ALGEBRAIC TOPOLOGY

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1. Chronological list of topics

The time estimate is very approximate. In practice I think that "week" will take longer then an actual week (on average).

1.1. Basic Homotopy theory.

Week 1.
- (a) Motivation and overview.
- (b) Basic Homotopy theory: homotopy, homotopy category, homotopy equivalence, pointed topological space. [GH, 1,2], [FF, 1], [HAT, 0].
- (c) Operation with spaces: product, bouquet, quotient, smash product, suspension, join, loop space,(mapping) cylinder and (mapping) cone. [GH, 7], [FF, 1], [HAT, 0]

Week 2. fundamental group π_1: definition, homotopy invariance, coverings, universal covering (existence and uniqueness), relation between coverings and π_1, examples. [GH, 4-6], [FF, 4,5], [HAT, 1.1,1.3].

Week 3. π_1 of a bouquet product and suspension, Seifert-van Kampen theorem, equivalent definitions of π_1, fundamental groupoid. [HAT, 1.2].

Week 4.
- (a) Higher homotopy groups π_n (basic facts): definition, commutativity, homotopical groups and co-groups. π_n of products, coverings and loop spaces, difficulties of computation of π_n of bouquets and suspensions. Weak homotopy equivalence of topological spaces, examples. [GH, 7], [FF, 6], [HAT, 4.1].
- (b) $\pi_k(S^n) = 0$

1.2. Basic Homology theory.

Week 5. Euler theorem

Week 6. Simplicial complexes: definition, realization. Barycentric subdivision, $Sing$, Product

Week 7.
- (a) Euler characteristic of a simplicial complex.
- (b) Homologies of a simplicial complex: definitions, examples. [GH, 10], [HAT, 2.1].
- (c) Singular homologies

Week 8. homological algebra

Week 9. Axiomatic approach to homologies: Definition, Barratt-Puppe sequence, relative homologies. Some corollaries and equivalent axioms: Mayer-Vietoris theorem, excision theorem, H_n of bouquet, long exact sequence of a triple, examples, uniqueness, Generalized Homology theories, problems with H_n of loop space. [GH, 16-17], [FF, 12], [HAT, 2.2, 2.3].

Week 10. Singular homologies: proof of axioms. [GH, 14-15], [FF, 11], [HAT, 2.1].

1.3. Advance Homotopy theory.

Week 11. $\pi_n(S^n)$; [FF, 9], [HAT, 4.2].

Week 12. CW complexes: definition, Homothopy extantion, cellular approximation, CW aproximation, Whitehead theorem, computation of π_1 and of homologies of CW complexes, obstacles to computation of π_n of CW complexes. [GH, 21], [FF, 3], [HAT, 0, 4.1].

Week 13. Simplicial sets. Definition, realisation, Kan condition. combinatorial description of homotopy classes of maps between realisations of Kan simplicial sets.

Week 14.

(a) long exact sequence of (Serre) fibration. Examples. [FF, 7,8], [HAT, 4.2].
(b) Eilenberg-MacLane spaces [FF, 2], [HAT, 4.2].

Week 15.

(a) relative homotopy groups and long exact sequence a pair. [FF, 8], [HAT, 4.1].
(b) Excision and corolaries: Hurewicz theorem, Freudenthal suspension theorem, stable homotopy groups [FF, 9], [HAT, 4.2].

1.4. Advanced Homology theory.

Week 16. Kunneth theorem. [GH, 29], [HAT, 3.2,3.B].

Week 17. Universal coefficient theorem [GH, 29], [FF, 15], [HAT, 3.1, 3.A]

Week 18. Cohomology: definition, cup product, duality to homologies. [GH, 23, 24], [FF, 14], [HAT, 3.1].

Week 19. Cohomology with compact support and Borel-Moore homology. [GH, 26], [HAT, 3.3]. Cech (co-)homology. [HAT, 3.3].

Week 20. Orientation and Poincare duality [GH, 22, 26], [HAT, 3.3].

Week 21. relation to Eilenberg-MacLane spaces [FF, 2], [HAT, 4.3]

1.5. Advanced topics.

Week 22.

(a) Sheaf cohomology.
(b) Spectral sequences.
(c) the stable homotopy category and spectra.
(d) Alexander duality
(e) Cohomology operations
(f) Bott periodicity theorem
(g) K-theory
(h) Bordisms

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2. Textbooks

The literature for the course is [GH, FF, HAT]. The course will follow a "convex combination" of [GH] and [FF]. We will use [HAT] as a source of examples, problems and additional information.

[GH] is the easiest one of the three, but it doesn't cover all of the required information. [FF] contains almost everything we will need, but omits too many details in some proofs. Also, the order of the topics in the course will be something between [GH] and [FF]. Additionally, [FF] is highly recommended for its illustrations. Finally, [HAT] is the most detailed of these three books, but it is too big to serve as a textbook for a first course in algebraic topology.

[GH] Greenberg and Harper, *Algebraic topology: a first course*.
[FF] Fomenko and Fuks, *Homotopic topology*.
[HAT] Hatcher, *algebraic topology*.

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