MATHEMATICAL INSTITUTIONAL ECONOMICS

Presentation at Workshop

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- There are two highly different skills in the development of a robust mathematical economics. They are;
  1. The building of the model by abstracting in an appropriate manner from the outside reality, and conjecturing the properties of interest.
  2. Exploiting the model structure developing and proving the theorems about the relevant properties.
- This talk is devoted primarily to the first.

Statics or Dynamics?

- An adequate understanding of broad economic dynamics requires the presence of money and financial institutions. Due to the plethora of institutions in any advanced economy a synthetic approach is required. There is no magic set of equations of motion that fits all economies.

- A useful overall synthesis of the many aspects of underlying macro-micro-economic structure can be organized utilizing **strategic market games.**
- **A strategic market game provides a well-defined game theoretic model of a closed dynamic economy that is** loosely coupled**.**
- **It is a natural generalization of Static, non-strategic General Equilibrium models.**

- Presented here is the sketch of a search for an adequate basis for understanding economic dynamics and the means for control of an economy.
- In a book Strategy and Market Structure (SMS), I utilized the noncooperative equilibrium approach to oligopolistic market structures.
- The original intent had been, first to study equilibrium with little concern for time, assets and explicit dynamics, and then try to develop an economic dynamics. I found that the former was relatively easy to do.

- The latter was far more difficult to achieve than I had expected. The inordinate proliferation of plausible models led me to suggest that the study of the dynamics of economic systems be called Mathematical Institutional Economics.
- This title stressed the role of institutions as carriers of process. They appear as parts of rules of the game.

- Obtaining a model is not enough. One has to have a solution concept and be able to derive the solution with the analysis. This is where the mathematics enters.
- I formulated a class of games called **Games of Economic Survival.**
- The concept behind a GES is that is that the corporation and its owners, each may have different payoffs and may have different levels of strategic control over the actions of the firm.

- With this dichotomy one can model situations in which a limited liability controlling stockholder can use the firm to take risks involving great gains or bankruptcy that he would not risk himself.
- The work noted above I had utilized primarily the NCE solution. I believe that the theory of games will never offer a single general solution concept that covers all strategic situations in which a game theoretic analysis of the problem appears to be of value.

- There is still much to be done with cooperative theory , especially with the **CORE** and **VALUE** solutions.
- In working with Shapley we were curious to study the links between the **COMPETITIVE EQUILIBRIUM OUTCOMES** in an **exchange economy** and the cooperative game solutions to an exchange economy in **coalitional form.**

- We had shown that the CORE, VALUE and NUCLEOLUS all converged to the **CE** under replication.
- In late 1970 I came to RAND for an extended time to work with Shapley.
- Independently from my work with Lloyd I had mentioned to Oscar Morgenstern that the one problem that fascinated me the most was to try for **a mathematical basis for a theory of money.**

- During 1960-1970 I butchered dozens of models without any success. When I arrived at RAND and reviewed my failure I decided to abandon my #1 problem and to concentrate on what was my #2 problem.
- In the work with Lloyd we had concentrated on the link between **EXCHANGE ECONOMIES** and MARKET GAMES which were in coalitional form and we had not studied **GAMES IN STRATEGIC FORM.**

- The Cournot model already illustrated both the noncooperative equilibrium and its convergence, but it was in the setting of a partial equilibrium or open economy.
- If in a closed economy convergence of the type symmetric noncooperative eqilibria (TSNE) could be established a sensitivity analysis of the CE in terms of both coalitional form and strategic form game theory would be available.

- In looking for the model to illustrate convergence of the NCE I decided to use a model where initially there were n individuals and n commodities. All individuals had a symmetric utility function in all the *n* commodities Thus one should be able to build an intrinsically symmetric game.
- With a complete market structure the number of markets would be $n(n-1)/2$ if every pair of goods could be exchanged with each other directly.

- I was concerned with whether I could construct a symmetric game with a minimal amount of simple markets.
- A price would be a ratio q_i/q_j. If I wanted to select a minimal network of markets that could permit efficient trade at least one market for each good was required. This called for n-1 markets.

- However I observed that I needed *n* independent strategy sets, but there were only $n-1$ independent prices.
- If one insisted that all players were to denominate the quantity of good offered for sale in terms of a specific commodity, then the player whose commodity is selected plays a nonsymmetric role.

- I overcame this difficulty by considering a somewhat different game with *n* monopolists trading in *n* + 1 commodities where each monopolist held one unit of his special good and a large enough supply of *n* + 1st good.
- I set the price of the *n* +1st good equal to one and had the strategies of each player *i* be an offer q_i for the sale of his commodity combined with bids (b_{i1}, b_{i2}, …,b_{in}) to purchase all commodities.
- I then tried this out on a specific simple model and considered the replication … it worked

- I took my preliminary model to Shapley who constructively tore it to bits in many different ways. After each tearing up it was possible to rebuild and strengthen it.
- Lloyd pointed out that my formulation was vulnerable to a division by zero, I noted that the specialist's role on the New York exchange required that he make an “orderly market,” i.e., that he has a small inventory available for sale.

- This can be treated mathematically by defining an “epsilon-related” game to the game under consideration where some small amount “epsilon” is available in each market. This cuts out the singularity.
- I was so intent on finishing the convergence package with Lloyd that for several weeks I thought only of the noncooperative game convergence.

- After some time it dawned on me that in fact this model provided the entry into the development of a general theory of money.
- The *n* +1st commodity that had been introduced could be regarded as a commodity money, and the condition that individuals were required to bid using this commodity imposed on each individual optimization a set of **CASH FLOW CONSTRAINTS**.
- If each individual did not have enough of the *n* + 1st commodity they could not necessarily attain the optimum outcome that could be obtained in the CE utilizing only **WEALTH CONSTRAINTS**.

- Mathematically these extra conditions stated that there was enough money if none of the cash flow constraints had a positive shadow price. In other words no constraint was binding.
- **My second most important problem turned out to solve my most important problem.**

- The model provided a key into creating a host of strategic process models consistent with the structure of the general equilibrium economy.
- Heeding the warning of Kenneth Arrow that one can easily be swamped by a plethora of models in attempting to deal generally with economic dynamics I considered how to cut down on the number of potential models in a reasonable manner. I devised the concept of **MINIMAL INSTITUTION.**

- It is a mechanism that is just able to perform a function or set of functions for which it has been designed. The removal of any part of the mechanism disables it from performing its function.
- A simple example of a minimal institution is provided by a price formation mechanism.

- Suppose, for any set *N* of agents trading in a set *M* of commodities plus a commodity money an agent *I* has an initial endowment of $(q_1^i, q_2^i, \ldots, q_m^i, q_{m+1}^i)$ where $q_{m+1}^i \geq 0$ is agent *i*'s holding of commodity money. All individuals are required to offer all of their commodities for sale (a tax collector's dream). A strategy of an individual *i* is a vector of *m* dimensions $(b_1^i, b_2^i, \ldots, b_m^i)$ where b_j^i is the bid in money by individual *i* for good *j*. The sum of all bids $\sum_{j=1}^{m} b_j^i \leq q_{m+1}^i$, the amount of money she has on hand. The price of good *j* is determined by the amount of money bid for the goods offered.

$$p_j = \frac{\sum_{i=1}^{n} b_j^i}{\sum_{i=1}^{n} q_j^i}$$

- This is a cash only economy with no credit indicated. The price formation explicitly spells out many of the assumptions that are implicit "cash-in-advance" models that there is a finite period under consideration where the market meets only once and the earnings from sales are not available for use until next period.

Necessary Functions and Sufficient Institutions

- The underlying theme in the portrayal of economic process models calls for the specification of **functional need** such as the need to be able to borrow, with the construction of the simplest institutional form, such as a **money market** that provides for the function.
- Elementary observations indicate the functions; economy and parsimony justify and limit the mechanisms and institutional observations match them with the economic reality for which they provide an abstraction.

- Even casual empiricism indicates that that there are costs associated with the running of institutions.
- Instead of using $m(m + 1)/2$ markets, if one commodity is designated as a money and it is in sufficient supply so that all "can pay cash" one needs only m markets. But often even the rich do not have enough cash on hand to meet their immediate payments.
- The economy needs to make sure that there is enough money and that it can be lent and borrowed, hence some form of **CREDIT MARKET** is called for.
- In general if the economy is to be flexible it must take into account error and uncertainty, but this may lead to states where an individual cannot repay the debt owed. This calls for the invention of **bankruptcy and reorganization** laws.

Dynamics and a Loosely Coupled System

- The mathematics of general equilibrium is without equations of motion.
- It provides existence proofs of equilibrium overwhelmingly in terms of equations and interior solutions.
- As Koopmans phrased it, it is pre-institutional. In contrast strategic market games, are process models and cannot avoid institutions.
- The institutions are the carriers of process they arise naturally in the definition of the rules of the game.

- Although one might wish to study equilibrium market prices, the manner of price formation must be made sufficiently explicit that the model could be utilized as a **PLAYABLE EXPERIMENTAL GAME.** In order to meet this criterion the mathematics must be able to supply how every position in the state space including all boundaries can be obtained regardless of the existence of an equilibrium.

Fiat Money or Gold?

- With the growth of the state and its role in enforcing commercial, contract, taxation and other laws gold was replaced by fiat money. The key feature in contrasting fiat money and gold is to understand the locus of the supply, the flexibility of removing money from the economy and the key to its maintenance of value. Smith and Shubik (2011) provide a detailed analysis.

- Even without invoking dynamics the recouping of the expenses of the governmental infrastructure required to support the issue of fiat are sufficient to justify an equilibrium with a fiat money with positive worth.

Debt, Bankruptcy and Reorganization Accommodate Disequilibrium

- A viable dynamic structure must be able to absorb divergence from the position of equilibrium. In a dynamic market economy with money and debt and independent decision-makers some combination of the functions of bankruptcy, reorganization or renegotiation is a logical necessity to resolve otherwise inconsistent outcomes.
- The existence of credit and bankruptcy and reorganization are the key factors in being able to construct an economy as a loosely coupled system.

The Change in Paradigm

- The combination of considerations of economy, parsimony, efficiency and control all conspire to create financial control processes to facilitate production and exchange.
- The legal, enforcement and government monetary mechanisms appear as a public good of sufficient size and power that the government must be added as an agent of considerable size.
- Technically the economy together with its control processes need to be modeled as a game with **one atomic player** and a **measure of small players.**

Summary Notes

- The paper touches on 12 further extensions of the application of SMG. They are:
- The Paper Trail
- Real and other Legal Persons
- Time and Uncertainty***
- Overlapping Generations and Dynasties
- Fiduciary Behavior
- Utility Bankruptcy and Default

- Money Credit and Conservation
- Where is Schumpeter? Innovation***
- What is a Theory?
- Edgeworth's warning***
- Why unify Macro and Micro-Economics?
- From Statics to Dynamics

- Only two are briefly noted: The innvolve Wealth distribution under uncertainty, and
- The role of innovation and the breaking of the circular flow of money.

Time and Uncertainty

- The initial development of the general equilibrium was based on an ingenious abstraction of both time and uncertainty from the economic structure. (Debreu 1959). Technically this can be done for finite partitions of time and uncertainty by multiplying the dimensions of the state space. For any finite period of time the overall economy may be presented as a game in strategic form.

- Since the late 1970s a body of literature has sprung up in macroeconomics utilizing dynamic programming methods for producing highly aggregated closed stochastic models of the economy. This has been led primarily by Robert Lucas.
- More or less at the same time Karatzas, Shubik, Sudderth and others considered mathematically similar models that, however differed in several basic ways.

- The former were aimed simultaneously at macroeconomic policy and with the aim to put a sound mathematical microeconomic foundation under macroeconomics.
- The latter were concerned with building logically completely defined strategic market game dynamic models consistent with general equilibrium that illustrated basic properties of the financial system such as goods, markets, money markets, bankruptcy structures, an elementary central bank and other minimal institutions. Specific instances of these games can be constructed as playable experimental games.

- An instance of the type of game being analyzed is given by the value function.
- $$V(s;p) = \sup_{0 \le b \le s} \left[u\left(\frac{b}{p}\right) + \beta EV(s - b + pY; p) \right]$$

  where the state is described by the initial amount of money is held by an individual and the initial market price p. b is the bid and Y is a random variable. Under more or less standard microeconomic assumptions it is shown that such a game with a fixed amount of money has an equilibrium wealth distribution that maps into itself.

Where is Schumpeter?

- A fundamental fact of economic life is still missing in the discussion up to this point. That is the role of innovation and the breaking the circular flow of money in an innovating economy. Schumpeter put forth this observation first in his thesis then in his book.
- Although it over a hundred years since Schumpeter argued that innovation is essentially a disequilibrium phenomenon and that it is critical to capitalistic competition, it has defied adequate mathematization.

- The reasons for this delay are that even at the highest level of abstraction a model must have a clear description of cash flows and the money supply.
- How to model innovation and what is operationally meant by breaking the circular flow. At any point of time all resources are given.

- Thus in a closed system an innovation must involve a reallocation of existing resources . The idea for an innovation and the implementation of the innovation are separate, The genesis of the idea may require relatively few resources, it involves primarily the perception of the existence of a new algorithm or formula for a projected new product and possibly some preliminary development. This may lead to a feasibility investigation followed by the commitment of financing to divert the resources from their current use.

Mathematical Institutional Economics: A Reprise

- When I first coined the phrase “Mathematical Institutional Economics” I had only a vague idea of why I named it such.
- It took me many years to appreciate the fact that the dynamics of a nation state or world could not be studied fruitfully without considering the monetary and financial control system that was in place to control and coordinate the system.

- The opening up of general equilibrium meant that the economic theorist is overwhelmed with a myriad of ways to complete and well define any dynamic model rising from being required to attach equations of motion to the static mathematical abstraction.
- It has to be clothed with some set of institutions that are the carriers of process.

- In picking the institutions the economic modeler has face up to two problems. They are the empirical validity of the assumptions and the desire for abstraction in order to help analysis.
- This leads to the concept of a **minimal institution**, the selection of which requires both institutional knowledge and the ability to abstract.

- The proliferation of alternative sets of sufficiently good institutions in some contexts signals the approach of economics to ecology and biology. Especially when one considers innovation.
- It is probably premature to speculate how the inevitable development of connections between economics, ecology and biology will develop. It is already fairly clear that the stress on Pareto optimal is disappearing to be replaced by concepts of viability within a context.

- I am an admirer of formal theories with clean axioms, interesting theorems and proofs and concerned with invariant properties. I suggest that the approach requires melding the modeling of context and institutions with analysis. These take the needed steps towards a mathematical institutional economics suitable to provide better insight in understanding the financial control of the overall economy in an evolutionary sociopolitical environment in which all economies must function.