The Hardness of Approximating Hereditary Properties

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Abstract

We consider the following class of problems: Given a graph G, find a maximum vertex induced subgraph of G satisfying a nontrivial hereditary property π. We show that this problem cannot be approximated for any such property π, within a factor of $n^{1-\epsilon}$ for any $\epsilon > 0$, unless $NP = ZPP$. This improves the result in [LY93] where it was shown that for any nontrivial hereditary property, the maximum subgraph problem cannot be approximated within a factor of $2^{(\log n)^c}$ for some $c > 0$, unless $NP \subseteq QP$.

1 Introduction

A graph property π is a collection of graphs. A property π is called nontrivial if there are infinitely many graphs for which π holds and infinitely many graphs for which π does not hold. A nontrivial graph property is said to be *hereditary* if whenever a graph G satisfies property π then also every vertex induced subgraph of G satisfies π. The maximum subgraph with property π problem is defined in the following manner: Given a graph G find the maximum vertex induced subgraph of G which satisfies property π. The maximum subgraph problem is NP-hard for any nontrivial hereditary property [LY80].

It is shown in [LY93] that for any nontrivial hereditary property π which is false for some complete multipartite graph, the maximum subgraph with property π problem cannot be approximated within a factor of n^ϵ for some $\epsilon > 0$ unless $P = NP$. In particular this theorem applies to the following graph properties: complete graph, independent set, planar, outerplanar, bipartite, complete bipartite, acyclic, max degree, interval, chordal.

Furthermore it was proven in [LY93] that for every nontrivial hereditary property π, the maximum subgraph with property π problem cannot be approximated within a factor of $2^{(\log n)^c}$ for some $c > 0$, unless $NP \subseteq QP$. Here QP is the class of languages which can be recognized in quasipolynomial time, i.e. time $2^{(\log n)^d}$ for some constant d. The conclusion of this theorem applies to the graph properties stated above and the following graph properties: comparability, permutation, perfect, circular-arc, circle, line graph.

In [Has99] it was shown that max-clique cannot be approximated within a factor of $n^{1-\epsilon}$ for any $\epsilon > 0$, unless $NP = ZPP$. We prove the following result:

Theorem 1.1. *For every nontrivial hereditary property π and for every $\epsilon > 0$, the maximum subgraph with property π problem cannot be approximated within a factor of $n^{1-\epsilon}$, unless $NP = ZPP$.*

For nontrivial hereditary properties which are false for some clique or independent set, this result follows in a straightforward manner by combining Hastad's result with the proof described in [LY93]. Thus the main contribution of this paper is in showing that this hardness result holds even for nontrivial hereditary properties which hold for all cliques and all independent sets.

A hereditary property π for which feasibility can be decided in time at most exponential in the size of the input is called a feasible hereditary property. In [Hal00] it was shown that for each feasible hereditary property π, the maximum subgraph with property π problem can be approximated within a factor of $n/\log n$. The maximum hereditary subgraph problem can be approximated within a factor of $O(n(\log\log n/\log n)^2)$, for feasible properties that fail for some clique or independent set (Theorem 2.6 of [Hal00]).

In certain situations we may wish to find a subgraph which does not only satisfy a property π but is also connected. A property π is called nontrivial on connected graphs if it holds only for connected graphs, and there are infinitely many connected graphs for which π holds and infinitely many connected graphs for which π does not hold. A nontrivial graph property on connected graphs is said to be *hereditary* if whenever a connected graph G satisfies property π then also every vertex induced connected subgraph of G satisfies π. The maximum connected subgraph with property π problem is defined in the following manner: Given a graph G find the maximum vertex induced connected subgraph of G which satisfies property π. The maximum connected subgraph problem is NP-hard for any nontrivial hereditary problem [Yan79]. Examples of properties that are hereditary and nontrivial on connected graphs include: clique, star, complete bipartite, path, tree, planar, outerplanar, bipartite, chordal, interval, max degree and others.

It is shown in [LY93] that for every property that is nontrivial and hereditary on connected graphs, the maximum connected subgraph problem cannot be approximated with ratio $2^{(\log n)^c}$

for some $c > 0$, unless $NP \subseteq QP$. Furthermore it is stated in [LY93] that if π is a nontrivial hereditary property on connected graphs which is satisfied by all paths and does not hold for some complete bipartite graph, then the maximum connected subgraph with property π problem cannot be approximated within a factor of $n^{1-\epsilon}$ for every $\epsilon > 0$, unless $P = NP$. We prove the following results:

Theorem 1.2. *For every property that is nontrivial and hereditary on connected graphs and for every $\epsilon > 0$, the maximum connected subgraph problem cannot be approximated with ratio $n^{1-\epsilon}$, unless $NP = ZPP$.*

Theorem 1.3. *Let π be a nontrivial hereditary property on connected graphs which is satisfied by all paths and does not hold for some star. Then the maximum connected subgraph with property π problem cannot be approximated within a factor of $o(n/\log n)$, unless 3-SAT can be solved in time $2^{o(n)}$.*

The reduction used in the proof of theorem 1.3 is similar to the one used in theorem 1 of [Yan79] to show the NP-hardness of the maximum connected subgraph problem. It is interesting to notice that it follows from theorem 1.3 that for certain hereditary properties it is harder to approximate the maximum connected subgraph problem then the maximum subgraph problem. For example by theorem 1.3 the maximum connected subgraph of degree smaller then k for every $k \geq 3$, cannot be approximate within a factor of $o(n/\log n)$ (under the assumption that there is no subexponential time algorithm for 3-SAT). On the other hand by Theorem 2.6 of [Hal00] the maximum subgraph of degree smaller then k for every $k \geq 3$, can be approximated within a factor of $O(n(\log\log n/\log n)^2)$ and thus it is easier to approximate then its connected counterpart.

One can also consider hereditary properties in directed graphs as well as in undirected graphs. Examples of such properties are: acyclic, transitive, symmetric, antisymmetric , tournament, max degree, line digraph. It was proved in [LY93] that for every nontrivial hereditary property on directed graphs, the maximum subgraph problem cannot be approximated with ratio $2^{(\log n)^c}$ for some $c > 0$, unless $NP \subseteq QP$. we prove the following:

Theorem 1.4. *For every nontrivial hereditary property on directed graphs and for every $\epsilon > 0$, the maximum subgraph problem cannot be approximated with ratio $n^{1-\epsilon}$, unless $NP = ZPP$.*

2 Hardness of Hereditary Properties

Let $\alpha(G)$ denote the size of the maximum independent set in G. Let $\alpha_{K_c}(G)$ denote the size of the maximum K_c-free vertex induced subgraph of G, where K_c is a clique on c vertices. For example $\alpha(G) = \alpha_{K_2}(G)$. Finally denote by $\alpha_H(G)$ the size of the maximum H-free induced subgraph of G, i.e. the size of the maximum induced subgraph which does not contain H as a vertex induced subgraph. From now on π shall denote a nontrivial hereditary graph property. For every property π we can define the complementary property π^c as follows: a graph G satisfies π^c if and only its complement G^c satisfies π. Notice that π^c is nontrivial and hereditary if and only if π is. It follows from Ramsey's theory that every nontrivial hereditary property π is satisfied either by all cliques or by all independent sets [LY93], as for any positive k, l a large enough graph G contains either an independent set of size l or a clique of size k. Thus for every nontrivial hereditary property π, either π or π^c is satisfied by all independent sets. We may assume w.l.o.g that π is satisfied by all independent sets. Let H be a forbidden graph with respect to property π. Notice that since π is satisfied by all independent sets H must contain at least one edge. In this section we shall prove the following theorem.

Theorem 2.1. *The following holds for every non-empty graph H. If for some $0 < \epsilon < \frac{1}{2}$ there is a randomized polynomial time algorithm which, when given an input graph G with n nodes does the following:*

- *if $\alpha(G) > n^{1-\epsilon}$ then the algorithm outputs 'case 1'.*
- *if $\alpha_H(G) < n^{\epsilon}$ then the algorithm outputs 'case 2' with constant probability.*

then $NP = ZPP$.

As any nontrivial hereditary graph property π which is satisfied by all independent sets has some forbidden non-empty graph H, i.e. H does not satisfy π, we get the following corollary.

Corollary 2.2. *For any nontrivial hereditary property π, the maximum subgraph with property π problem cannot be approximated within a factor of $n^{1-\epsilon}$ for any $\epsilon > 0$, unless $NP = ZPP$*

The reminder of this section is dedicated to proving theorem 2.1. Our starting point is Hastad's theorem [Has99].

Theorem 2.3. *If for some constant $0 < \epsilon < \frac{1}{2}$ there is a randomized algorithm which, when given an input graph G with n nodes does the following:*

- *If $\alpha(G) > n^{1-\epsilon}$ then the algorithms outputs 'case 1'*

- *If $\alpha(G) < n^{\epsilon}$ then the algorithm outputs 'case 2' with constant probability.*

then $NP = ZPP$.

We shall require the following well known version of the Ramsey theorem.

Lemma 2.4. *Let $k \geq 2$ be an integer. Then for every large enough n, any graph on n vertices contains a clique set of size k or an independent set of size at least $n^{1/k}$.*

Proof: We have the following theorem on Ramsey numbers ([ES35]): any graph on $\binom{k+t-2}{k-1}$ vertices contains a clique of size k or an independent of size t. Setting $t = \lceil n^{1/(k-1)} \rceil - k$ we get

$$\begin{aligned}\binom{k+t-2}{k-1} &\leq (k+t-2)^{k-1} \\ &\leq n\end{aligned}$$

Thus any graph on n vertices contains a clique of size k or an independent set of size $\lceil n^{1/(k-1)} \rceil - k \geq n^{1/k}$ for large enough n. ■

Theorem 2.5. *The following holds for any integer $c \geq 2$. If for some $0 < \epsilon < \frac{1}{2}$ there is a randomized polynomial time algorithm which, when given an input graph G with n nodes does the following:*

- *if $\alpha(G) > n^{1-\epsilon}$ then the algorithm outputs 'case 1'.*

- *if $\alpha_{K_c}(G) < n^{\epsilon}$ then the algorithm outputs 'case 2' with constant probability.*

then $NP = ZPP$.

Proof: Fix some $c \geq 2$ and suppose that for some $0 < \epsilon < \frac{1}{2}$ there is a randomized polynomial time algorithm $A(G)$ which, when given an input graph G on n vertices has the following behavior

- if $\alpha(G) > n^{1-\epsilon}$ then $A(G) = 1$ (the algorithm outputs 'case 1').

- if $\alpha_{K_c}(G) < n^{\epsilon}$ then $A(G) = 2$ (the algorithm outputs 'case 2') with constant probability.

If $\alpha(G) < n^{\epsilon/c}$ then by lemma 2.4 every subgraph of G of size n^{ϵ} contains a clique of size c and thus $\alpha_{K_c}(G) < n^{\epsilon}$. We conclude that

- If $\alpha(G) > n^{1-\epsilon/c}$ then $A(G) = 1$.
- If $\alpha(G) < n^{\epsilon/c}$ then $A(G) = 2$ with constant probability.

Thus by theorem 2.3 we are done. ■

We note that theorem 2.5 can also be derived in a relatively straightforward manner from Hastad's theorem and the proof of theorem 1 of [LY93], but our proof of theorem 2.5 is much simpler.

At this point our analysis diverges from the one in [LY93]. While the analysis in [LY93] for the case of general nontrivial hereditary properties is based upon partitioning large cliques into edge disjoint cliques of fixed size, our analysis involves partitioning a graph into edge disjoined subgraphs whose size is strongly dependent on the size of the original graph.

Lemma 2.6. *The following holds for any $n \geq k \geq 2$. If S is a set of n elements, then S contains at least $\frac{n^2}{k^4}$ subsets of size k, such that the intersection of any two such subsets contains at most one element.*

Proof: The claim holds trivially when $k \geq \sqrt{n}$, thus we may assume from now on that $k < \sqrt{n}$.

Given a subset $S_1 \subseteq S$ of size k we have that there are at most $\binom{k}{2} \cdot \binom{n}{k-2} - 1$ subsets of size k such that their intersection with S_1 contains at least two elements. As there are $\binom{n}{k}$ subsets of size k of S and each one of them which is chosen invalidates at most $\binom{k}{2} \cdot \binom{n}{k-2} - 1$ others we can find a collection of subsets of size at least

$$\begin{aligned}
\frac{\binom{n}{k}}{\binom{k}{2} \cdot \binom{n}{k-2}} &\geq \frac{2(n-k)^2}{k^4} \\
&\geq \frac{2(n-\sqrt{n})^2}{k^4} && \text{as } k < \sqrt{n} \\
&\geq \frac{n^2}{k^4} && \text{for } n \geq 12
\end{aligned}$$

such that the intersection of any two of them contains at most one element. Furthermore it is easy to verify that the claim holds for $2 \leq n \leq 11$ and thus we are done. ■

Proof of Theorem 2.1:

Let c be the number of vertices in graph H, i.e. $c = |H|$. Suppose that for some $0 < \epsilon < \frac{1}{2}$ there is a randomized polynomial time algorithm $A(G)$ which, when given an input graph G on n vertices has the following behavior

- if $\alpha(G) > n^{1-\epsilon}$ then $A(G) = 1$.

- if $\alpha_H(G) < n^\epsilon$ then $A(G) = 2$ with constant probability.

We will use algorithm $A(G)$ to build a randomized polynomial time algorithm $B(F)$, which given as an input a graph F on n vertices has the following behavior

- If $\alpha(F) > n^{1-\epsilon/8}$ then $B(F) = 1$.
- If $\alpha_{K_c}(F) < n^{\epsilon/8}$ then $B(F) = 2$ with constant probability.

Notice that by theorem 2.5 if such an algorithm B exist then $NP = ZPP$. Algorithm $B(F)$ consists of two steps.

1. construct a new graph G by retaining each edge of F with probability $\frac{1}{2}$.
2. return $A(G)$.

If $\alpha(F) > n^{1-\epsilon/8}$ then $\alpha(G) > n^{1-\epsilon}$ as any independent set in F is also an independent set in G. Suppose now that $\alpha_{K_c}(F) < n^{\epsilon/8}$. By lemma 2.6 any vertex induced subgraph of F of size n^ϵ contains at least $\frac{n^{2\epsilon}}{n^{\epsilon/2}} = n^{3\epsilon/2}$ vertex sets of size $n^{\epsilon/8}$ such that any two of them intersect in at most one vertex. As each vertex set of size $n^{\epsilon/8}$ contains a clique of size c, we conclude that each subgraph of size n^ϵ in F contains at least $n^{3\epsilon/2}$ edge disjoint cliques of size c. In graph G each such clique becomes an H-subgraph with probability $\gamma = 2^{-c^2}$ at the least. Thus the probability that a certain subgraph of size n^ϵ of G does not contain a H-subgraph is at most

$$(1-\gamma)^{n^{3\epsilon/2}}$$

As the number of vertex induced subgraphs of size n^ϵ in G is at most

$$\begin{aligned} \binom{n}{n^\epsilon} &\leq n^{n^\epsilon} \\ &\leq 2^{n^\epsilon \cdot \log n} \\ &\leq 2^{n^{5\epsilon/4}} && \text{for large enough } n \end{aligned}$$

We have that the probability that graph G contains an H-free vertex induced subgraph of size n^ϵ is at most

$$(1-\gamma)^{n^{3\epsilon/2}} \cdot 2^{n^{5\epsilon/4}} \leq \frac{1}{2} \qquad \text{for large enough } n$$

Thus if $\alpha_{K_c}(F) < n^\epsilon/8$ then $\alpha_H(G) < n^\epsilon$ with constant probability. This together with the fact that if $\alpha(F) > n^{1-\epsilon/8}$ then $\alpha(G) > n^{1-\epsilon}$ concludes our proof. ■

3 Related problems and generalizations

In certain situations we may wish to find a subgraph which does not only satisfy a property π but is also connected. Examples of properties that are hereditary and nontrivial on connected graphs include: clique, star, complete bipartite, path, tree, planar, outerplanar, bipartite, chordal, interval, degree bounded and others. The following theorem was proved in [LY93].

Theorem 3.1. *For every property that is nontrivial and hereditary on connected graphs, the maximum subgraph problem cannot be approximated with ratio* $2^{(\log n)^c}$ *for some* $c > 0$*, unless* $NP \subseteq QP$*.*

We will prove that for every property that is nontrivial and hereditary on connected graphs and for every $\epsilon > 0$, the maximum connected subgraph problem cannot be approximated with ratio $n^{1-\epsilon}$, unless $NP = ZPP$.

Proof of Theorem 1.2:
Let π be a nontrivial hereditary property on connected graphs. The following is shown in [LY93]

Theorem 3.2. *If* π *is a nontrivial hereditary property on connected graphs, then* π *is satisfied by all cliques or by all stars or by all induced paths.*

The proof of this claim is given in Appendix A for completeness. Let G be the instance of the graph in which we seek the maximum connected subgraph satisfying π. If π is satisfied by all cliques then the complementary property π^c is satisfied by all independent sets and π^c has some forbidden non-empty graph H, and thus the theorem follows directly from theorem 2.1. If π is satisfied by all stars then once again by adding a vertex which is connected to all the vertices of the graph G the theorem follows directly from theorem 2.1 by noticing that each independent set in the original graph corresponds to a star in the new graph.

If not all stars or all cliques satisfy π then it must be satisfied by all induced paths. It was shown in [LY93] that if π is a nontrivial hereditary property on connected graphs which is satisfied by all induced paths and does not hold for some star, then the maximum connected subgraph with property π problem cannot be approximated within a factor of $n^{1-\epsilon}$ for every $\epsilon > 0$, unless $P = NP$, and thus we are done. $\blacksquare$

In fact we can strengthen the theorem above for certain properties. We will prove that if π be a nontrivial hereditary property on connected graphs which is satisfied by all paths and does not hold for some star, then the maximum connected subgraph with property π problem cannot be approximated within a factor of $o(n/\log n)$, unless 3-SAT can be solved in time $2^{o(n)}$.

Proof of Theorem 1.3:
Let φ be an instance of 3-SAT with n variables. A 3-SAT formula in which the number of clauses is linear in the number of the variables will be called a *linear* 3-SAT formula. The sparsification lemma of [IPZ01] states that for any $\epsilon > 0$ we may convert φ into a collection of $2^{\epsilon n}$ linear 3-SAT formulas, where the original formula is satisfied if and only if at least one of the linear formulas is satisfied. Henceforth we shall assume that the number of clauses in φ is linear in the number of variables of φ. The instance φ consists of r clauses $C_0, C_1, ..C_{r-1}$. We construct a graph G from φ in the following manner. Graph G consists of vertices $v_{i,j}$, where $0 \leq i \leq t$ for some t to be determined later and $1 \leq j \leq 3$. Vertex $v_{i,j}$ corresponds to literal j of clause $i \pmod r$. Denote by T_i the triple $v_{i,1}, v_{i,2}, v_{i,3}$. For all i, j, k, l if $v_{i,j}$ and $v_{k,l}$ are such that the literals corresponding to them contradict each other then we put an edge between $v_{i,j}$ and $v_{k,l}$, and call such an edge a 'bad' edge. Next for all i, j we put edges between $v_{i,j}$ and $v_{k,l}$ for all k, l such that $k \equiv i \pmod r$ and $l \neq j$. Once again we call such edges 'bad' edges. Finally for all $i < t$ and all j we connect vertex $v_{i,j}$ to vertices $v_{i+1,l}$ for all l. If such an edge did not already exist we call it a 'good' edge.

Claim 3.3. *If φ is satisfiable then G contains an induced path of length t.*

Proof: As there is a satisfying assignment A for φ, we can pick in each triple T_i a vertex which corresponds to a literal in clause $i \pmod r$ which is satisfied by assignment A. Furthermore we always pick the same literal for triples i, k such that $i \equiv k \pmod r$. These vertices span an induced path of length t in G. ■

Claim 3.4. *If φ is unsatisfiable then each connected induced subgraph of G of size cr contains a star of size $c/36$*

Proof: Let H be a connected induced subgraph of G, with at least cr vertices. As φ contains r clauses, at least c of the vertices in H correspond to the same clause in φ, thus at least $c/3$ of them corresponds to the same literal in that clause. Suppose w.l.o.g that these vertices are $v_{i_1,1}, v_{i_2,1}, \ldots, v_{i_{c/3},1}$. Choose out of them $c/6$ vertices $v_{i_1,1}, v_{i_2,1}, \ldots, v_{i_{c/6},1}$ such that for all $1 \leq k < m \leq c/6$, $|i_k - i_m| \geq 2r$. For all k, let D_k be the shortest path in H consisting only of good edges from $v_{i_k,1}$ to a vertex which is adjacent to a bad edge in H. Denote the vertices of D_k by $\{v_{a_1,b_1}, v_{a_2,b_2}, \ldots, v_{a_l,b_l}\}$ where $v_{a_1,b_1} = v_{i_k,1}$. Notice that since D_k is a shortest path one of the two statements below must hold:

1. For all $1 \leq i \leq l,\ a_i \leq i_k$, denote such a path as a left path
2. For all $1 \leq i \leq l,\ a_i \geq i_k$, denote such a path as a right path

Suppose w.l.o.g that for all $1 \leq k \leq c/12$ each path D_k is a right path. Furthermore suppose w.l.o.g that D_1 is the shortest of these paths. Each such path is indeed an induced path as it is a shortest path and thus its length is bounded by r as if it were longer this path would induce a satisfying solution to φ. We conclude that each of these paths ends in a different vertex as for all $1 \leq k < m \leq c/12$ $|i_k - i_m| \geq 2r$. Suppose that for all k, path D_k ends in the vertex v_{q_k,w_k}. By our assumption that D_1 is the shortest path we have that $q_k \geq q_1$ for all $k \geq 1$, and thus every path D_k contains a vertex which corresponds to the same clause as vertex v_{q_1,w_1}. To conclude, we have $c/12$ vertices that correspond to the same clause and one of these vertices, namely vertex v_{q_1,w_1} has a bad edge connecting it to some vertex u. If at least third of these vertices correspond to the same literal as v_{q_1,w_1} then we have a star of size $c/36$ as each one has a bad edge to u. Otherwise we have that at least $c/36$ vertices correspond to a literal different than v_{q_1,w_1}, but then we have once again a star of size $c/36$ since v_{q_1,w_1} touches these vertices. ■

Theorem 1.3 follows from claims 3.3 and 3.4 by taking $t = 2^{\delta n}$ where δ is arbitrarily small. ■

It follows from theorem 1.3 that for certain hereditary properties it is harder to approximate the maximum connected subgraph problem then the maximum subgraph problem. For example by theorem 1.3 the maximum connected subgraph of degree smaller then k for every $k \geq 3$, cannot be approximated within a factor of $o(n/\log n)$ (under the assumption that there is no subexponential time algorithm for 3-SAT). On the other hand by Theorem 2.6 of [Hal00] the maximum subgraph of degree smaller then k for every $k \geq 3$, can be approximated within a factor of $O(n(\log\log n/\log n)^2)$ and thus it is easier to approximate than its connected counterpart.

One can consider hereditary properties in directed graphs as well as in undirected graphs. Examples of such properties are: acyclic, transitive, symmetric, antisymmetric , tournament, degree-constrained, line digraph. The following theorem was proved in [LY93].

Theorem 3.5. *For every nontrivial hereditary property on directed graphs, the maximum subgraph problem cannot be approximated with ratio $2^{(\log n)^c}$ for some $c > 0$, unless $NP \subseteq QP$.*

Using the techniques developed in [LY93] and section 2 of this paper, one can prove in a straightforward manner the following.

Theorem 3.6. *For every nontrivial hereditary property on directed graphs, the maximum subgraph problem cannot be approximated with ratio $n^{1-\epsilon}$ for any $\epsilon > 0$, unless $NP = ZPP$.*

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A Proof of theorem 3.2

We will need the following claim.

Theorem A.1. *For any natural number t, there exist a natural number $n = R(t)$ such that in any connected graph on n or more vertices, there exists a clique of t vertices or a star of t vertices or an induced path of t vertices.*

Proof: Let G be a connected graph. By Ramsey's theorem ([ES35]) there exist a natural number n_0 such that any graph with at least n_0 vertices contains a clique of size t or an independent set of size t. If a graph G contains a vertex of degree n_0, then it has either a clique of size t or a star of size t. If G has maximum degree $\leq n_0$, then taking $n > (n_0)^{t+1}$, graph G will have an induced path of length t. ■

We conclude that if π is a nontrivial hereditary property on connected graphs then π is satisfied by all cliques or by all stars or by all induced paths, for otherwise by theorem A.1 it will be satisfied only by finitely many graphs.

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