Weizmann Institute of Science
Faculty of Mathematics and Computer Science

RECENT ADVANCES IN NONLINEAR EVOLUTIONARY EQUATIONS AND ANALYSIS OF MULTI-SCALE PHENOMENA

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Ziskind Building, Room 1

Abstracts

Gaussian Beam Decomposition of High Frequency Wave Fields

Gil Ariel

Gaussian beam approximations of high frequency waves has been gradually developing into a useful simulation tool with application in seismic migration, computational electro-magnetics, semiclassical approximations in quantum mechanics and more. As the term ``high frequency'' suggests, such applications involve many wave oscillations in the domain of interest and thus, direct numerical methods of the wave propagation are prohibitively computationally costly.

One of the difficulties in using Gaussian beam approximations is that the initial wave field needs to be expressed in a particular basis. Accordingly, we present a numerical method for approximating highly oscillatory wave fields as a superposition of Gaussian beams. The method estimates the number of beams and their parameters automatically. This is achieved by an Expectation-Maximization algorithm that fits real, positive Gaussians to the energy of the highly oscillatory wave fields and a weighted Fourier transform of the field.ד The EM data is processed to find beams parameters.

Joint work with Bjorn Engquist, Nick Tasuchev and Richard Tsai (UT).

The Young Measures Limit for Multi-scaled Differential Equations

Zvi Artstein

We examine singularly perturbed ordinary differential systems where the fast contribution may not yield a stationary point; then a Young measure provides an adequate description of the limit behavior; how to track the evolution of this limit, both analytically and numerically, is our main concern.

Comparing large time behavior of regularity criteria
for the $2d$ Euler equation

Claude Bardos

Different type of estimates are at present available for weak and strong solution of the $2d$ Euler Equation. In $2d$ the $W^{1,p}$ norm, of weak solutions of the Euler equations, for $1<p<\infty$, is time independent. Similarly, the L^∞ norm of voriticity of the unique solution is also time independent. The norm in the Besov space $B^{1}_{1,\infty}$, however, is bounded uniformly by $e^{\gamma t^2}$. Eventually for the $C^{1,\alpha}$ norm the only available estimate supposed to be crude, and it exhibits double exponential behavior of the form $e^{Ce^{\gamma t}}$. Finally, as it was proved in an old paper with Said Benachour we show that the strip of spatial analyticity shrinks at most like $e^{-Ce^{\gamma t}}$.

Decay Estimates and Vanishing Viscosity for Viscous Hamilton-Jacobi Equations

Matania Ben-Artzi

Sharp temporal decay estimates are established for the gradient and time derivative of solutions to the Hamilton-Jacobi equation $\partial_t v + H(|\nabla_x v|)=\varepsilon\ \Delta v$, the parameter ε being either positive or zero. Special care is given to the dependence of the estimates on ε. As a by-product, we obtain convergence of the solutions , as $\varepsilon\to 0,$ to a viscosity solution, the initial condition being only continuous and either bounded or non-negative. The main requirement on H is that it grows superlinearly or sublinearly at infinity, including in particular $H(r)=r^p$ for $r\in [0,\infty)$ and $p\in (0,\infty)$, $p\ne 1$. (Joint work with S. Benachour and Ph. Laurencot).

Averaging Evolution Equations via Convolution

Ido Bright

We introduce an averaging framework, where the solution of a time-varying equation with a small amplitude is approximated by the solution of a slowly varying auxiliary system, generated by convolving the original equation with a kernel function. The effect of the convolution is smoothing of the equation, thus, making it more amenable to numerical computations. We present tight results on the approximation error for general classes of vector fields and kernels.

Analysis of Models of Complex Fluids

Peter Constantin

The models of mixtures of fluids and particles described will be of the type of Nonlinear Fokker Planck equations coupled to Navier-Stokes equations. Some of the issues of blow up versus regularity will be presented and illustrated on simple didactic models.

From a Generalized Davey-Stewartson System to the Almost Cubic Nonlinear Schrödinger Equation

Alp Eden

In this talk, I will try to highlight some of the central results in our work on a generalized Davey-Stewartson system. In the purely elliptic case, the Generalized Davey-Stewartson system can also be considered as an almost cubic nonlinear Schrödinger equation that shares many similarities with the two dimensional cubic nonlinear Schrödinger equation. The class of almost cubic nonlinear Schrödinger equations also includes the usual Davey-Stewartson system in the elliptic-elliptic case, as well as some cases of the Zakharov-Schulmann equations. The usual analysis of the cubic nonlinear Schrödinger equation rests on how the local cubic nonlinearity acts between different function spaces allowing the use of the Strichartz inequalities. Although the nonlinearity of the almost cubic nonlinear Schrödinger equation is non-local in nature its similar action on various function spaces allow a similar analysis for the local well-posedness in various Sobolev spaces. The problem of demarcation of the focusing and defocusing cases of the generalized Davey-Stewartson system when viewed this way becomes a much moretransparent problem. It is no surprise that this is also the demarcation for the existence af standing waves, whose existence will also be discussed.

Continuations of NLS Solutions Beyond the Singularity

Gadi Fibich

The continuation of NLS solutions beyond the singularity has been an open problem for many years. In this talk I will present several novel approaches to this problem, and discuss their consequences. Joint work with Moran Klein

Some Mathematical Problems in a Neoclassical Theory of Electric Charges

A. Babin and A. Figotin

We study a number of mathematical problems related to our recently introduced neoclassical theory for electromagnetic phenomena in which charges are represented by complex valued wave functions as in the Schrödinger wave mechanics. In the non-relativistic case the dynamics of elementary charges is governed by a system of nonlinear Schrödinger equations coupled with the electromagnetic fields, and we prove that if the wave functions of charges are well separated and localized their centers converge to trajectories of the classical point charges governed by Newton's equations with the Lorentz forces. We also found exact solutions in the form of localized accelerating solitons. Our studies of a class of time multiharmonic solutions of the same field equations show that they satisfy Planck-Einstein relation and that the energy levels of the nonlinear eigenvalue problem for the hydrogen atom converge to the well-known energy levels of the linear Schrödinger operator when the free charge size is much larger than the Bohr radius.

On the Well-Posedness of the Vacuum Einstein's Equations

Lavi Karp

The vacuum Einstein's equations can be transferred to coupled hyperbolic-elliptic systems. I will discuss these systems on asymptotically flat space-times and the well-posedness of the coupled system in weighted Sobolev spaces.

On Multi-weighted Parabolic Systems in Sobolev Spaces

Oleg Kelis

The most known parabolic equation is the heat equation

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x_1^2}-\ldots-\frac{\partial^2 u}{\partial x_n^2}=0.$$

Differentiation this equation with respect to the time t has a weight *2* whereas differentiation with respect to the spacial coordinates has weight 1. Similarly for general parabolic operators the t-differentiation has a constant weight $2b$ with some natural b, whereas differentiation with respect to the spacial variables has weight 1.

I.G. Petrovskii (1938), T. Shirota (1955) and V. Solonnikov (1965), introduced successively more general definitions of parabolic systems (see e.g. [2], [3], [4]). Although the entries of matrix operator have different orders with respect to the spacial variables, the t-differentiation has a constant weight $2b$ with some natural b. Therefore, all these systems may be considered as single-weighted parabolic systems.

The purpose of the present study is to investigate more general form of parabolic systems when the weight $2b$ is not constant. These systems, are called multi-weighted parabolic systems.

We study the uniqness and existence of the solution for initial-boundary value problems for multi-weighted parabolic systems in appropriate Sobolev spaces.

Taking into account the solvability of elliptic systems of Douglis-Nirenberg type developed in [1], [5], [6], [7], we are going to apply similar methods to parabolic systems.

Keywords: Parabolic systems, Parabolic initial-boundary value problems, Sobolev spaces, Douglis-Nirenberg systems.

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The Euler-α Regularization of Two-dimensional Vortex Patch and Vortex Sheet Dynamics

Jasmine Linshiz

We present an analytical study of the two-dimensional incompressible Euler-alpha, an inviscid second-grade fluid, equations, for vortex patch and vortex sheet motion. We show the convergence of the solutions of the Euler-alpha equations to solutions of the Euler equations, and estimate the convergence rate for vortex patch with smooth boundaries. For the vortex sheet dynamics we present an alpha-regularization of the Birkhoff-Rott equation, induced by the Euler-alpha equations.

Energy Cascades in Quantum Turbulence

V. L´vov, S. Nazarenko and O. Rudenko

I will present our new approach to the theory of quantum turbulence in superfluids, based on the fact that the physics of interacting Kelvin Waves (KWs) is highly non-trivial and cannot be understood on the basis of pure dimensional reasoning only. A consistent theory of KW turbulence should be based on explicit knowledge of the details of their interactions. I will overview first our detailed calculation and comprehensive analysis of the interaction coefficients for KWs, thereby fixing previous mistakes and show that the previously suggested Kozik-Svistunov (KS) energy spectrum of KWs, which has been often used for analysis of experimental and numerical data in superfluid turbulence, is irrelevant, because it is based on an erroneous assumption of the locality of the energy transfer through scales.

Our second step toward a consistent theory of quantum turbulence is a derivation of new kinetic equation for KWs on quantized vortex filaments with random large-scale curvature, that describes step-by-step (local) energy cascade over scales caused by 4-wave interactions. Resulting new L'vov-Nazarenko (LN) energy spectrum of KWs with exponent -5/3 (the same as in Kolmogorov-Obukhov spectrum of hydrodynamic turbulence) must replace in future theory of quantum turbulence the previously used KS-spectrum.

As a next step we reconsidered our theory of superfluid turbulence that describes the bottleneck accumulation of the energy spectrum near the inter-vortex scales, replacing KW-spectrum of KWs on the LN-spectrum and generalized the theory on the case of non-zero temperatures, accounting for the effect of the mutual friction. Resulting temperature dependence of the effective viscosity is in full qualitative agreement with the observations in Rochester spin-down experiments on superfluid 4He. I will also show that theoretically found energy accumulation near the intervortex scales is in a good agreement with large-scale simulation of quantum turbulence (2048-cube) performed within the Gross-Pitaevskii model on the Japan EARTH simulator.

Bacteria--Phagocytes Dynamics, Axiomatic Modelling and Mass-Action Kinetics

Roy Malka and Vered Rom-Kedar

Axiomatic modeling is ensued to provide a family of models that describe bacterial growth in the presence of phagocytes, or, more generally, prey dynamics in a large spatially homogenous eco-system. We show that naive models may miss important features that are associated with the biologically reasonable limited growth curve of the bacteria (prey) and the saturation associated with the phagocytosis (predator kill) term. Notably, these features appear at relatively low concentrations, much below the saturation range. These observations imply that models that invoke mass-action kinetics for the kill term and in which the phagocytes (predator) dynamics is much slower than that of the bacteria (prey) rule out a whole class of biologically relevant models.

Boundary and Initial Value Problems with Measure Data for Semilinear Elliptic and Parabolic Equations

Moshe Marcus

We shall discuss boundary value problems for equations of the form $-\Delta u+g(u)=0$ and initial value problems for the corresponding evolution equation $\partial u/\partial t-\Delta u+g(u)=0$.

Here g is a continuous, super-linear, monotone increasing function such that $g(0)=0$. Main examples: $g(t)=|t|^{q-1}t$ with $q>1$ and $g(t)=\exp^t-1$.

We shall concentrate on the special features of problems with measure data. We shall also discuss the connection between problems with power non-linearity and problems in probability involving branching processes and super-diffusions.

On Wigner and Bohmian Measures

Peter A. Markowich

We define a new class of measures on phase space which occur naturally when the so-called Bohmian approach to quantum mechanics is analysed. We discuss the significance of these measures with respect to semiclassical limits and compare to the well-established concept of Wigner measures.

The Vanishing Viscosity Limit in Channel and Pipe Flows

Anna L. Mazzucato

We study the vanishing viscosity limit for certain Taylor-Couette flows in channels and pipes. We establish convergence of the Navier-Stokes solution to the corresponding Euler solution as viscosity vanishes in various norms. We use both semiclassical analysis of the small-diffusion limit for a heat equation with drift, and Pradtl-type equations. This is joint work with Dongjuan Niu, Michael Taylor, and Xiaoming Wang.

Front Propagation in Anomalous Diffusion-Reaction Systems

Vladimir A. Volpert, Y. Nec and A.A. Nepomnyashchy

Reaction-diffusion systems describe numerous phenomena in nature. It has been recently understood that many diffusion processes are described by models of anomalous diffusion, which explain the observation of anomalously fast (superdiffusion) or slow (subdiffusion) growth of displacement moments of corresponding random walks. These models include a spatial non-locality or/and temporal memory and involve integral operators (e.g., fractional derivatives) in addition to differential operators. The interplay between the anomalous diffusion and the reactions is not yet well understood. In the present talk, we analyze the propagation of reaction fronts in systems with anomalous diffusion.

In the case of reaction-superdiffusion equations, we consider the exactly solvable case where the reaction term is a discontinuous piecewise linear function. Applying the Fourier transform, we find traveling fronts and pulses, and discuss the effect of superdiffusion on the solutions. Specific problems that we consider include FitzHugh-Nagumo equations, domain wall pinning, systems of waves and others. A similar approach is applied to reaction-subdiffusion equations.

Study of the 3D Euler Equations by Clebsch Potentials

Koji Ohkitani

We study how the vortex stretching process in 3D Euler flows is constrained in the class of flows with Clebsch potentials. Classical differential geometry of surfaces are fully exploited for the characterisation. In particular, we identify the dual mechanisms for nonlinearity depletion.

Large Time Behavior of the Heat Kernel

Yehuda Pinchover

In the study of heat conduction and diffusion, the minimal positive heat kernel is the fundamental solution to a second-order parabolic initial value problem on a particular domain with (generalized) Dirichlet boundary condition. It is also one of the main tools in the study of the spectrum of second-order elliptic operators.

In this talk we will discuss large time behaviors of the heat kernel of a general time-independent second-order linear parabolic operator which is defined on a noncompact manifold. In particular, we will present strong ratio limit properties of the quotients of the heat kernels of subcritical and critical operators.

Statistical Physics and Scaling Concepts in Elasto-plasticity of Amorphous Solids

Itamar Procaccia

I will discuss the effect of finite temperature T and finite strain rate $\dot\gamma$ on the statistical physics of plastic deformations in amorphous solids made of N particles. We recognize three regimes of temperature where the statistics are qualitatively different. In the first regime the temperature is very low, $T<T_{\rm cross}(N)$, and the strain is quasi-static. This regime can be fully understood, including predicting where the system responses plastically (based on equilibrium measurements only). In this regime the elasto-plastic steady state exhibits highly correlated plastic events whose statistics are characterized by anomalous exponents. In the second regime $T_{\rm cross}(N)<T<T_{\rm max}(\dot\gamma)$ the system-size dependence of the stress fluctuations becomes normal, but the variance depends on the strain rate. The physical mechanism of the cross-over is different for increasing temperature and increasing strain rate, since the plastic events are still dominated by the mechanical instabilities (seen as an eigenvalue of the Hessian matrix going to zero), and the effect of temperature is only to facilitate the transition. A third regime occurs above the second cross-over temperature $T_{\rm max}(\dot\gamma)$ where stress fluctuations become dominated by thermal noise. Throughout the lecture we demonstrate that scaling concepts are highly relevant for the problem at hand, and finally we present a scaling theory that is able to collapse the data for all the values of temperatures and strain rates, providing us with a high degree of predictability.

Scaling in Turbulent Flows: Heuristics and Rigorous Results

Fabio Ramos

In this talk I will present some classical heuristics and related recent rigorous results for the Kolmogorov-Obukhov spectral scaling for fluid flows at high Reynolds number. In particular, I will describe how to obtain some universal bounds for the Littlewood-Paley first-order moments for weak solutions of the 3D Navier-Stokes equations.

Integral Solutions to a Class of Nonlocal Evolution Equations

Simeon Reich

We study the existence of integral solutions to a class of nonlinear evolution equations of the form

\begin{equation} \left\{\begin{array} u'(t)+A(u(t)) \ni f(t,u(t)), \
t \in(0,T), \\ u(0)=g(u), \end{array} \right. \end{equation}

where $A: D(A) \subseteq X \rightarrow 2^{X}$ is an m-accretive operator on a Banach space $X,$ and $f: [0,T] \times X \rightarrow X$ and $g: C(0,T; X) \rightarrow \overline{D(A)}$ are given functions.

We obtain sufficient conditions for this problem to have a unique integral solution. This is joint work with Jes\'us Garc\'{\i}a Falset.

Parabolic Resonance: A Route to Hamiltonian Spatio-Temporal Chaos

Vered Rom-Kedar

We show that initial data near an unperturbed stable plane wave can evolve into a regime of spatiotemporal chaos in the slightly forced conservative periodic one-dimensional nonlinear Schrödinger equation [1]. We demonstrate that this spatio-temporal chaos is intermittent: there are windows in time for which the solution gains spatial coherence. The parameters and initial profiles that lead to such intermittency are predicted by utilizing a novel geometrical description of the integrable unforced equation [2-3] and by analyzing the parabolic resonance instability [4].

Joint work with Eli Shlizerman [1-3] and D. Turaev [4].

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Brother, Can You Spare a Compacton?

Philip Rosenau

Unlike certain personal, or national, tragedies which may extend indefinitely, the patterns observed in nature are of finite extent. Yet the patterns predicted via mathematical modeling as a rule have infinite tails, a by product of their analytical nature. Rather then viewing it as an inherent inability of math to model physics, we adopt the opposite view: it reflects an inadequate modelling of nature. To generate a compact pattern one has to escape the curse of analyticity. Differently stated, one needs mechanism(s) which beget a local singularity. The resulting local loss of uniqueness enables to connect a smooth part of the solution with ground state and thus to form an entity with a compact support: the compacton. We shall describe a variety of singularity inducing mechanisms which enable to form compact solution of dispersive or dissipative multi-dimensional phenomena.

Extrinsic Geometric Flows on Foliated Manifolds

Vladimir Rovenski

We consider deformations of Riemannian metrics on a manifold equipped with a codimension-one foliation subject to quantities expressed in terms of its second fundamental form. We prove the local existence and uniqueness theorem and present solutions to some particular cases of the non-linear problem. The key step of the solution procedure is to find (from a system of quasi-linear PDE's) the power sums of the principal curvatures of the foliation. Soliton Riemannian metrics are considered. Applications to totally umbilical foliations and extrinsic Ricci flow are given.

Singular limits of symmetric hyperbolic systems with large variable-coefficient terms

Steve Schochet

Some results on uniform well-posedness and limit behavior of symmetric hyperbolic systems with large variable-coefficient terms will be presented. The study of such systems was initiated by the discovery of results for the equatorial shallow water equations over the past few years by a number of researchers.

Exponential Energy Growth in a Fermi Accelerator

Kushal Shah, Dmitry Turaev, Vered Rom-Kedar

An unbounded energy growth of particles bouncing off 2D smoothly oscillating polygons is observed. Notably, such billiards have zero Lyapunov exponents in the static case. For a special 2D polygon geometry - a rectangle with a vertically oscillating horizontal bar - we show that this energy growth is not only unbounded but also exponential in time. This is the first time that exponential energy growth in a billiard accelerator is observed. For the energy averaged over an ensemble of initial conditions, we derive an a-priori expression for the rate of the exponential growth as a function of the geometry and the ensemble type. We demonstrate numerically that the ensemble averaged energy indeed grows exponentially, at a close to the analytically predicted rate - namely, the process is controllable.

Shaping of Growing Thin Sheets

Eran Sharon

Many natural structures are made of soft tissue that undergoes complicated continuous shape transformations due to the distribution of local *active* deformation of their "elements". Currently, the ability to mimic this shaping mode in manmade structures is poor. I will present some results of our study of actively deforming thin sheets. We describe the local active "growth" by a prescribed non-Euclidean "target metric" tensor and suggest guidelines for the selection of equilibrium configurations. We use environmentally responsive gel sheets that adopt prescribed metrics upon induction by environmental conditions. With this system we study the shaping mechanism and energy scaling in different cases of imposed metrics. Our observations include the generation of multi-scale configurations and helical strips in hyperbolic sheets. Finally, we study how shaping by active deformation is manifested in natural organs, such as leaves and pods.

Analysis and Computation of a Discrete KdV-Burgers Type Equation with Fast Dispersion and Slow Diffusion

Marshall Slemrod

The long time behavior of the dynamics of an example of a fast-slow system of ordinary differential equations is examined. The system is derived from a spatial discretization of a Korteweg-de Vries-Burgers type equation, with fast dispersion and slow diffusion. The discretization is based on a model developed by Goodman and Lax, that is composed of a fast system drifted by a slow forcing term. A difficulty to invoke available multi-scale methods arises since the underlying system does not posses a natural split to fast and slow state variables. Our approach depicts the limit behavior as a Young measure with values being invariant measures of the fast contribution to the flow. The slow contribution to the dynamics causes these invariant measures to drift. We keep track of this drift via slowly evolving observables. Averaging equations for the latter lead to computation of characteristic features of the motion and the location the invariant measures. Such computations are presented in the paper.

Existence and Equilibration of Global Weak Solutions to Kinetic Models of Dilute Polymers

Endre Süli

We establish the existence of global-in-time weak solutions to a general class of coupled microscopic-macroscopic FENE-type bead-spring chain models that arise from the kinetic theory of dilute solutions of polymeric liquids with noninteracting polymer chains. The class of models involves the unsteady incompressible Navier-Stokes equations in a bounded domain in two and three space dimensions, for the velocity and the pressure of the fluid, with an elastic extra-stress tensor appearing on the right-hand side of the momentum equation. The extra-stress tensor stems from the random movement of the polymer chains and is defined by the Kramers expression through the associated probability density function that satisfies a Fokker-Planck type parabolic equation, a crucial feature of which is the presence of a center-of-mass diffusion term. We require no structural assumptions on the drag term in the Fokker-Planck equation; in particular, the drag term need not be corotational.

With a square-integrable and divergence-free initial velocity datum for the Navier-Stokes equation and a nonnegative initial probability density function for the Fokker-Planck equation, which has finite relative entropy with respect to the Maxwellian, we prove the existence of global-in-time weak solutions to the coupled Navier-Stokes-Fokker-Planck system, satisfying the initial condition, such that the velocity belongs to the classical Leray space and the probability density function has bounded relative entropy and square integrable Fisher information over any time interval. The key analytical tool in our proof is Dubinskii's compactness theorem in seminormed sets. It is also shown using the Csiszar-Kullback inequality that, in the absence of a body force, the global weak solution decays exponentially in time to the equilibrium solution, at a rate that is independent of the choice of the initial datum and of the centre-of-mass diffusion coefficient.

The talk is based on joint work with John W. Barrett (Department of Mathematics, Imperial College London).

Linear Equations in Critical Regularity Spaces: Hierarchical Construction of Their Nonlinear Solutions

Eitan Tadmor

We construct uniformly bounded solutions of the equations div(U)=f and curl(U)=f, for general f's in the critical regularity spaces L^d(R^d) and, respectively, $L^3(R^3)$. The equations are linear but construction of their solutions is not. Our constructions are special cases of a general framework for solving linear equations, L(U)=f, covered by the closed range theorem. The solutions are realized in terms of nonlinear hierarchical representations, U=sum(u_j), which we introduced earlier in the context of image processing. The u_j's are constructed recursively as proper minimizers, yielding a multi-scale decomposition of the solutions U.