Algorithmic Game Theory - handout2

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Course webpage: https://www.wisdom.weizmann.ac.il/∼feige/agt2024.html

Homework assignments are an integral part of the course and will be a significant part of the grade. Please hand in the written assignments two weeks after they are given.

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Homework. (Please keep the answers short and easy to read.)

1. In class we showed the well known algorithm for finding a stable matching (a.k.a. stable marriage) of Gale and Shapely.

   Consider the following game with $2n$ players, n men and n women, each having his/her own preference list over partners of the other sex. In this game, every man and every woman supplies a preference list (either their true preference list, or some other preference list), the outcome of the game is the matching produced by the stable matching algorithm when run on the supplied preference lists (the algorithm where the unengaged men propose), and the payoff for a player is the rank (in the player's list) of the partner assigned to the player. An interesting question is whether the players have incentives to play *truthfully* in this game. Namely, is it always to the benefit of a player to report his or her true preference list, or may the player win a better partner (from the player's point of view) by reporting a different preference list?

   It is known that for every man, reporting his true preference list is a dominant strategy. The proof of this is non-trivial. In this assignment you are required to answer some easier questions.

   (a) Show that all players following the strategy of reporting their true preference lists is not necessarily a Nash equilibrium of the game. Namely, show an example ($n = 3$ suffices for this purpose), where a woman can benefit (eventually be matched by the algorithm to a man that she prefers more) by reporting a preference list that is different from her true preference list.

   (b) Prove that this game always has some pure Nash equilibrium (though as question 1(a) shows, in this Nash equilibrium some players might not be reporting their true preferences).

2. **The surprise examination paradox.**

   A teacher announces that in the following week there will be a surprise examination. A clever student argues by backward induction that having a surprise examination is impossible. (The exam cannot be on the last day because by then the students will not be surprised by it. Having agreed that it cannot be on the last day, by the time the day before last arrives, the students expect it to be given on that day. And so on.) Here we consider a multi-round two player zero-sum game between a teacher and a student. In every round the teacher has two possible actions: either E (to give an exam) or N (not to give an exam). In every round, the student has two possible actions: either S (to study towards an exam) or N (not to study). In every round, both players play simultaneously.

   In the unbounded version of the game, the game ends on the first round on which at least one of the players does not play N. If on that round the play was (E, S) the student wins (he anticipated an exam on that day and was not surprised), and otherwise the teacher wins (the student failed to predict the day of the exam and was surprised). If the game never ends, no player wins (it is a tie).

   The bounded version of the game is similar to the unbounded version, except that the game is known to last for *at most* 4 rounds. If the game reaches round 4, then round 4 is special in the following sense: if the teacher plays N in round 4, the student wins regardless of what the student plays.

   (a) In the bounded game, what is a max-min strategy for the teacher? What is a max-min strategy for the student? How would you provide a simple proof that each of the strategies that you present is indeed a max-min strategy?

   (b) In the unbounded game, is there a max-min strategy for the teacher? For the student? Explain.