajectory of (1) when the realistic pressure field (2) and the realistic size of zonal-dependent pressure field ($\varepsilon = 0.0011$) are used. The value of $c$ is chosen according to (4) so that a (nearly flat) parabolic resonance occurs. Such a $c$ corresponds to a zonally traveling wave with a period of roughly 1 yr. Comparison with Figs. 2b,c demonstrates that such slowly varying waves affect the equatorial balloons' stability much more than the higher-frequency waves.

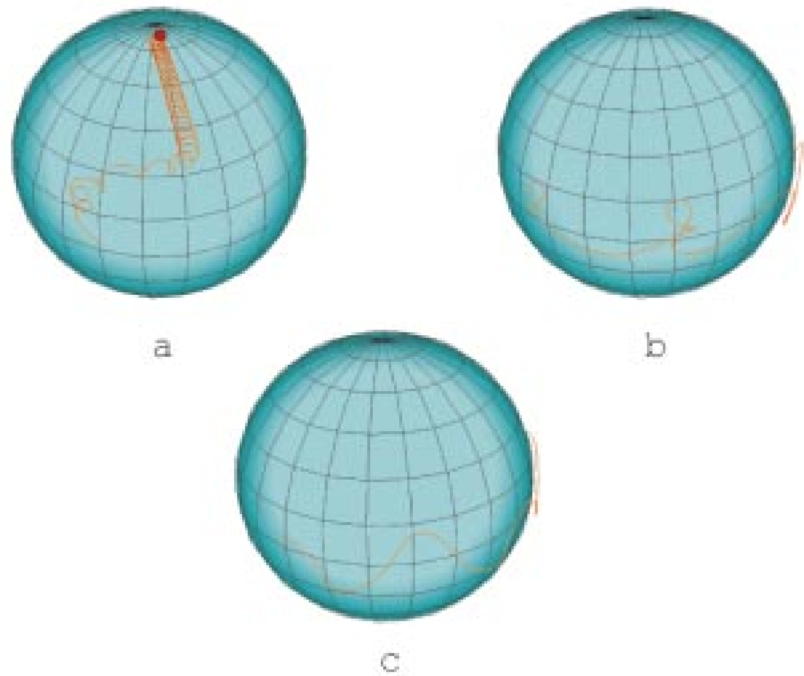


FIG. 2. Near-equatorial motion for the near-geostrophic model. Evolution of one initial condition in the ($\Lambda$, $\phi$) plane. $A(\phi) = \cos(\phi)$, $B(\phi) = -0.001 \tanh (\phi^2 - 0.5236^2)/0.05$ $t = 300$, $\varepsilon = 0.0011$, $k = 3$ $u_0 = 0.0049242$, $\upsilon_0 = 0.0011$, $\phi_0 = 0.12 = 6.9°$, $\lambda_0 = 1.855 = 106.3°$. (a) Under parabolic resonance conditions $c = 0.001$; (b) non-monotonic zonal behavior, $c = 0.01$; (c) far from parabolic resonance conditions, monotonic zonal motion, $c = 0.1$.

The orbits shown correspond to *rare* initial conditions for which northward squirts are found. More generally, several types of motion are possible near the equator under the parabolic conditions. First, as for the elliptic resonance case, the particle may perform relatively regular, essentially quasiperiodic, north–south excursions up to latitudes of order $\varepsilon^{1/2}$. Second, the particle may get trapped inside the parabolic resonance zone. When trapped, its motion consists of two different segments; one going eastward w.r.t. to the wave speed, corresponding to north–south excursions that reach latitudes of $O(\varepsilon^{1/2})$ and that are zonally limited to regions of order $2\pi k^{-1}$. The other segment corresponds to motion in one hemisphere, by which the particle oscillates about a mean northward (or southward) motion along the unperturbed stable equilibrium branch of solutions. These excursions are unlimited in latitude for $B(\phi) \equiv 0$, $c = 0$ and are of large magnitude for small (w.r.t. 1) $B'(\phi)$ and $|c - c_p|$. The smaller these are, the larger the latitude that may be reached by such a trapped trajectory. Moreover, long-time integration of such solutions contains, in many cases, the long squirts that are shown in Figs. 2a and 1 and, with them, the possible transfer to a neighboring resonance cell, corresponding to a different zonal region.

Finally, notice that to have long squirts $D$ must change by an $O(1)$ quantity. It follows from (3b) that the timescale for such squirts is at least $O(1/k\varepsilon)$. Indeed, we find numerically that doubling the value of $\varepsilon$ to 0.002 produces a squirt like the one shown in Fig. 1a that reaches the pole in 20 instead of 40 days. Moreover, the numerically observed squirts last about 350 nondimensional time units, which agrees well with $k\varepsilon = 0.0033$. Indeed, in the TWERL field experiment (Julian et al. 1977) balloon 1274 was launched on 1 September 1975 from American Samoa (14°S, 170°W) and crossed 60°S latitude on 1 October; that is, that balloon traveled nearly 45° in 30 days. Thus, we conclude that our model, in which the order of magnitude of all parameters, including $\varepsilon$, have been determined from atmospherical Eulerian observations, produces the observed timescale of the Lagrangian squirts.

Mathematically, the behavior of near-flat parabolic resonances provides a new type of strong chaotic instability for a two degrees of freedom (d.o.f.) Hamiltonian system (Rom-Kedar 1997). There, it was established that the occurrence of parabolic resonances is a codimension one phenomenon (i.e., it will typically occur in any one-parameter family of a near-integrable two d.o.f. Hamiltonian system); hence it is expected to appear in numerous applications. Moreover, the basic idea that the addition of another nonseparable d.o.f. may be considered from the bifurcation theory point of view as the addition of another parameter (Lerman and Umanskii 1994a,b) applies to higher d.o.f. as well; for example, we may think of the wave speed *c* as another conserved quantity of a larger system showing that for higher-dimensional systems the parabolic resonance case may be generic.

## 3. Conclusions

The mechanism of nearly flat parabolic resonance that appears in the proposed near-geostrophic model (1) offers an explanation for both peculiar observations on the balloons' trajectories; the large meridional velocity goes together with a very small zonal velocity during the times when the parabolic resonance dominates the flow. These resonant flows happen in highly limited longitudinal bands—thus the low probability of the balloons entering the resonance band—and last, this resonant flow of the balloons is not in geostrophic balance, so the pressure field does not have to adjust to the velocity field. Moreover, it has been demonstrated that using realistic magnitudes of meridional and zonal pressure gradients in the model produce poleward squirts that occur on the experimentally observed timescales. We also note that the proposed model (1) exhibits (as does almost any low-dimensional Lagrangian model for chaotic advection) exponential divergence of nearby trajectories on small timescales and possibly slowly decaying correlations on longer timescales, which are responsible for both super- and subdiffusive behavior. These well-known properties of typical chaotic low-dimensional Hamiltonian systems [see Schlesinger et al. (1993) and references therein] fit well with atmospherical observations on particle motion, much better than the traditional turbulent diffusional models.

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