# Category Theory Spring 2015 Exercise 7

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1. **[S]** Prove that any groupoid is equivalent to a disjoint union of $BG$'s.
2. **[S]** Compute the fiber product and pushout for groupoids of the form $BG$ in terms of the above description.
3. Try to define pushout and fiber product in the naive sense in the 1-category of (finite) groupoids and verify that it is a bad idea. For example it is not invariant with respect to equivalence.
4. **[S]** prove that the category of representations of $BG$ (or of $BG$-sets) is equivalent to the category of representations of $G$ (or $G$-sets).
5. Let $X$ be a topological space. Prove that the category of $\Pi_1(X)$-sets is equivalent to the category of covers of $X$.
6. **[S]** Let $F : X \to Y$ be a functor of groupoids. Show that if the induced functor from $Y$-sets to $X$-sets is an equivalence then $F$ is an equivalence.