Complexity Theory

Homework Set No. 8 (Review)

Date Due: ??

These are review question. The list will probably grow.

1. Prove that if $NP \subseteq BPP$ then $NP = RP$.

2. Prove that $Co - NEXP \subseteq NEXP/O(n)$, i.e. that in non-deterministic exponential time using $O(n)$ bits of advise you can recognize all langauge in co-non-deterministic exponential time.

3. Recall the class S_2^P where both players move simultaneously. Show that if $NP \subseteq P/Poly$ then $PH = S_2^P$.

4. For $x, y \in \{0,1\}^n$ let the set $D_{x,y}$ be the set of indices such that $x[i] \neq y[i]$. Suppose that Alice get x and Bob gets y and there goal is to find an $i \in D_{x,y}$. (i) what is the deterministic communication complexity of this task. (ii) What is the probabilistic communication complexity.

5. In the simultaneous messages model both for a communication protocol both parties Alice who receives input x and Bob who receives inputs y send a message to a referee Charlie who initially get no input and should compute the result of the desired function. One can consider deterministic, probabilistic and probabilistic with a shared random string protocols for this model. (a) determine bounds on the complexity of the $x > y$ function in all models. (b) How about the $x = y$ function?

6. Consider the class AC^0 of functions computable by constant depth circuits with polynomial number of gates and unbounded fan-in $\vee$ and $\wedge$. Show that any such circuits can be computed

by a depth three circuit of polynomial size with a single unbounded fan-in parity gate next to the output and polynomially many $\vee$ and $\wedge$ gates with $\log n$ fan-in.