Category Theory Spring 2015 Exercise 5

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1. Let $\mathcal{D}$ be a category with finite products and a terminal object. Let $\mathcal{C}$ be the category of groups equipped with the forgetful functor to **Set**. Prove that the category of $\mathcal{C}$ objects in $\mathcal{D}$ is equivalent to the category of group objects in $\mathcal{D}$.

2. Prove that the category of group objects in **Grp** is equivalent to **AbGrp**.

3. Prove that **Grp** objects in **Top** are equivalent to **Top** objects in **Grp**.

4. Is the same true when replacing **Top** with the category of smooth manifolds?

5. Prove that the category of **Vect** objects in sheaves on X is equivalent to sheaves of vector spaces on X.

6. Let $\mathcal{C}$ be a category and S a set of morphisms closed on composition and containing identities.

   Assume that

   (a) Any corner

   $$\begin{array}{ccc} B & \xrightarrow{s} & A \\ \downarrow & & \\ C & & \end{array}$$

   with $s \in S$ can be completed to a square

   $$\begin{array}{ccc} B & \xrightarrow{s} & A \\ \downarrow & & \downarrow \\ C & \xrightarrow{s'} & B' \end{array}$$

   with $s' \in S$.

   (b) For any $f, g : A \to B$ and $s \in S$ such that $f \circ s = g \circ s$ there is $s' \in S$ such that $s' \circ f = s' \circ g$.

   Prove that the localization $S^{-1}\mathcal{C}$ exists.