Multilevel Directional Aggregation Approach for Fast Field Evaluation

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The computational analysis of electromagnetic scattering is important in a variety of applications. In many cases, the problem is formulated using an integral equation for the induced current density. The integral equation is then discretized into a matrix equation using the moment method. For a two-dimensional dielectric scatterer, the number of unknowns, *N*, is proportional to the area of its cross section normalized to the wavelength. The matrix equation is then solved either by a direct or iterative method. A direct method requires $O(N^3)$ floating-point operations, while an iterative method employing direct matrix-vector multiplication requires $O(N^2)$ operations per iteration. The cost of iterative solvers can be reduced by accelerating the matrix vector multiplication representing discretized evaluation of the field produced by a given current distribution.

In this paper, we develop a novel multilevel directional aggregation (DA) technique that facilitates the numerically efficient evaluation of the field produced by a given current distribution. The proposed algorithm improves the one proposed by the second author (A. Brandt, *Comp. Phys. Comm*. 65, 24-38, 1991). This work extends our previous study of planar (O. Livne, A. Brandt, A. Boag, *Microwave Optical Technology Lett.,* 32, 454-458, 2002) and volumetric scatterers (Kh. Garb, A. Brandt, A. Boag, *URSI Radio Science Meeting*, Monterey, CA, June 2004.). In the DA scheme, at each level one distinguishes several "propagation directions", and performs a separate aggregation (anterpolation) process for each such direction. The proposed algorithm includes recursively the following 6 steps: a) Grid density oscillatory anterpolation that is performed for every direction; b) Angular density interpolation of the currents; c) Field calculation on the coarse grid; d) Angular field anterpolation; e) Field oscillatory interpolation; and, finally, f) Local field correction. The field produced by each aggregated domain of that process is intended to be calculated only within a specific *sector* of the radial grid emanating from that domain, namely, only in directions around the specific propagation direction. The number of propagation directions increases with the level, typically doubled upon each coarsening step in 2D problems. The asymptotic complexity attained by the DA scheme is $O(N\log N)$. For volumetric scatterers, the complexity can be further reduced to $O(N)$ by using anisotropic grids.

The proposed approach is illustrated by the evaluation of electric field that is created by the point current source in the dielectric cylinder of square cross section. The evaluation error is investigated as a function of the number of "propagation directions" and the range of local correction.