

Category Theory Spring 2015 Exercise 0

Rami Aizenbud, Adam Gal

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These exercises are not for mandatory submission, but still important.

1. Read about Yoneda's Lemma. It is a formulation of the statement that an object in a category is determined by how other objects map to it.
2. Read about natural transformations between functors.
3. Let F be the identity functor of the category of vector spaces over \mathbb{k} . Show that the set of natural transformations from F to F is canonically isomorphic to \mathbb{k} . ("canonically" means roughly, without making any choices)
4. Let $F : \mathbf{Vect}_{\mathbb{k}} \rightarrow \mathbf{Vect}_{\mathbb{k}}$ be the functor $F(V) = V^{**}$. Check that the canonical maps $V \rightarrow V^{**}$ define a natural transformation $\alpha : \text{Id} \rightarrow F$ and that V is finite dimensional iff α_V is an isomorphism.