

Category Theory Spring 2015 Exercise 4

Rami Aizenbud, Adam Gal

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1. [S] Deduce from the weak Nullstellensatz ("A field finitely generated over \mathbb{C} as algebra is equal to \mathbb{C} ") an equivalence between complex affine algebraic varieties and commutative unital algebras finitely generated over \mathbb{C} .
2. Prove Yoneda's lemma.
3. Prove that Grothendieck sheaves are equivalent to Leray sheaves (on a fixed variety X).
4. Write adjoints on both sides (when they exist) for the following functors. When the adjoint functor has an adjoint on the other side write it as well.
 - (a) [S] Forgetful functors
 - i. From **Top** to **Set**
 - ii. From compact top. spaces to T4 top. spaces
 - iii. From **Vect** to **Set**
 - iv. From **AbGrp** to **Grp**
 - v. From the category of pairs (A, S) of a commutative algebra and a subset of invertible elements to the category of pairs (A, S) of a commutative algebra and a subset of (general) elements.
 - (b) The pullback functor $\Phi^* : \text{Mod}_A \rightarrow \text{Mod}_B$ where $\Phi : B \rightarrow A$ is a morphism of algebras/groups.
 - (c) The functor of invariants from G -reps to **Vect** for some group G .
 - (d) The functors $X \mapsto \text{Hom}(X, A)$ and $X \mapsto \text{Hom}(A, X)$. For fixed $A \in \text{Mod}_R$, R an algebra.
5. [S] Show that a functor which has a left (right) adjoint commutes with all (co)limits.