# Algebraic Topology - Exercise 1

Solve the following questions, choose one of questions 1-3, do 4-8. You can solve additional questions for more points.

1. Classify the capital letters of the Latin alphabet (Cailibri font) according to their homotopical and topological types (topological type - i.e up to homeomorphism). Give rigorous proofs when two letter are not homeomorphic, and to 5-6 cases when you claim that two letters are homotopically equivalent or homeomorphic. You do not have to prove that two given letters are not homotopically equivalent, and reuse your arguments (it saves paper).
2. Show that $\mathbb{R}^n \backslash \{*\}$ is homotopically equivalent to $S^{n-1}$.
3. Show that if $f, g : S^n \to S^n$ are continuous maps such that $f(x) \neq -g(x)$ for all $x \in X$ then $f$ and $g$ are homotopic.
4. Show that the torus without a point is homotopically equivalent to a plane without two points.
5. Prove that a homotopy equivalence induces a bijection on path components and a homotopy equivalence of the corresponding components.
6. Show that a map $f : X \to Y$ is homotopic to a constant map $\iff f$ can be extended to the a map $\tilde{f} : C_f \to Y$ (the mapping cone of $f$).
7. Show that for a map $f : X \to Y$ we have that $M_f$ is homotopy equivalent to $Y$ (the mapping cylinder of $f$).
8. Show that $\Sigma(X) \cong S^1 \wedge X$ ($\Sigma(X)$ is the reduced suspension of $X$).

   Extra:
9. Show that $S^n * S^m \cong S^{n+m+1}$ (the join of the spaces).
10. Show that $X * Y \simeq S(X \wedge Y)$, i.e the join of the spaces is homotopically equivalent to the suspension of their smash product.