Handout 1: FVS and metric facility location

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The course has a large overlap with the book [2].

We saw in class two algorithms for feedback vertex set, one with a $4 \log n$ approximation ratio, the other with an approximation ratio of 2. They are based on the local ratio technique. Lecture notes for this appear on the home page of the course.

We shall also consider (uncapacitated) metric facility location. Last semester we presented a deterministic algorithm with approximation ratio 4, and a randomized algorithm with approximation ratio 3. We shall present in class a deterministic primal-dual algorithm with an approximation ratio of 3, and an algorithm based on randomized rounding of an LP, with approximation ratio $1 + \frac{2}{e}$.

In the homework we shall explore greedy-like algorithms whose analysis is based on *dual-fitting*, and derivation of quantitative bounds on their approximation ratio is based on *factor-revealing LPs*. Those who wish to receive credit on this course should hand in the homework by May 15.

The questions below refer to pages 795–804 and 807–808 in [1], but you are of course welcome to also read other parts of that paper.

1. Present Algorithm 1 from [1], and explain how it can be implemented in polynomial time.

2. Present the LP relaxation of Balinsky, its dual, and its relation to Algorithm 1.

3. Explain what *dual fitting* is, and why it is relevant to the analysis of the approximation ratio of Algorithm 1.

4. Explain the *factor revealing LP* and why it captures the approximation ratio of Algorithm 1. How can it be used in order to establish lower bounds on the approximation ratio of Algorithm 1? How can it be used in order to provide an upper bound on the approximation ratio of Algorithm 1? (You do not need to derive any solutions to the factor revealing LP. Just explain what solutions, if found, can tell us about the approximation ratio of Algorithm 1.)

5. Present Algorithm 2 from [1], and explain how it can be implemented in polynomial time.

6. Consider the statement: “for every instance of metric facility location, Algorithm 2 provides a solution of cost not higher than that given by Algorithm 1”. Do you think that this statement is true? Can you prove it? Can you provide a counter example?

References

[1] Kamal Jain, Mohammad Mahdian, Evangelos Markakis, Amin Saberi, Vijay V. Vazirani: Greedy facility location algorithms analyzed using dual fitting with factor-revealing LP. J. ACM 50(6): 795–824 (2003)

[2] David P. Williamson, David B. Shmoys: The Design of Approximation Algorithms. Cambridge University Press 2011. https://www-cambridge-org.ezproxy.weizmann.ac.il/core/books/design-of-approximation-algorithms/88E0AEAEFF2382681A103EEA572B83C6