LIE ALGEBRAS: HOMEWORK 3
DUE: 13 APRIL 2010

(1) Let I be an ideal in a Lie algebra L. Prove that the quotient algebra L/I is abelian if and only if $[L, L] \subseteq I$.

(2) Prove that L is solvable if and only if there exists a chain of subalgebras
$$L = L_0 \supset L_1 \supset L_2 \supset \cdots \supset L_k = 0,$$
such that L_{i+1} is an ideal in L_i and such that each quotient L_i/L_{i+1} is abelian.

(3) Prove that a Lie algebra L is semisimple if and only if L has no abelian ideals.

(4) Suppose that L is a Lie algebra with $\dim L = 3$ and $L = [L, L]$. Prove that L is simple. (Hint: First, show that if L is a Lie algebra with $L = [L, L]$, then any homomorphic image of L also equals its derived algebra.)

(5) Classify (up to isomorphism) 3 dimensional Lie algebras with $[L, L] \subseteq Z(L)$. Prove your list is complete. Realize each algebra as a linear Lie algebra (i.e. as as a subalgebra of $\mathfrak{gl}_n$ for an appropriate choice of n).

6 April 2010