SPRING SCHOOL
ON ORBITS, PRIMITIVE IDEALS AND QUANTUM GROUPS

FEBRUARY 25 – MARCH 1

Abstracts

Mini-courses

Stephane Launois

Title: Primitive ideals in quantum matrices, and beyond.

Abstract:

1) Primitive ideals in quantum affine spaces;
2) Cauchon's deleting-derivations algorithm;
3) Primitive ideals in quantum matrices: dimension of the strata;
4) Link with combinatorics.

Markus Reineke

Title: Quiver Grassmannians.

Abstract: Quiver Grassmannians are projective varieties parametrizing compatible configurations of subspaces of vector spaces. We will review examples appearing in Algebraic Lie Theory, derive their basic geometric properties and explain the role of the representation theory of quivers in the study of their geometry. We will discuss universality, single out certain classes of "well-behaved" quiver Grassmannians, and construct desingularizations.

Hans-Juergen Schneider

Title: Hopf algebras and root systems.

Abstract: I will give an introduction to some recent results on the classification of Hopf algebras and the relationship between (a certain class of) Hopf algebras and generalized root systems and Weyl groupoids. The basic notion is the Nichols (or quantum symmetric) algebra of a braided vector space and of a Yetter-Drinfeld module over some Hopf algebra. Important special cases of Nichols algebras are the plus parts of deformed universal enveloping algebras. The Weyl groupoid for diagonal braidings was introduced by I. Heckenberger. The talks are based on joint papers with N. Andruskiewitsch, N. Andruskiewitsch and I. Heckenberger, and with I. Heckenberger.

Research Talks

Evgeny Feigin

Title: Degenerate $\mathfrak{s}l(n)$: representations and flag varieties.

Abstract: We will define the PBW degeneration of a simple Lie algebra and discuss a notion of a highest weight representation in the degenerate settings. In type A we will introduce a class of natural highest weight representations and describe a connection with the geometry of the degenerate flag varieties and their desingularizations. Several conjectures on the structure of the natural highest weight representations of the degenerate algebras will be stated.

Michael Finkelberg

Title: Quantization of symplectic zastava and Yangians.

Abstract: For a Lie algebra g, zastava space Z(g) is a certain closure of the space of based maps from the projective line to the flag variety of g. It carries a natural Poisson structure. In case g is of type A or C, Z(g) can be constructed as a quiver variety. This provides a natural quantization of the Poisson structure which is closely related to the corresponding Yangians (joint work with Leonid Rybnikov).

Maria Gorelik

Title: On the characters of vacuum modules over affine Lie superalgebras.

Abstract: Vacuum modules over affine Lie (super)algebra are irreducible highest weight modules with the highest weight proportional to the 0th fundamental weight. I will discuss a character formula for vacuum modules over affine Lie superalgebras. The talk is based on a joint work with V. Kac.

Istvan Heckenberger

Title: Nichols algebras over groups with finite root system of rank two.

Abstract: This is a joint work with Leandro Vendramin. We classify all non-abelian groups G such that there exists a pair (V, W) of absolutely simple Yetter-Drinfeld modules over G such that the Nichols algebra of the direct sum of V and W is finite-dimensional

under two assumptions: the square of the braiding between V and W is not the identity, and G is generated by the support of V and W. As a corollary, we prove that the dimensions of such V and W are at most six. As a tool we use the Weyl groupoid of (V, W).

Crystal Hoyt

Title: Finite weight modules for the Lie superalgebra $D(2, 1, a)$.

Abstract: Let $\mathfrak{g}$ be a basic Lie superalgebra. A weight module over $\mathfrak{g}$ is called finite if all of its weight spaces are finite dimensional, and it is called bounded if there is a uniform bound on the dimension of a weight space. The minimum bound is called the degree of this $\mathbb{Z}/2\mathbb{Z}$-graded module. We prove for $\mathfrak{g} = D(2, 1, a)$ that Verma modules and simple finite weight modules are always bounded. We prove that a simple cuspidal module M has degree bounded by $(8, 8)$, and that this bound is attained if and only if M belongs to a $(\mathfrak{g}, \mathfrak{g}_0)$-coherent family for some typical weight (as defined by Mathieu and Grantcharov)

Ivan Penkov

Title: On ideals in $\mathrm{U}(\mathfrak{g}_\infty)$ for $\mathfrak{g}_\infty = \mathfrak{sl}_\infty, \mathfrak{so}_\infty, \mathfrak{sp}_\infty$.

Abstract: With the development of the representation theory of infinite-dimensional finitary Lie algebras $\mathfrak{g}_\infty$, the understanding of ideals, in particular primitive ideals, of $\mathrm{U}(\mathfrak{g}_\infty)$ becomes increasingly relevant. In this talk I will review known results on ideals of $\mathrm{U}(\mathfrak{g}_\infty)$, due mainly to A. Zhilinskii, A. Petukhov and the speaker, and will discuss some open problems. Two such problems are whether $\mathrm{U}(\mathfrak{g}_\infty)$ is a two-sided Nötherian and the lack of an analogue of Duflo's Theorem. Joint work with A. Petukhov.

Charles Torossian

Title: Kashiwara-Vergne problem and associators

Abstract: The Kashiwara-Vergne (KV) conjecture is a property of the Campbell-Hausdorff series put forward in 1978. It has been settled in the positive by E. Meinrenken and A. Alekseev 2006. We will explain how this problem is related with the Grothendieck-Teichmueller group GRT. Actually we introduce a family of infinite dimensional groups KV_n, and an extension $\hat{KV}_2$ of the group KV_2. We show that the group $\hat{KV}_2$ contains GRT and acts freely and transitively on the set of solutions of the KV problem $Sol(KV)$ which contains the set of solutions of the pentagon equation with values in the group KV_3. In particular any Drinfeld's associators give rise to a solution of the KV problem.