Algebraic Topology - Exercise 5

Solve the following questions:

0. Construct a non-normal cover of $S^1 \vee S^1$ (No need to hand in if you did so in Exercise 4.).

1. Let X, Y and Z be topological manifolds, and let $p : Y \to X$ be an n-cover of X, show that you can construct an n cover of $X \# Z$ by $Y \# \underbrace{Z \# \ldots \# Z}_{n}$ (We say that X is a topological manifold if it is a topological space which looks locally like $\mathbb{R}^n$ for some $n \in \mathbb{N}$).

2. Let X be a set with two binary operations $*_1$ and $*_2$, having an identity element. Show that if $(a *_1 b) *_2 (c *_1 d) = (a *_2 c) *_1 (b *_2 d)$ we have that $* = *_1 = *_2$ and $*$ is commutative and associative.

   Extra points:

3. Compute the fundamental group of the space obtained from two tori $S1 \times S1$ by gluing a circle $S1 \times \{x_0\}$ in one torus along the corresponding circle $S1 \times \{x_0\}$ in the other torus.