Algebraic Topology - Exercise 7

Solve the following questions:

1. Prove that the boundary of an n-simplex is homeomorphic to S^{n-1} for $n \geq 1$.

2. Find a triangulation (i.e. a homeomorphism to a simplicial complex) of an n-dimensional polytope, that is the convex hull of a finite set of vertices in $\mathbb{R}^n$ (which atleast n of them don't lie in the same $n-1$-dimensional space). No need to prove claims from Euclidean geometry.

   (a) Using Barycentric subdivisions.

   (b) Using homeomorphisms to a disk.

3. Find a triangulation to the following spaces:

   (a) The Klein bottle.

   (b) The Mobius strip.

   (c) The projective plane.

   (d) A surface of genus 2.

   Extra:

4. Show that the realization of a barycentric subdivision of a simplicial complex S is homeomorphic to the geometric realization of S.