Algebraic Topology - Exercise 9

Solve the following questions:

1. Compute the simplicial homology of the following spaces:
   (a) A closed surface of genus 2.
   (b) The Klein bottle.
   (c) The 2-torus $\mathbb{T}^2$.
   (d) $\mathbb{R}^3$.

2. Let $f : C \to D$ be an injective map of chain complexes. Show that the projection from C_f to D/C induces a quasi-isomorphism (Note that C_f is the chain complex mapping cone and D/C is isomorphic to $D/f[C]$ as f is injective.)

3. Let $0 \to C \to D \to E \to 0$ be a short exact sequence of chain complexes, prove the existence of a long exact sequence of homology groups.

4. Let S be a simplicial complex, show that $H_0(S)$ is generated by the path components of S.

5. Show that if $f - g$ is null-homotopic then f and g induce the same homomorphism on the homologies.

6. Show that $\widetilde{H}_k(X) = H_k(X)$ for $k > 0$ and $\widetilde{H}_0(X) \oplus \mathbb{Z} = H_0(X)$ where $\widetilde{H}_k(X)$ stands for the k-th reduced homology of X.