Introduction to Complexity Theory

Notes for a One–Semester Course

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Preface

Complexity Theory is a central field of Theoretical Computer Science, with a remarkable list of celebrated achievements as well as a very vibrant present research activity. The field is concerned with the study of the *intrinsic* complexity of computational tasks, and this study tend to aim at *generality*: It focuses on natural computational resources, and the effect of limiting those on the *class of problems* that can be solved. Put in other words, Complexity Theory aims at understanding the *nature of efficient computation*.

Topics: In my opinion, a introductory course in complexity theory should aim at exposing the students to the basic results and research directions in the field. The focus should be on concepts and ideas, and complex technical proofs should be avoided. Specific topics may include

- Revisiting NP and NPC (with emphasis on search vs decision);
- Complexity classes defined by one resource-bound – hierarchies, gaps, etc;
- Non-deterministic Space complexity (with emphasis on NL);
- Randomized Computations (e.g., ZPP, RP and BPP);
- Non-uniform complexity (e.g., P/poly, and lower bounds on restricted circuit classes);
- The Polynomial-time Hierarchy;
- The counting class #P, approximate-#P and uniqueSAT;
- Probabilistic proof systems (i.e., IP, PCP and ZK);
- Pseudorandomness (generators and derandomization);
- Time versus Space (in Turing Machines);
- Circuit-depth versus TM-space (e.g., AC, NC, SC);
- Communication complexity;
- Average-case complexity;

Of course, it would be hard (if not impossible) to cover all the above topics (even briefly) in a single-semester course (of two hours a week). Thus, a choice of topics has to be made, and the rest may be merely mentioned in a relevant lecture or in the concluding lecture. The choice may depend on other courses given in the institute; in fact, my own choice was strongly effected by this aspect.

Prerequisites: It is assumed that students have taken a course in computability, and hence are familiar with Turing Machines.

Model of Computation: Most of the presented material is quite independent of the specific (reasonable) model of computation, but some material does depend heavily on the locality of computation of Turing machines.

The partition of material to lectures: The partition of the material to lectures reflects only the logical organization of the material, and does not reflect the amount of time to be spent on each topic. Indeed, some lectures are much longer than other.

State of these notes: These notes provide an outline of an introductory course on complexity theory, including discussions and sketches of the various notions, definitions and proofs. The latter are presented in varying level of detail, where the level of detail does not reflect anything (except the amount of time spent in writing). Furthermore, the notes are neither complete nor fully proofread.

Related text: The current single-semester introductory course on complexity theory is a proper subset of a two-semester course that I gave in 1998–99 at the Weizmann Institute of Science. Lectures notes for that course are availalbe from the webpage

`http://www.wisdom.weizmann.ac.il/~oded/cc.html`

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