Statistics and geometry in high-Schmidt number scalar mixing

Jörg Schumacher[1], Dan Kushnir[2], Achi Brandt[2], Katepalli R. Sreenivasan[3] and Herwig Zilken[4]

[1] Dept. of Physics, Philipps University, D-35032 Marburg, Germany
[2] Dept. of Computer Science and Applied Mathematics, The Weizmann Institute of Science, 76100 Rehovot, Israel
[3] International Centre for Theoretical Physics, 34014 Trieste, Italy
[4] Visualization Laboratory, Central Institute for Applied Mathematics, Research Centre Jülich, D-52425 Jülich, Germany

The mixing of substances is present in various turbulent systems. Examples arise in reacting flows and combustion, mixing of salt and plankton in oceans and of chemical pollutants in the stratosphere [1]. The physics of scalar mixing depends strongly on the ratio of the kinematic viscosity ν of the fluid to the diffusivity κ of the scalar. It is given by the Schmidt number $Sc = \nu/\kappa$. For the following, we focus to the so-called Batchelor regime of scalar mixing [2], i.e. $Sc > 1$. High-resolution simulations are used to explore some geometrical and statistical properties of the gradients of passive scalar fields, $\nabla\theta(\mathbf{x}, t)$ (for more details, see also [3, 4]). The strong resolution requirements that significantly exceed usually adopted conditions result in rather low Taylor microscale Reynolds numbers ($R_\lambda \leq 63$) of the advecting flow. The Schmidt numbers are 8 and 32.

Large scalar gradients are associated with regions of locally intensive mixing and can be quantified by the scalar dissipation rate. This field will be of interest for the following and is defined as

$$\epsilon_\theta(\mathbf{x}, t) = \kappa(\nabla\theta(\mathbf{x}, t))^2 \,. \tag{1}$$

Figure 1 illustrates the shape and spatial distribution of its largest amplitude events. We see that regions with large dissipation rate are organized in thin extended sheets for cases $Sc > 1$, in contrast to the maxima of the energy dissipation rate which are plotted in the same panels. The figure indicates also a Reynolds number dependence of the mixing. With increasing value the sheets become smaller, but more numerous. This is attributed to the local flow patterns which are responsible for their formation. Eddy sizes vary over a larger range of scales with growing Reynolds number.

The tail of the probability density function (PDF) of the scalar dissipation rate determines the statistical distribution of those intensive mixing events.

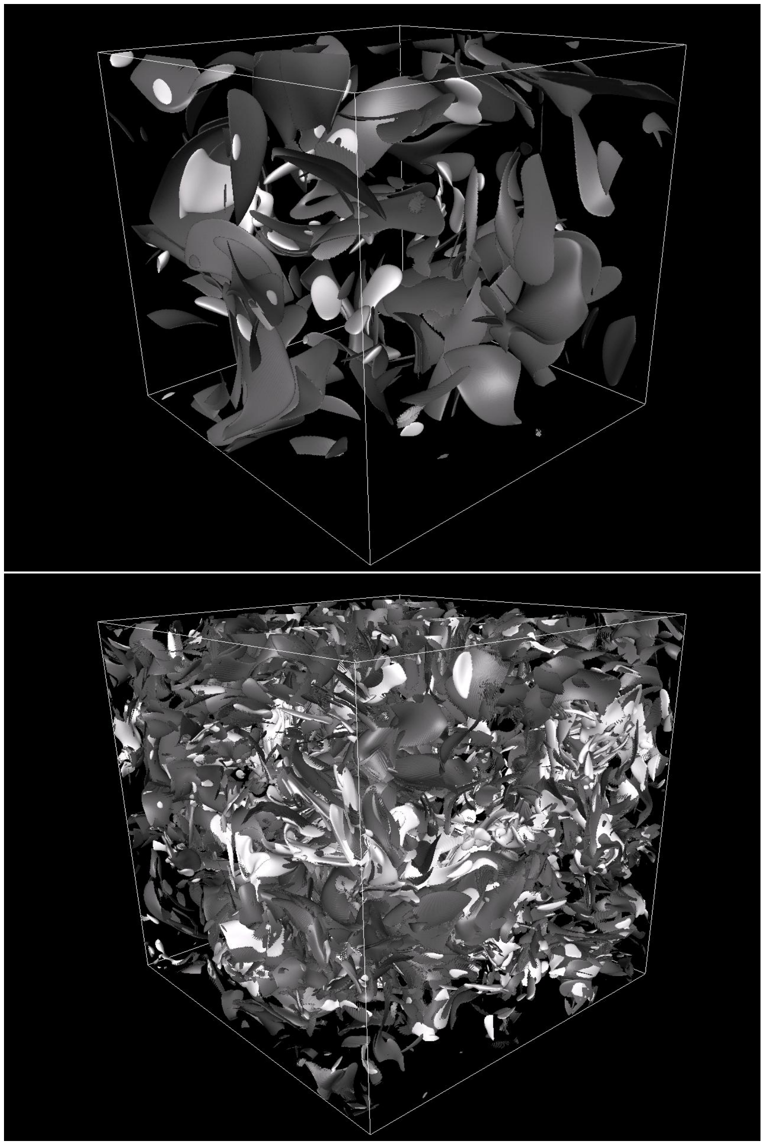

Fig. 1. Joint isovolume plots of the energy dissipation field $\epsilon(\mathbf{x},t)$ (light) and the scalar dissipation rate $\epsilon_\theta(\mathbf{x},t)$(dark). The data are obtained from very well resolved pseudospectral simulations in a periodic box with grid sizes of $N^3 = 1024^3$ points. The spectral resolution criterion is $k_{max}\eta_B = 11.84$ (upper panel) and $k_{max}\eta_B = 3.39$ (lower panel) with $k_{max} = \sqrt{2}N/3$ and the Batchelor scale η_B. The advecting turbulence is homogeneous and isotropic and is maintained stationary by stochastic forcing at low wavenumbers. The passive scalar fluctuations are kept stationary by a constant mean scalar gradient in y direction. The isovolume levels for both pictures are $5\times\langle\epsilon\rangle$ for the energy dissipation rate and $5\times\langle\epsilon_\theta\rangle$ for the scalar dissipation rate. The Schmidt number was $Sc = 8$. Upper picture: $R_\lambda = 24$. Lower picture: $R_\lambda = 63$.

The PDF is plotted in Fig.2 for two Reynolds numbers. We find considerable deviations from lognormality, which exceed those previously reported (see [4] for a more detailed discussion). Such deviations are detected for all Reynolds and Schmidt numbers studied here. In the figure, the tails of PDF were fitted with a stretched exponential

$$p(\epsilon_\theta \gg \langle\epsilon_\theta\rangle) \sim \epsilon_\theta^{-1/2} \exp\left(-C_2\epsilon_\theta^{\alpha/2}\right) , \tag{2}$$

Such statistics were derived analytically for scalar advection in smooth and white-in-time flows in the limit of an infinite Péclet number. An exponent $\alpha = 2/3$ was found [5]. The tails remain below that limit, but above $\alpha = 1/2$ which corresponds with an exponential distribution of $|\nabla\theta|$.

Interesting for the small-scale modeling of mixing, e.g. for flamelets in combustion, is the cross-section thickness scale of the dissipation sheets. It determines the scale across which the most intensive mixing events are present. The thickness is analysed here by a fast multiscale clustering algorithm [6], applied to two-dimensional planar cuts through snapshots of the scalar dissipation field. The sheets appear in the cuts as filaments. A local principal component analysis is applied to subpieces of each separate filament and the smaller eigenvalue is then nothing else but the local filament thickness, l_d. Their distribution is shown in Fig. 3. The PDF is supported by all scales within the viscous-convective range. Only a smaller number of the sheets have a thickness close to the Batchelor scale η_B, which is the finest scale in the turbulent mixing process. The collapse of the distributions in each of the panels suggests that the most probable thickness – the maximum of the PDF – varies as the Batchelor scale η_B with Sc at fixed Reynolds number (left) and as the Kolmogorov scale η with R_λ at fixed Schmidt number (right).

The numerical simulations were carried out on the IBM JUMP cluster at the John von Neumann Institute for Computing and we are grateful for their steady support. JS wishes to thank the Deutsche Forschungsgemeinschaft for support. Four of us (JS, DK, AB and KRS) were also supported by the US National Science Foundation.

References

1. Warhaft Z, (2000), Annu. Rev. Fluid Mech., 32:203-240
2. Batchelor GK, (1959), J. Fluid Mech., 5:113-133
3. Schumacher J, Sreenivasan KR and Yeung PK, (2005), J. Fluid Mech., 531:113-122
4. Schumacher J and Sreenivasan KR, (2005), Phys. Fluids, in press
5. Chertkov M, Falkovich G and Kolokolov I, (1998), Phys. Rev. Lett., 80:2121-2124
6. Kushnir D, Galun M and Brandt A, (2005) Pattern Recognition, submitted

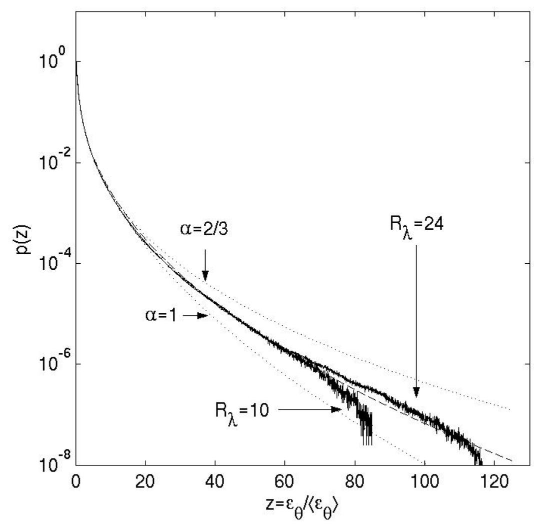

Fig. 2. Log-linear plot of the probability density function of the scalar dissipation rate, normalized to the mean value. Data are for two different Taylor microscale Reynolds numbers at $Sc = 32$. Fits to the data for $z \geq 10$ with the stretched exponential term of (2) are also plotted and the corresponding exponents α are shown. The dashed line is the optimum of a least square fit resulting in $\alpha = 0.86$.

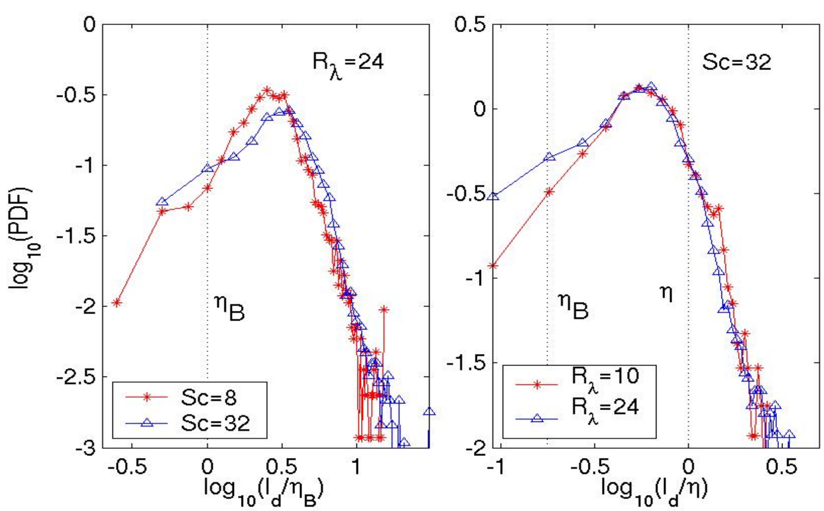

Fig. 3. Distribution of the local cross-section thickness l_d of the scalar dissipation rate filaments for $\epsilon_\theta \geq 4\langle\epsilon_\theta\rangle$. Left panel: Probability density function (PDF) $p(l_d/\eta_B)$ for two different Schmidt numbers at $R_\lambda = 24$. Right panel: PDF $p(l_d/\eta)$ for two different Reynolds numbers at $Sc = 32$.