VANISHING OF CERTAIN EQUIVARIANT DISTRIBUTIONS ON p-ADIC SPHERICAL SPACES, AND NON-VANISHING OF SPHERICAL BESSEL FUNCTIONS.

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Abstract. We prove vanishing of distribution on p-adic spherical spaces that are equivariant with respect to a generic character of the nilradical of a Borel subgroup and satisfy a certain condition on the wave-front set. We deduce from this non-vanishing of spherical Bessel functions for Galois symmetric pairs.

1. Introduction

Let $\mathbf{G}$ be a reductive group, quasi-split over a non-Archimedean local field F of characteristic zero. Let $\mathbf{B}$ be a Borel subgroup of $\mathbf{G}$, and let $\mathbf{U}$ be the unipotent radical of $\mathbf{B}$. Let $\mathbf{H}$ be a closed subgroup of $\mathbf{G}$. Let G, B, U, H denote the F-points of $\mathbf{G}, \mathbf{B}, \mathbf{U}, \mathbf{H}$ respectively. Suppose that $\mathbf{H}$ is an F-spherical subgroup of $\mathbf{G}$, i.e. that there are finitely many $B \times H$-double cosets in G. Let $\mathfrak{g}, \mathfrak{h}$ be the Lie algebras of G, H respectively. Let ψ be a non-degenerate character of U and let χ be a character of H. For $x \in G$ denote $H^x := xHx^{-1}$ and denote by χ^x the character of H^x defined by conjugation of χ. For a $B \times H$-double coset $\mathcal{O} \subset G$ define

$$\mathcal{O}_c := \{x \in \mathcal{O} \mid \psi|_{H^x \cap U} = \chi^x|_{H^x \cap U}\}.$$

Let

$$Z := \bigcup_{\mathcal{O} \text{ s.t. } \mathcal{O} \neq \mathcal{O}_c} \mathcal{O}.$$

Identify T^*G with $G \times \mathfrak{g}^*$ and let $\mathcal{N}_{\mathfrak{g}^*}$ be the set of nilpotent elements in $\mathfrak{g}^*$.

In this paper we prove the following theorem.

Theorem A (see Section 3). *Let $\xi \in \mathcal{S}^*(G)^{(U \times H, \psi \times \chi)}$ be a (U, ψ)-left equivariant and (H, χ)-right equivariant distribution on G. Suppose that the wave-front set (see section 2.3) $WF(\xi)$ lies in $G \times \mathcal{N}_{\mathfrak{g}^*}$ and $\operatorname{Supp}(\xi) \subset Z$. Then $\xi = 0$.*

In the case when $\mathbf{H}$ is a subgroup of Galois type we can prove a stronger statement. By a subgroup of Galois type we mean a subgroup $\mathbf{H} \subset \mathbf{G}$ such that

$$(\mathbf{G} \times_{\operatorname{Spec} F} \operatorname{Spec} E, \mathbf{H} \times_{\operatorname{Spec} F} \operatorname{Spec} E) \simeq (\mathbf{H} \times_{\operatorname{Spec} F} \mathbf{H} \times_{\operatorname{Spec} F} \operatorname{Spec} E, \Delta\mathbf{H} \times_{\operatorname{Spec} F} \operatorname{Spec} E)$$

for some field extension E of F, where $\Delta\mathbf{H}$ is the diagonal copy of $\mathbf{H}$ in $\mathbf{H} \times_{\operatorname{Spec} F} \mathbf{H}$.

Corollary B (see Section 4). *Let $\mathbf{H} \subset \mathbf{G}$ be a subgroup of Galois type, and let χ be a character of H. Let S be the union of all non-open $B \times H$-double cosets in G. Let $\xi \in \mathcal{S}^*(G)^{(U \times H, \psi \times \chi)}$. Suppose that $WF(\xi) \subset G \times \mathcal{N}_{\mathfrak{g}^*}$ and $\operatorname{Supp}(\xi) \subset S$. Then $\xi = 0$.*

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Note that if χ is trivial, we can consider the distribution ξ as a distribution on G/H. Considering $\mathbf{G}' := \mathbf{G} \times \mathbf{G}$ and taking $\mathbf{H}$ to be the diagonal copy of $\mathbf{G}$ we obtain the following corollary for the group case.

Corollary C (see Section 4). *Let ψ_1 and ψ_2 be non-degenerate characters of U. Let $B \times B$ act on G by $(b_1, b_2)g := b_1 g b_2^{-1}$. Let S be the complement to the open $B \times B$-orbit in G. For any $x \in G$, identify $T_x G$ with $\mathfrak{g}$ and $T_x^* G$ with $\mathfrak{g}^*$. Let*

$$\xi \in \mathcal{S}^*(G)^{U \times U, \psi_1 \times \psi_2}$$

and suppose that $WF(\xi) \subset S \times \mathcal{N}_{\mathfrak{g}^*}$. Then $\xi = 0$.

1.1. **Applications to non-vanishing of spherical Bessel functions.** Let π be an admissible representation of G (of finite length). Let $\mathbf{H} \subset \mathbf{G}$ be an algebraic spherical subgroup and let χ be a character of H. For equivariant functionals $\phi \in (\pi^*)^{(U,\psi)}$ and $v \in (\widetilde{\pi}^*)^{(H,\chi)}$define the *spherical Bessel distribution* by

$$\xi_{v,\phi}(f) := \langle v, \pi^*(f)\phi \rangle.$$

By [AGS, Theorem A] we have $WF(\xi_{v,\phi}) \subset G \times \mathcal{N}_{\mathfrak{g}^*}$.

The *spherical Bessel function* is defined to be the restriction $j_{v,\phi} := \xi_{v,\phi}|_{G-S}$, where S is the union of all non-open $B \times H$-double cosets in G. One can easily deduce from [AGS, Theorem A] and Lemma 3.1 that $j_{v,\phi}$ is a smooth function. Theorem A and Corollary B imply the following corollary.

Corollary D. *Suppose that π is irreducible and v, ϕ are non-zero. Then*

(i) $\xi_{v,\phi}|_{G \setminus \overline{Z}} \neq 0$.

(ii) If $\mathbf{H}$ is a subgroup of Galois type then $j_{v,\phi} \neq 0$.

For the group case this corollary was proven in [LM, Appendix B].

1.2. **Related results.** In [AG] a certain Archimedean analog of Theorem A is proven (see [AG, Theorem A]). This analog implies that the Archimedean analog of Corollary D(ii) holds for any spherical pair (G, H) (see [AG, Corollary B]).

Corollary C together with [AGS, Theorem A] can replace [GK75, Theorem 3] in the proof of uniqueness of Whittaker models [GK75, Theorem C].

Theorem A can be used in order to study the dimensions of the spaces of H-invariant functionals on irreducible generic representations of G (see [AG, §1.3] for more details). It can also be used in the study of analogs of Harish-Chandra's density theorem (see [AGS, §1.7] for more details).

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2. Preliminaries

2.1. **Conventions.**

- We fix $F, \mathbf{G}, \mathbf{B}, \mathbf{U}, \mathbf{X}$ and ψ as in the introduction.

- All the algebraic groups and algebraic varieties that we consider are defined over F. We will use capital bold letters, e.g. $\mathbf{G}, \mathbf{X}$ to denote algebraic groups and varieties defined over F, and their non-bold versions to denote the F-points of these varieties, considered as l-spaces or F-analytic manifolds (in the sense of [Ser64]).
- When we use a capital Latin letter to denote an F-analytic group or an algebraic group, we use the corresponding Gothic letter to denote its Lie algebra.
- We denote by G_x the stabilizer of x and by $\mathfrak{g}_x$ its Lie algebra.

2.2. **Vanishing of equivariant distributions.** The following criterion for vanishing of equivariant distributions follows from [BZ76, §6] and [Ber83, §§1.5].

Theorem 2.1 (Bernstein-Gelfand-Kazhdan-Zelevinsky). *Let an algebraic group* $\mathbf{H}$ *act on an algebraic variety* $\mathbf{X}$*, both defined over* F*. Let* χ *be a character of* H*. Let* $Z \subset X$ *be a closed* H*-invariant subset. Suppose that for any* $x \in Z$ *we have*

$$\chi|_{H_x} \neq \Delta_H|_{H_x} \Delta_{H_x}^{-1},$$

where Δ_H *and* Δ_{H_x} *denote the modular functions of the groups* H *and* H_x*. Then there are no non-zero* (H, χ)*-equivariant distributions on* X *supported in* Z*.*

2.3. **Wave front set.** In this section we give an overview of the theory of the wave front set as developed by D. Heifetz [Hef85], following L. Hörmander (see [Hör90, §8]). For simplicity we ignore here the difference between distributions and generalized functions.

Definition 2.2.

(1) *Let* V *be a finite-dimensional vector space over* F*. Let* $f \in C^\infty(V^*)$ *and* $w_0 \in V^*$*. We say that* f vanishes asymptotically in the direction of w_0 *if there exists* $\rho \in \mathcal{S}(V^*)$ *with* $\rho(w_0) \neq 0$ *such that the function* $\phi \in C^\infty(V^* \times F)$ *defined by* $\phi(w, \lambda) := f(\lambda w) \cdot \rho(w)$ *is compactly supported.*

(2) *Let* $U \subset V$ *be an open set and* $\xi \in \mathcal{S}^*(U)$*. Let* $x_0 \in U$ *and* $w_0 \in V^*$*. We say that* ξ *is* smooth at (x_0, w_0) *if there exists a compactly supported non-negative function* $\rho \in \mathcal{S}(V)$ *with* $\rho(x_0) \neq 0$ *such that the Fourier transform* $\mathcal{F}^*(\rho \cdot \xi)$ *vanishes asymptotically in the direction of* w_0*.*

(3) *The complement in* T^*U *of the set of smooth pairs* (x_0, w_0) *of* ξ *is called the* wave front set of ξ *and denoted by* $WF(\xi)$*.*

(4) *For a point* $x \in U$ *we denote* $WF_x(\xi) := WF(\xi) \cap T^*_x U$*.*

Remark 2.3. Heifetz defined $WF_\Lambda(\xi)$ for any open subgroup Λ of $F^\times$ of finite index. Our definition above is slightly different from the definition in [Hef85]. They relate by

$$WF(\xi) - (U \times \{0\}) = WF_{F^\times}(\xi).$$

Proposition 2.4 (see [Hör90, Theorem 8.2.4] and [Hef85, Theorem 2.8]). *Let* $U \subset F^m$ *and* $V \subset F^n$ *be open subsets, and suppose that* $f : U \to V$ *is an analytic submersion. Then for any* $\xi \in \mathcal{S}^*(V)$*, we have*

$$WF(f^*(\xi)) \subset f^*(WF(\xi)) := \left\{(x, v) \in T^*U \,|\, \exists w \in WF_{f(x)}(\xi) \text{ s.t. } d^*_{f(x)} f(w) = v\right\}.$$

Corollary 2.5. *Under the assumption of Proposition 2.4 we have*

$$WF(f^*(\xi)) = f^*(WF(\xi)).$$

Proof. The case when f is an analytic diffeomorphism follows immediately from Proposition 2.4. This implies the case of open embedding. It is left to prove the case of linear projection $f : F^{n+m} \to F^n$. In this case the assertion follows from the fact that $f^*(\xi) = \xi \boxtimes 1_{F^m}$ where 1_{F^m} is the constant function 1 on F^m. □

Corollary 2.6. *Let X be an F-analytic manifold. We can define the wave front set of any distribution in $\mathcal{S}^*(X)$, as a subset of the cotangent bundle T^*X.*

Theorem 2.7. *(Corollary from* [A13, Theorem 4.1.5]*) Let an F-analytic group H act on an F-analytic manifold Y and let χ be a character of H. Let $\xi \in \mathcal{S}^*(Y)^{(H,\chi)}$. Then*
$$WF(\xi) \subset \{(x, v) \in T^*Y | v(T_x(Hx)) = 0\}.$$

Theorem 2.8 ([A13, Theorem 4.1.2])**.** *Let $Y \subset X$ be F-analytic manifolds and let $y \in Y$. Let $\xi \in \mathcal{S}^*(X)$ and suppose that* $\operatorname{Supp}(\xi) \subset Y$*. Then $WF_y(\xi)$ is invariant with respect to shifts by the conormal space $CN^X_{Y,y}$.*

Corollary 2.9. *Let M be an F-analytic manifold and $N \subset M$ be a closed algebraic submanifold. Let ξ be a distribution on M supported in N. Suppose that for any $x \in N$, we have $CN^M_{N,x} \not\subseteq WF_x(\xi)$. Then $\xi = 0$.*

Proof. Suppose $\xi \neq 0$ and let $x \in \operatorname{Supp}(\xi)$. Then $(x, 0) \in WF_x(\xi)$. But then from Theorem 2.8 we have $CN^M_{N,x} \subseteq WF_x(\xi)$ which contradicts our assumption on ξ. □

3. Proof of Theorem A

Lemma 3.1. *Let $x \in G$. Let ξ be a (U, ψ)-left equivariant and (H, χ)-right equivariant distribution on G such that $WF(\xi) \subset G \times \mathcal{N}_{\mathfrak{g}^*}$. Then $WF_x(\xi) \subset CN^G_{BxH,x}$.*

Proof. Let $\mathfrak{t}$ be the Lie algebra of a maximal torus contained in B, and let $\mathfrak{h}, \mathfrak{u}$ be the Lie algebras of H, U respectively. Identify T^*_xG with $\mathfrak{g}^*$ using the right multiplication by x^{-1}. We have $CN^G_{BxH,x} = (\mathfrak{t} + \mathfrak{u} + ad(x)\mathfrak{h})^\perp$. Since ξ is $\mathfrak{u}$-equivariant, by Theorem 2.7 we have $WF_x(\xi) \subset \mathfrak{u}^\perp$. Similarly, since ξ is $\mathfrak{h}$-equivariant on the right, we have $WF_x(\xi) \subset (ad(x)\mathfrak{h})^\perp$. By our assumption $WF_x(\xi) \subset \mathcal{N}_{\mathfrak{g}^*}$. Now, $\mathfrak{u}^\perp \cap \mathcal{N}_{\mathfrak{g}^*} = (\mathfrak{t} + \mathfrak{u})^\perp$ and thus
$$WF_x(\xi) \subset (ad(x)\mathfrak{h})^\perp \cap \mathfrak{u}^\perp \cap \mathcal{N}_{\mathfrak{g}^*} = (ad(x)\mathfrak{h})^\perp \cap (\mathfrak{u}+\mathfrak{t})^\perp = (\mathfrak{t}+\mathfrak{u}+ad(x)\mathfrak{h})^\perp = CN^G_{BxH,x}.$$
□

Proof of Theorem A. Suppose that there exists a non-zero right (U, ψ)-equivariant and left (H, χ)-equivariant distribution ξ supported on Z such that $WF(\xi) \subset G \times \mathcal{N}_{\mathfrak{g}^*}$.We decompose G into $B \times H$-double cosets and prove the required vanishing coset by coset. For a $B \times H$-double coset $\mathcal{O} \subset G$ define $\mathcal{O}_s := \mathcal{O} \setminus \mathcal{O}_c$ and stratify $\mathcal{O}_c$ to a union of smooth locally closed F-analytic subvarieties $\mathcal{O}^i_c$. The collection
$$\{\mathcal{O}^i_c \,|\, \mathcal{O} \text{ is a } B \times H\text{-double coset}\} \cup \{\mathcal{O}_s \,|\, \mathcal{O} \text{ is a } B \times H\text{-double coset}\}$$
is a stratification of G. Order this collection to a sequence $\{S_i\}_{i=1}^N$ of smooth locally closed F-analytic submanifolds of G such that $U_k := \bigcup_{i=1}^k S_i$ is open in G for any $1 \leq k \leq N$. Let k be the maximal integer such that $\xi|_{U_{k-1}} = 0$. Suppose $k \leq N$ and let $\eta := \xi|_{U_k}$. Note that $\operatorname{Supp}(\eta) \subset S_k$. We will now show that $\eta = 0$, which leads to a contradiction.

Case 1. $S_k = \mathcal{O}_s$ for some orbit $\mathcal{O}$. Then $\eta = 0$ by Theorem 2.1 since η is $(U \times H, \psi \times \chi)$-equivariant.

Case 2. $S_k \subset \mathcal{O} = \mathcal{O}_c$ for some orbit $\mathcal{O}$. Then $S_k \subset G \backslash Z$ and $\eta = 0$ by the conditions.

Case 3. $S_k \subset \mathcal{O}_c \nsubseteq \mathcal{O}$ for some orbit $\mathcal{O}$. In this case $\dim \mathcal{O}_c < \dim \mathcal{O}$ and thus

$$CN^G_{S_k,x} \supsetneq CN^G_{\mathcal{O},x}.$$

By Lemma 3.1 we have, for any $x \in S_k$,

$$WF_x(\eta) \subset CN^G_{\mathcal{O},x} \text{ and thus } CN^G_{S_k,x} \nsubseteq WF_x(\eta).$$

By Corollary 2.9 this implies $\eta = 0$.

□

4. Proof of Corollaries B and C

Let $\mathbf{U}'$ denote the derived group of $\mathbf{U}$.

Lemma 4.1. *Let $\overline{W}$ be the Weyl group of $\mathbf{G}$. Let $\overline{w} \in \overline{W}$ and let $w \in G$ be its representative. Suppose that $w\mathbf{U}w^{-1} \cap \mathbf{U} \subset \mathbf{U}'$. Then $\overline{w}$ is the longest element in $\overline{W}$.*

Proof. Let $\mathfrak{u}$ be the Lie algebra of $\mathbf{U}$. On the level of Lie algebras the condition $wUw^{-1} \cap U \subset U'$ means that $(Ad(w)\mathfrak{u}) \cap \mathfrak{u} \subset \mathfrak{u}'$. The algebra $\mathfrak{u}$ can be decomposed as

$$\mathfrak{u} = \bigoplus_{\alpha \in \Phi^+} \mathfrak{g}_\alpha.$$

It is easy to see that

$$(Ad(w)\mathfrak{u}) \cap \mathfrak{u} = \sum_{\alpha \in \Phi^+, \overline{w}^{-1}(\alpha) \in \Phi^+} \mathfrak{g}_\alpha.$$

Let $\Delta \subset \Phi^+$ be the set of simple roots in Φ^+. Then from the condition of the lemma we obtain that $\overline{w}^{-1}(\Delta) \subset \Phi^-$, and as a consequence $\overline{w}^{-1}(\Phi^+) \subset \Phi^-$. Let $\overline{w}_0$ be the longest element in $\overline{W}$. Then $\overline{w}_0\overline{w}^{-1}(\Phi^+) \subset \Phi^+$. Since Φ^+ is a finite set and $\overline{w}_0\overline{w}^{-1}$ acts by an invertible linear transformation, we get $\overline{w}_0\overline{w}^{-1}(\Phi^+) = \Phi^+$. Since simple roots are the indecomposable ones, it follows that $\overline{w}_0\overline{w}^{-1}(\Delta) = \Delta$. This implies that $\overline{w}_0\overline{w}^{-1} = 1$ (see e.g. [Hum72, §10.3]), and thus $\overline{w}_0 = \overline{w}$. □

Corollary 4.2. *Let $\mathbf{H}$ be a reductive group. Assume $\mathbf{G} = \mathbf{H} \times \mathbf{H}$ and let $\Delta\mathbf{H} \subset \mathbf{G}$ be the diagonal copy of $\mathbf{H}$. Denote $\mathbf{X} = \mathbf{G}/\mathbf{H}$ and let $x \in X$ be such that $\mathbf{U}_x \subset \mathbf{U}'$. Then the orbit $\mathbf{B}x$ is open in $\mathbf{X}$.*

Proof. We can identify $\mathbf{X}$ with $\mathbf{H}$ using the projection on the first coordinate. We can assume that $\mathbf{B} = \mathbf{B_H} \times \mathbf{B_H}$ where $\mathbf{B_H}$ is a Borel subgroup of $\mathbf{H}$. Let $\overline{W}$ be the Weyl group of $\mathbf{H}$ and W be a set of its representatives. By Bruhat decomposition,

$$\mathbf{H} = \bigsqcup_{w \in W} \mathbf{B_H} w \mathbf{B_H}$$

It is well-known that the only open $\mathbf{B_H} \times \mathbf{B_H}$ orbit in $\mathbf{H}$ is $\mathbf{B_H} w_0 \mathbf{B_H}$, where $w_0 \in W$ is the representative of the longest Weyl element. Let $w \in W$. Let $\mathbf{U_H}$ be the nilradical of $\mathbf{B_H}$. Then

$$\mathbf{U}_w = \{(u_1, u_2) | u_1 w u_2 = w, \ \ u_1, u_2 \in \mathbf{U_H}\},$$

and we see that for a pair $(u_1, u_2) \in \mathbf{U}_w$ we have $u_1 = wu_2w^{-1} \in w\mathbf{U_H}w^{-1}$. Therefore,

$$\mathbf{U}_w \cong \mathbf{U_H} \cap w\mathbf{U_H}w^{-1}.$$

Let

$$R = \{x \in \mathbf{X} \,|\mathbf{U}_x \subset \mathbf{U}'\} = \{x \in \mathbf{H}|\mathbf{U_H} \cap x\mathbf{U_H}x^{-1} \subset \mathbf{U_H}' = [\mathbf{U_H}, \mathbf{U_H}]\},$$

and let $\mathbf{R}$ be the corresponding algebraic variety. Since $\mathbf{U}$ and $\mathbf{U}'$ are normal in $\mathbf{B}$, we obtain that $\mathbf{R}$ is $\mathbf{B}$-invariant. The corollary follows now from Lemma 4.1. □

Corollary 4.3. *Let $\mathbf{H} \subset \mathbf{G}$ be a subgroup of Galois type. Then for every non-open B-orbit $\mathcal{O} \subset G/H$ there exists $y \in \mathcal{O}$ such that $\psi(U_y) \neq 1$.*

Proof. Let $\mathcal{O} \subset G/H$ be a non-open B-orbit and $x \in \mathcal{O}$. Since $\mathcal{O}$ is open in G/H if and only if $\mathbf{B}x$ is (Zariski) open in $\mathbf{G/H}$, Corollary 4.2 implies $\mathbf{U}_x \not\subset \mathbf{U}'$. Thus, there exists a non-degenerate character φ of U such that $\varphi(U_x) \neq 1$. For a fixed $x \in \mathcal{O}$, the set of characters φ' of U such that $\varphi'(U_x) \neq 1$ is Zariski-open, thus dense in the l-space topology and thus intersects the B-orbit of ψ. Thus there exists $y \in Bx = \mathcal{O}$ such that $\psi(U_y) \neq 1$. □

Proof of Corollary B. By Theorem A it is enough to show that $S \subset Z$. Let $\mathcal{O} \subset S$ be a $B \times H$ double coset. Corollary 4.3 implies that there exists $x \in \mathcal{O}$ such that $\psi|_{U \cap H^x} \neq 1$. Since H^x is reductive and U is unipotent, we have $\chi^x|_{U \cap H^x} = 1$, and thus $\mathcal{O} \subset Z$. □

Proof of Corollary C. Define $\mathbf{G}' = \mathbf{G} \times \mathbf{G}$, $\mathbf{H}' = \Delta(\mathbf{G}) \subset \mathbf{G}'$ and $\mathbf{B}' = \mathbf{B} \times \mathbf{B}$. The non-degenerate characters ψ_1, ψ_2 define a non-degenerate character of the nilradical U' of B'. Note that $\mathbf{H}' \subset \mathbf{G}'$ is a subgroup of Galois type and that G'/H' is naturally isomorphic to G. Let η be the pull-back of ξ to G' under the projection $G \to G'/H' \cong G$. Then we have $\operatorname{Supp} \eta \subset S'$, where S' is the union of all non-open $B' \times H'$-double cosets in G'. Also, by Corollary 2.5 we have $WF(\eta) \subset G' \times \mathcal{N}_{\mathfrak{g}'^*}$. By Corollary B we obtain $\eta = 0$ and thus $\xi = 0$. □

Remark 4.4. Corollary B can not be generalized literally to arbitrary symmetric pairs. The reason is that neither can Corollary 4.2. For example consider the pair $\mathbf{G} = \mathbf{GL}_{2n}, \mathbf{H} = \mathbf{GL_n} \times \mathbf{GL_n}$, where the involution is conjugation by the diagonal matrix with first n entries equal to 1 and others equal to -1. Let x be the coset of the permutation matrix given by the product of transpositions

$$\prod_{i=0}^{\lfloor (n-1)/2 \rfloor} (2i+1, 2n-2i),$$

and let $\mathbf{B}$ consist of upper-triangular matrices. Then $\mathbf{U}_x \subset \mathbf{U}'$, while $\mathbf{B}x$ is of middle dimension in $\mathbf{G/H}$. It can be shown that there exists a (U, ψ)-left equivariant, H-right invariant distribution ξ on G supported in BxH and satisfying $WF(\xi) \subset G \times \mathcal{N}_{\mathfrak{g}^*}$.

However, Corollary D(ii) might hold for any spherical subgroup $\mathbf{H}$. In fact, this is the case over the archimedean fields, see [AG, Corollary B].

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