Final assignment

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1. Recall that an n point metric space (over a finite set V, with distance function d) is an ℓ_2^2 metric (also called *negative type* metric) if there is an embedding $f : V \to R^n$ such that for every $i, j \in V$ their ℓ_2 distance satisfies $||f(i) - f(j)||_2 = \sqrt{d(i,j)}$. Show that every finite ℓ_1 metric space in which all points are vectors with integer coefficients is an ℓ_2^2 metric space.

2. Recall the uniform sparsest cut problem

$$\min \frac{c(E[S, V \setminus S])}{|S| \cdot |V \setminus S|}$$

with edge costs $c : E \to R^+$. For it, we presented in class the $O(\sqrt{\log n})$ approximation algorithm of Arora, Rao and Vazirani ([1], appears also in [3], and in more recent lecture notes of Rothvoss [2]). However, to simplify the presentation, we assumed in class that in the optimal solution $|S| = \frac{|V|}{2}$. Explain how the algorithm and its analysis can be adapted to handle input instances in which the optimum is obtained for a set S of a much smaller size (that might be sub-linear in $|V|$).

3. In the *minimum simultaneous subgraph cut* (min-SSC) problem, the input is an n vertex graph $G(V, E)$ with non-negative edge costs $c : E \to R^+$, and a list of k sets $S_1, \ldots S_k \in V$, where every set contains exactly r vertices. The goal is to remove a minimum cost set of edges $F \subset E$ so that in the remaining graph, there no input set S_i that has all its vertices in the same connected component. You may assume that $1 \leq k \leq n$ and that $3 \leq r \leq n - 1$. Design polynomial time algorithms for min-SSC (there may be different algorithms for different ranges of parameters) that together cover the whole range of parameters. For each algorithm prove an upper bound on its approximation ratio, as a function of n, k, r. (For some limited ranges of the parameters there are algorithms that return the optimal answer.) Provide also hardness of approximation results for min-SSC (they need not match the approximation ratios achieved by your algorithms).

References

[1] Sanjeev Arora, Satish Rao, Umesh V. Vazirani: Expander flows, geometric embeddings and graph partitioning. J. ACM 56(2): 5:1–5:37 (2009).

[2] Thomas Rothvoss: Lecture Notes on the ARV Algorithm for Sparsest Cut. CoRR abs/1607.00854 (2016).

[3] David P. Williamson, David B. Shmoys: The Design of Approximation Algorithms. Cambridge University Press 2011.