Algorithmic Game Theory - handout3

Uriel Feige

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The following table explains how to obtain the dual of a primal LP that is in general form. Here vectors are column vectors, A_i denotes row i of matrix A and A^j denotes column j.

$\min c^T x$		$\max b^T y$
$A_i x \geq b_i$	$i \in I^+$	$y_i \geq 0$
$A_i x = b_i$	$i \in I^=$	y_i free
$x_j \geq 0$	$j \in J^+$	$y^T A^j \leq c_j$
x_j free	$j \in J^=$	$y^T A^j = c_j$

Reading. Linear programming is a very useful algorithmic technique that has a well developed theory behind it. If this subject is new to you, it is recommended that you read more about it.

The surprise examination paradox.

A teacher announces that in the following week there will be a surprise examination. A clever student argues by backward induction that having a surprise examination is impossible. (The exam cannot be on the last day because by then the students will not be surprised by it. Having agreed that it cannot be on the last day, by the time the day before last arrives, the students expect it to be given on that day. And so on.) Here we consider a multi-round two player zero sum game between a teacher and a student. In every round the teacher has two possible actions: either A (to give an exam) or B (not to give an exam). In every round, the student has two possible actions: either A (to study towards an exam) or B (not to study). In every round, both players play simultaneously. In the unbounded version of the game, the game ends on the first round on which at least one of the players plays A. If on that round both play A the student wins, and if only one of them played A the teacher wins. If the game never ends, no player wins (it is a tie). In the bounded version of the game, the game is known to last for at most 4 rounds. If no player wins by the 4th round (no player played A in any of the first 4 rounds), then the student wins.

Please keep the answers to the following questions short and easy to read, though there is no need to hand in this homework.

1. In the bounded game, what is a max-min strategy for the teacher? What is a max-min strategy for the student? How would you provide a simple proof that each of the strategies that you present is indeed a max-min strategy?
2. In the unbounded game, is there a max-min strategy for the teacher? For the student? Explain.

Further food for thought about the unbounded game:

1. We said in class that a two player game of perfect information can be assumed not to repeat the same position twice (when it does the game can be declared a tie). Does this observation apply to the surprise examination game?
2. Does the surprise examination game have a mixed Nash equilibrium? If not, then what solution concept would you suggest for this game?