LONGWAVE MARANGONI INSTABILITY WITH SORET EFFECT.

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Investigation of the Marangoni convection in binary fluids in the framework of a linear stability theory was started several decades ago\textsuperscript{[1]}. Recently, the appearance of long wavelength monotonic and oscillatory instabilities in a layer with a fixed heat flux in a solid substrate and for small\textsuperscript{[2]} and finite\textsuperscript{[3]} values of the Biot number at the upper free deformable surface has been found. In the present talk, the case of arbitrary Biot numbers on both layer boundaries is discussed.

We consider a system consisting of a layer of an incompressible binary liquid with a deformable free surface, and a solid substrate layer heated or cooled from below. Surface tension is assumed to linearly depend upon both temperature and solute concentration. The Soret effect is taken into account. It is assumed that the layer is sufficiently thin, so the effect of buoyancy can be neglected as compared to the Marangoni effect. The Dufour effect is neglected.

We investigate the long wavelength Marangoni instability in the case of asymptotically small Lewis $L$ and Galileo $G$ numbers for finite surface tension, finite Biot numbers at the upper surface and finite conductivity of the solid substrate. In the framework of long-wave linear stability theory we find both long wavelength monotonic and oscillatory modes of instability in various parameter domains of Biot and Soret $\chi$ numbers.

A set of long-wave nonlinear evolution equations that governs the spatiotemporal dynamics of a thin binary-liquid film is derived in the domain of

$$ k = O(\varepsilon) = O(\sqrt{L}) \ll 1. $$

Our analysis shows that the set of equations is well-posed when the Marangoni number is below the monotonic instability threshold, i.e.,

$$ M < 48L / \chi. $$

The weakly nonlinear analysis is carried out in the limit of the low conductivity of the solid substrate. In the leading order of the problem we consider a particular solution corresponding to a pair of traveling waves with complex amplitudes $H_+$, and angle $\theta$ between wave vectors. We obtain a set of two Landau equations that govern evolution of wave amplitudes:

$$ \frac{dH_+}{d\tau} = \delta H_+ + \alpha(X,Z)|H_+|^2 H_+ + \beta(X,Z,\theta)|H_+|^2 H_-, $$

$$ \frac{dH_-}{d\tau} = \delta H_- + \alpha(X,Z)|H_-|^2 H_- + \beta(X,Z,\theta)|H_-|^2 H_+, $$

where $\alpha = \alpha_+ + i\alpha_-, \beta = \beta_+ + i\beta_-, X$ is rescaled Galileo number, $Z$ is rescaled squared wave number. In the case of small gravity ($X = 0$) the weakly nonlinear theory predicts the appearance of stable supercritical solutions if
(i) $\max \beta_r(\theta) < \alpha_r < 0$ (traveling waves); this case takes place for $3.033 < Z < 4.756$ and $37.335 < Z < 44.892$;

(ii) $\alpha_r < 0, \alpha_r < \beta_r(\theta), \alpha_r + \beta_r(\theta) < 0$ (superposition of two waves with the angle $\theta_{\text{max}}$ between wave vectors, the angle $\theta_{\text{max}}$ corresponds to the maximum value of $\beta_r(\theta)$; this case takes place for $1.632 < Z < 3.033$ and $44.892 < Z < 44.938$. In all other cases unstable subcritical solutions appear, hence the weakly nonlinear theory is not sufficient for finding stable solutions. It is shown that the two-wave solution with the angle $\theta_{\text{max}}$ between wave vectors, corresponding to the maximum of value $\beta_r(\theta)$, is linearly stable with respect to the excitation of new waves. In the two-dimensional case the predictions of the analytical theory have been justified by numerical simulations.

In the case of small gravity three-dimensional stable supercritical wavy patterns are found. Also, bifurcations of spatially periodic solutions and spatio-temporal regimes in a long computational region are investigated numerically.

In the case of the finite conductivity of the solid substrate linear stability analysis in the domain of finite wave numbers $k = O(1)$ in the case of asymptotically small Lewis and Galileo numbers reveals that there are no additional minima of the monotonic neutral curve in the this domain.

Studying of the asymptotic behaviors of the long-wave monotonic neutral curve obtained in the region $k = O(L^{1/2}) = O(\varepsilon)$ in the limit of large wave numbers and also of the monotonic neutral curve obtained in the case of finite wave numbers $k = O(1)$ in the limit when $k \to 0$, we reveal a novel long-wave intermediate asymptotic limit $k = O(L^{1/4}) = O(\sqrt{\varepsilon})$, where depending on the parameter values the minimum of the full monotonic neutral curve can be present.

A weakly nonlinear amplitude evolution equation describing the dynamics of the long-wave monotonic instability in the case of asymptotically small Lewis and Galileo numbers, is derived in the domain

$$\sqrt{L} \leq k \leq 1$$

(4)

using new scaling of the wave number which is different from that in the case of the oscillatory instability. The form of the obtained nonlinear evolution equation is very similar to that obtained for the surface-tension driven convection in a horizontal liquid layer confined between poorly conducting boundaries\textsuperscript{[4]}.