

Category Theory Spring 2015 Exercise 1

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Categories:

- **Set** - category of sets
- **Grp** - category of groups
- **AbGrp** - category of abelian groups
- **UCalg_k** - category of unital commutative algebras over k
- **Top** - category of topological spaces
- **Ban** - category of Banach spaces
- **Vect_k** - category of vector spaces over k
- $\text{Open}(T)$ - category of open sets of the topological space T .

1. For each of the above categories, explain (at least to yourself) what are products, equalizers, fiber products, limits over a sequence and monomorphisms. Explain also the dual notions (i.e. coproduct, ...).
2. Find a morphism of rings, both of which are unital, which does not preserve the unit.
3. Find a category which does not have an initial object.
4. Let C be a category. Prove that the following are equivalent:
 - (a) C has arbitrary limits.
 - (b) C has all products and equalizers.
 - (c) C has all fiber products and a final object.
5. Prove also the corresponding statement about functors.
6. Let **FinSet**, **FinSet_{*}** be the categories of finite sets and pointed finite sets. Show that the obvious functor **FinSet** \rightarrow **FinSet_{*}** (adjoining a basepoint) preserves all fiber products but not all limits.