

efficient computation ~ poly-time comput.

The search version $R \subseteq \{0,1\}^* \times \{0,1\}^*$

poly-bounded $\equiv \exists \text{ poly } p \text{ s.t. } (x,y) \in R \Rightarrow |y| \leq p(|x|)$

Notation: $R(x) \equiv \{y : (x,y) \in R\}$, $S_R \equiv \{x : R(x) \neq \emptyset\}$

poly Find $\equiv \{R : \exists \text{ poly-time alg } A \text{ s.t. } \forall x \in S_R \exists y \in R(x) \text{ s.t. } A(x,y) = 1\}$

poly Check $\equiv \{R : \exists \text{ poly-time alg } A \text{ s.t. } \forall x \in S_R \forall y \in R(x) \text{ s.t. } A(x,y) = 1\}$

poly-time computable $\equiv \{R : \exists \text{ poly-time alg } A \text{ s.t. } \forall x \in S_R \exists y \in R(x) \text{ s.t. } A(x,y) = 1\}$

The P vs NP question as $P \subseteq \text{PFF}$?

REPF $\equiv \{R : \exists \text{ poly-time alg } A \text{ s.t. } \forall x \in S_R \exists y \in R(x) \text{ s.t. } A(x,y) = 1\}$

$P \neq \text{PC} : R \equiv \{ (x,y) : x \in \{0,1\}^* \} \cup \{ (x, \sigma^x) : x \in \{0,1\}^* \}$

The decision version

$P \equiv \{S : \exists \text{ poly-time } A \text{ s.t. } A(x) = 1 \text{ iff } x \in S\}$

NP $\equiv \{S : \exists \text{ poly } p \text{ s.t. } \exists \text{ off. verifiable process } \text{ of length } \leq p(|x|) \text{ for } x \in S\}$

$\Rightarrow \exists p \exists \text{ poly-time procedure } V$

witness (complete) $\forall x \in S \exists w \in \Sigma^* \text{ s.t. } V(x,w) = 1$

(sound) $\forall x \notin S \forall w \in \Sigma^* \text{ s.t. } V(x,w) = 0$

poly-time computable $\equiv \{S : \exists \text{ poly-time alg } A \text{ s.t. } A(x) = 1 \text{ iff } x \in S\}$

NP \subseteq P \Leftrightarrow NP = P \Leftrightarrow NP = P