Infinite ergodic theory and related fields

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Bergelson: *A selection of open problems in finite and infinite ergodic theory*

Abstract: We will present and discuss some old and new results and natural open problems which focus on (multiple) recurrence and convergence in finite and infinite measure preserving systems.

Björklund: *Random hyperbolic geometry and quasimorphisms*

Abstract: Let G be a countable group. A real-valued function f on G is called a quasimorphism if the derivative: $df(g,h) = f(gh) - f(g) - f(h)$ is uniformly bounded on $G \times G$. The study of the asymptotic properties of quasimorphisms (on hyperbolic groups) was initiated by Sarnak, and Calegari-Fujiwara, a few years ago. The two latter authors were able to prove a (possible degenerate) central limit theorem for the values of a Hölder continuous quasimorphism along a simple random walk on a word- hyperbolic group. Similar results were also obtained by Horsham-Sharp. In this talk I will discuss a recent preprint with T. Hartnick in which we prove a general (non-degenerate) central limit theorem for the values of a quasimorphism (not necessarily Hölder continuous) along random walks (under weak integrability conditions) on a general countable group (not necessarily hyperbolic). If time permits I will mention some applications to random hyperbolic geometry (areas of random hyperbolic polygons). Joint work with T. Hartnick (ETH).

Conze: *Ergodic cocycles, conformal measures and points with non uniform distribution in some infinite billiards*

Abstract: The dynamic of the directional billiard with periodically distributed rectangular obstacles in the plane reduces to that of a skew product defined by a cocycle with values in $\mathbb{Z}^2$ over some interval exchange transformations.

This is a motivation to investigate the construction of recurrent ergodic cocycles over interval exchange transformations. In the first part of the talk, we will present a class of such cocycles over IET of periodic type (joint work with Krzysztof Frączek).

For the models provided by the billiard, there are special directions where the directional flows in the periodic rectangular billiard can be reduced to a cocycle over an irrational rotation. In a second part, we will present ergodicity results for this particular model (joint work with Eugene Gutkin).

Finally, we will apply to this model the construction of conformal measures for a rotation, and show the existence of points with non uniformly distributed orbits in the plane.

Cyr: *Conformal Measures for Transient Markov Shifts*

Abstract: In this talk we will explore the thermodynamic formalism of countable Markov shifts exhibiting transience. The main goal will be to construct a con formal measure for locally compact transient shifts using some insight from the theory of Martin boundaries for Markov chains; examples will be given where it is non-unique (corresponding to a "phase transition" for the system). If time permits, we will discuss some recent results concerning the general (non locally compact) case.

Denker: *Almost sure convergence for infinite measure preserving transformations*

Abstract: This talk will present some results on the almost sure behavior of the limsup for partial sums $\sum_{k<n} f\circ\theta^k$ of an infinite measure preserving transformation θ. I will discuss connections to probability, log averaging and large deviation theory. This is a talk based on joint work with J. Aaronson several years ago.

Einsiedler: *Measure rigidity and Diophantine approximation in the Cantor set*

Abstract: We will discuss an application of the partial classification of geodesic invariant measures by E. Lindenstrauss to Diophantine approximation within the standard middle third Cantor set. Using this connection we show that a.e. point in the Cantor set is well approximable. This is joint work with L. Fishman and U. Shapira.

Fisher: *Log average ergodic theorems and self-similar return times for some infinite measure transformations*

Abstract: For a finitely generated Fuchsian group, when the Riemann surface defined by the group has infinite area, the geodesic flow still preserves a natural ergodic probability measure (the Patterson-Sullivan measure) but the corresponding measure for the horocycle flow has become infinite. Something similar happens for a quite different class of transformations, related to renewal processes. In joint work with Marina Talet, we prove that paths of the renewal process are attracted to those of the Mittag-Leffler process along its scaling flow (the "geodesic flow") and then derive a log average ergodic theorem for the corresponding increment flow (the "horocycle flow").

Furstenberg: *A Multiple Recurrence Theorem for Actions of* $\mathrm{SL}_2(\mathbb{R})$

Abstract: This lecture is based on joint work with Eli Glasner. The underlying idea is that even for actions of non-amenable group on a compact space, in which case there may not be an invariant measure, there will be so-called "stationary" measures. We can study recurrence properties in this context. As an application we formulate a Szemeredi theorem regarding geometric progressions in large subsets of $\mathrm{SL}_2(\mathbb{R})$.

Glasner: *Topological groups with Rohlin properties*

Abstract: In a classical paper P. R. Halmos shows that weak mixing is generic in the measure preserving transformations. Later, in his book, he gave a more streamlined proof of this fact based on a fundamental lemma due to V. A. Rohlin. For this reason the name of Rohlin has been attached to a variety of results, old and new, relating to the density of conjugacy classes in topological groups. I will review some of the new developments in this area and its connections to model theory. (A joint work with Benjy Weiss.)

Gorodnik: *Mixing for automorphisms of nilmanifolds*

Abstract: Automorphisms of nilmanifolds present an interesting and rich family of examples of partially hyperbolic dynamical systems. In this talk we will discuss mixing properties of such automorphisms. This is a joint work with R. Spatzier.

Gouëzel: *Almost sure invariance principle by spectral methods*

Abstract: The (vector-valued) almost sure invariance principle is a strong reinforcement of the central limit theorem. I will explain how to prove it for Birkhoff sums of dynamical systems, solely under a spectral assumption on perturbed transfer operators. Contrary to many classical situations, this assumption is not formulated in terms of the behavior of a perturbed eigenvalue (I do not even require the existence of such an eigenvalue).

Guivarc'h: *Strong ergodicity and spectral gap properties for affine group actions on tori and nilmanifolds*

Abstract: Let X be a compact nilmanifold, $A(X)$ the group of affine maps of X into itself, Γ a closed subgroup of $A(X)$. Let p be a finitely supported probability measure on $A(X)$ such that $\mathrm{supp}\,(p)$ generates Γ. We are interested in the spectral properties of the convolution operator on $\mathbb{L}^2(X)$ defined by p, and by the ergodicity properties of associated skew products. In particular, if $\Gamma \subset \mathrm{Aut}(X)$, we give necessary and sufficient conditions for spectral gaps, in terms of the Γ-action on the maximum torus factor of X. We show also ergodicity of a class of corresponding skew products with infinite invariant measure.

Hochman: *Zero temperature limits of Gibbs states*

Abstract: Let f be a Holder potential on the full one-sided shift $\{0,1\}^{\mathbb{N}}$, and let μ_β denote the Gibbs measure for f at inverse temperature β (existence and uniqueness are classical, as is the smooth dependence on β). I will discuss the question of existence of the limit of these measures as β tends to infinity, i.e. as the temperature tends to zero. Although over infinite state spaces it was known, due to work of Van Enter and Ruszel, that the limit may not exist, it was believed that over finite state spaces the measures should converge. In particular, for finite alphabets Bremont proved that convergence does take place when f takes only finitely many values. I will present joint work with Jean-Rene Chazottes in which we construct a counterexample and discuss some of its features. If time permits I will also discuss results in the multidimensional case.

Keane: *An example of a strong type of nonsingular orbit equivalence*

Abstract: For two specific type III_λ transformations, we present a measure preserving invertible mapping from one to the other which preserves orbits. The spaces are symbolic and the mapping and its inverse are both finitary. This is joint work with Karma Dajani.

Kifer: *Advances in nonconventional limit theorems*

Abstract: The polynomial ergodic theorem (PET) which attracted substantial attention in ergodic theory studies the limits of expressions having the form

$$1/N \sum_{n=1}^{N} T^{q_1(n)} f_1 \cdots T^{q_\ell(n)} f_\ell$$

where T is a weakly mixing measure preserving transformation, f_i's are bounded measurable functions and q_i's are polynomials taking on integer values on the integers. Motivated partially by this result and generalizing my previous result we obtain together with Varadhan a functional central limit theorem for even more general expressions in both discrete time

$$\frac{1}{\sqrt{N}}\sum_{n=1}^{[Nt]}\big(F\big(X_1(n),X_2(2n),...,X_m(mn),\\ X_{m+1}(q_{m+1}(n)),X_{m+2}(q_{m+2}(n)),\cdots,X_\ell(q_\ell(n))\big)-\bar{F}\big)$$

and in the continuous time for the corresponding expressions of the form

$$\frac{1}{\sqrt{T}}\int_0^{Tt}\big(F\big(X_1(s),X_2(2s),...,X_m(ms)X_{m+1}(q_{m+1}(s)),\\ X_{m+2}(q_{m+2}(s)),\cdots,X_\ell(q_\ell(s))\big)-\bar{F}\big)ds$$

where X_i's are sufficiently fast mixing processes with some stationarity properties, F is a Hölder continuous function, $\bar{F}=\int Fd(\mu_1\times\cdots\times\mu_\ell)$, μ_j is the distribution of $X_j(0)$ and $q_i, i>m$ are positive functions taking on integer values on integers with some growth conditions which are satisfied, for instance, when q_i's are polynomials of growing degrees. This result can be applied in the case when $X_i(n)=T^nf_i$ where T is a mixing subshift of finite type, a hyperbolic diffeomorphism or an expanding transformation taken with a Gibbs invariant measure, as well, as in the case when $X_i(n)=f_i(\xi_n)$ where ξ_n is a Markov chain satisfying the Doeblin condition considered as a stationary process with respect to its invariant measure. In the continuous time case the result can be applied to $X_i(t)=f_i(\xi_t)$ where ξ_t is a nondegenerate diffusion on a compact manifold.

Lemańczyk: *On the minimal self-joining property for flows over irrational rotations*

Abstract: We will study the minimal self-joining property of so called von Neumann's special flows T^f over irrational rotations. That is, we will show some minimality property of the set of invariant measures with "right" marginals for the product system $T^f\times T^f$. The key arguments consist in showing that such measures are in one-to-one correspondence with some locally finite measures of some $\mathbb{Z}^2$-cylindrical transformations and make use of some recent results in non-singular ergodic theory. This is a joint work with K. Frączek.

Lindenstrauss: *On quantum unique ergodicity and recurrence*

Abstract: The quantum unique ergodicity conjecture of Rudnick and Sarnak states that if ϕ_i is a sequence of L^2-normalized eigenfunctions of the Laplacian on a compact negatively curved manifold M $|\phi_i|^2dvol$ tends weakly to the uniform measure on M.

This has been established for arithmetic hyperbolic surfaces under the assumption that the ϕ_i respect certain symmetries of these surfaces. I will explain how this question is related to ergodic theory, infinite ergodic theory, and to eigenfunctions of the Laplacian on graphs. Based in part on joint work with S. Brooks.

Meyerovitch: *Ergodicity of Poisson products and* FROL *transformations*

*Abstract:*Let $T : \mathbb{R}_+ \to \mathbb{R}_+$ be a conservative Lebesgue-measure preserving transformation. The Poisson suspension T_* of T acts naturally on discrete countable subsets of $\mathbb{R}$. A *First Return of Leftmost (FROL)* transformation is defined by applying the Poisson suspension of T repeatedly, until the image of the left most point is the left most point of the image. We will show how a proof of this transformation's ergodicity follows from ergodicity of the Poisson product $T \times T_*$, and prove the latter statement.

Nakada: *On continued fraction mixing*

Abstract: We survey some results arising from continued fraction mixing property such as trimmed sums, the existence of Darling-Kac sets etc. A number of these results have been done by Jon Aaronson and his collaborators. We also compare this mixing property with some other mixing properties.

Nogueira: *Approximation to points on* $\mathbb{R}^2$ *by orbits of the linear action of* $\mathrm{SL}(2,\mathbb{Z})$

Abstract: The orbit $\mathrm{S}L(2,\mathbb{Z})\mathbf{x}$ is dense in $\mathbb{R}^2$ if the initial point $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2$ has irrational slope. We look at the latter property from a diophantine perspective. For any target point $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2$, we introduce two exponents $\mu(\mathbf{x},\mathbf{y})$ and $\hat{\mu}(\mathbf{x},\mathbf{y})$ that measure the approximation to $\mathbf{y}$ by elements $\gamma\mathbf{x}$ of the orbit in terms of the size of the matrix $\gamma \in SL(2,\mathbb{Z})$. We estimate both exponents under various conditions. Our results are optimal if the target point $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ equals the origin, or its slope is a rational number or $y_1 = 0$. In those cases the exponents $\mu(\mathbf{x},\mathbf{y})$ and $\hat{\mu}(\mathbf{x},\mathbf{y})$ are given in terms of the irrationality measure of the slope of $\mathbf{x}$.
(Joint work with Michel Laurent (arXiv:1004.1326v1 [math.NT]).)

Park: *Examples and properties of entropy dimension*

Abstract: Entropy dimension is introduced to classify the complexity of entropy zero systems. We present the topological and measurable examples of fractional entropy dimension. We consider the notions of entropy generating sequence and dimension set. We also discuss some of the open problems.

Picco: *Recent results on the one-dimensional long range Ising model*

Abstract: In this talk I will review recent results concerning the one dimensional long range Ising model. The hamiltonian with empty boundary conditions is

$$H(\sigma_\Lambda)[\omega] = \frac{1}{2} \sum_{(i,j)\in\Lambda\times\Lambda} J(|i-j|)(1-\sigma_i\sigma_j) - \theta \sum_{i\in\Lambda} h_i[\omega]\sigma_i.$$

where

$$J(n) = \begin{cases} J(1) \gg 1; \\ \frac{1}{n^{2-\alpha}} \quad \text{if} \quad n > 1, \quad \text{with } \alpha \in [0,\alpha_+]. \end{cases}$$

with $\alpha_+ = \frac{\log 3}{\log 2} - 1$ and $h_i(\omega)$ is a realization of a familly of iid symmetric random variables (Bernoulli, Gaussian, subgaussian)

Adding a boundary conditions η

$$W(\sigma_\Lambda, \eta_{\Lambda^c}) = \sum_{i\in\Lambda}\sum_{j\in\Lambda^c} J(|i-j|)(1-\sigma_i\eta_j)$$

one defines Gibbs measures as usual.

In collaboration with Cassandro and Orlandi we have proved that there are at least to Gibbs states (the so-called + state and the − state) when $1/2 < \alpha \leq \alpha_+$. (CMP 288,731-744 (2009).

In a recent work with the same collaborators we give upper bound and lower bound on the length of runs of + or − in the case $0 \leq \alpha \leq 1/2$.

We review these results introducing basic facts about rigorous statistical mechanics. Some open problems will be mentioned.

Pollicott: *Escape rates for Gibbs measures*

Abstract: We extend results of Bunimovich-Yurchenko and Keller-Liverani to give asymptotic estimates on the rate at which a Gibbs measure for a subshift of finite type escapes through a small cylinder set. This involves modifying a result of Hirata on Poisson return times and has a variety of applications via symbolic dynamics. This is joint work with Andrew Ferguson.

Przytycki: *Nice inducing schemes and thermodynamical formalism for rational maps*

Abstract: For a large class of rational maps f I will prove the existence of equilibria for the potentials $-t\log|f'|$ and the real analytic dependence of the pressure $P(-t\log|f'|)$ on t. The tool is a return map which gives an Infinite Iterated Function System. This yields the analyticity of the Hausdorff dimension spectrum for Lyapunov exponents. Other results on this spectrum for all rational maps will be also discussed. These are joint results with Juan Rivera-Letelier, Katrin Gelfert and Michal Rams.

Schapira: *Behaviour of half-horocycles on hyperbolic surfaces*

Abstract: We present an example of dynamical system : the horocyclic flow on infinite volume hyperbolic surface, where the dynamical behaviours in positive and negative time of the orbits are very different. One half of the orbit is dense and equidistributed whereas the other half of the orbit is not even recurrent.

Szász: *(Super)diffusive asymptotics for perturbed Lorentz or Lorentz–like processes*

Abstract: After the first success in establishing the diffusive, Brownian limit of planar, finite-horizon, periodic Lorentz processes, in 1981 Sinai turned the interest toward studying models when

Here the invariant measure is, of course, infinite. The 1981 solution for a stochastic random-walk-model only led in 2009 to that for the locally perturbed, finite-horizon Lorentz process (by Dolgopyat, Varjú and the present author). Beside reporting on these results we also analyze the first steps in extending the superdiffusive limit obtained for the infinite horizon Lorentz process to locally perturbed ones (results by Nándori, Paulin, Varjú and the speaker).

Thouvenot: *A fact concerning rank one transformations which preserve an infinite measure*

Abstract: to a question due to E. Roy, as to whether King's weak closure theorem was valid for rank one transformations which preserve an infinite measure, we shall give a (very) partial answer: in the "mixing " case, the centralizer of such a transformation is just its powers. This is a joint work with Valery Ryzhikov.

Volný: *Quenched limit theorems*

Abstract: Let (X_i) be Markov Chain with a state space S and stationary measure ν, f is a measurable function on S. The process $(f(X_i))$ is stationary and reciprocally, each (strictly) stationary process can be represented in this way. A limit theorem for the process $(f(X_i))$ is called "quenched" if for ν almost all x from S, the limit theorem remains true for δ_x as the starting measure (δ_x meaning the Dirac measure at x). The problem was posed by Kipnis and Varadhan in 1986 and it is still not known whether their CLT is quenched. Here, several "classical" central limit theorems for stationary processes will be studied, whether they are quenched or not.

Weiss: *On the ergodic theory of infinite measure spaces*

Abstract: The title is that of the thesis submitted by Jon Aaronson to the Hebrew University of Jeruslaem in 1977. About twenty years later the AMS published a monograph by Jon bearing a similar title. I will try to describe the background of the thesis beginning with the St. Petersburg paradox, and the remarkable developments that have taken place since, due in no small measure to the efforts of Jon.

Zweimüller: *Return- and hitting-time distributions of small sets*

Abstract: For probability preserving dynamical systems with some mixing properties, there is a wealth of results showing that return- and hitting-time distributions of natural small sets (small cylinders or centered balls around typical points) asymptotically have an exponential distribution. I will discuss recent joint work with F. Pene and B. Saussol (Brest, France) which shows that for a variety of infinite measure preserving systems the corresponding limit law is that of a stable subordinator at an independent exponential time. This can be understood as a combination of the finite-measure results mentioned above, and the Darling-Kac theorem for infinite measure systems. In fact, a variant of Jon Aaronson's notion of pointwise dual ergodicity used in his proof of the Darling-Kac theorem plays a prominent role in some of our results.